

Cost Analysis of a Hybrid-Movement-Based and Time-Based Location Update Scheme in Cellular Networks

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Abstract—A movement-based location update (MBLU) scheme is effective and has low operation complexity. On the other hand, cellular networks use periodic LUs that are intrinsically time-based LUs (TBLUs) to detect the presence of a user equipment (UE) to avoid wasting network resources to deliver incoming calls to an abnormally detached UE. When implemented in a cellular network, the MBLU scheme must incorporate TBLUs to detect the UE presence. Thus, a hybrid-movement-based and time-based LU (HMTBLU) scheme arises naturally. Under the HMTBLU scheme, an LU occurs either when the movement threshold for MBLUs is reached or when the time threshold for TBLUs is reached. In this paper, we develop an embedded Markov chain approach to calculate the signaling cost of the HMTBLU scheme. We derive analytical formulas for the signaling cost and formulas useful for designing an optimal paging scheme. The accuracy of these formulas is validated through simulation. Based on these formulas, we conduct a numerical study to investigate the influence of various parameters on the signaling cost. The derived formulas can guide the implementation of the HMTBLU scheme in cellular networks, including Long-Term Evolution (LTE). Moreover, the developed mathematical approach can be applied to studying other LU schemes such as the LU scheme used in LTE.

Index Terms—Embedded Markov chain, hybrid location update, modeling, movement-based location update (MBLU), periodic location update, time-based location update (TBLU).

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I. INTRODUCTION

A. Background

CELLULAR communications have widespread applications in the transportation area, as evidenced by the fact that a large percentage of mobile data is contributed by passengers on moving vehicles. Moreover, in environments of high-speed movement, cellular networks face a plethora of challenges that are not severe issues in low speed or static environments, e.g., the location management issue to be studied in this paper. A cellular network must keep track of the location of a moving user equipment (UE) for the successful delivery of incoming calls to the UE. The network tracks the UE location through a mechanism called location management. Location management comprises two essential operations, namely, location update (LU) and paging. LU is the operation initiated by a UE to update toward a network its registration of location. Paging is the operation initiated by a network to determine the accurate location of UE when an incoming call to the UE arrives. The network pages the UE by broadcasting a paging message in cells close to the cell where the last LU before the call arrival occurs. Based on whether an LU scheme can adapt itself to different mobility and traffic characteristics, an LU scheme can be classified as a static scheme or a dynamic scheme. Under a static LU scheme, the coverage of a network is partitioned into registration areas (RAs), each of which consists of a number of cells. Moreover, a UE performs an LU when moving into a new RA. The LU schemes IS-41 [1] and GSM MAP [2] used in existing second-generation (2G) and third-generation (3G) cellular networks are in nature two static LU schemes. A static LU scheme is cost ineffective because it fails to consider the diverse mobility and traffic characteristics of different UEs. Imagine an example where two UEs cross RAs at the same rate. Suppose that one UE has frequent incoming calls, whereas the other has no incoming calls. Under a static LU scheme, the two UEs perform LUs at the same rate. However, it is intuitive that the UE with frequent incoming calls should perform LUs more frequently, whereas the other UE does not need to perform LU at all. Therefore, a cost-effective LU scheme should be dynamic enough to adapt itself to different mobility and traffic characteristics.

To cope with the above drawback of static LU schemes, three dynamic LU schemes, namely, a distance-based LU (DBLU) scheme, a movement-based LU (MBLU) scheme, and a

time-based LU (TBLU) scheme, were proposed in [3]. Under the three schemes, an LU is performed when the traveled distance in terms of cells, the number of crossed cells, and the elapsed time since the last LU reach, respectively, a distance threshold, a movement threshold, and a time threshold. Given a traffic model (i.e., a call arrival process) and a mobility model (i.e., a cell residence time distribution and a model of UE movement among cells), the signaling cost of a dynamic LU scheme due to the operations of LU and paging is a downward convex function of the corresponding threshold [4]–[7]. That is, there is an optimal threshold that can minimize the signaling cost. Therefore, these LU schemes are dynamic because they can dynamically adjust their thresholds to best match specific traffic and mobility patterns to minimize the signaling cost. In contrast, under a static LU scheme, there is no adjustable parameter. Among these dynamic LU schemes, the DBLU scheme has the least signaling cost but requires a network to provide a UE the information about the distance between two arbitrary cells. This requirement cannot be met in existing cellular networks. The TBLU scheme is the simplest but may produce unnecessary LUs. For example, a stationary UE does not need to perform LU before it moves. The MBLU scheme is the most practical because it is effective and has low operation complexity. The operation complexity is a movement counter at the UE to tally the number of crossed cells. A plethora of studies on the MBLU scheme have been carried out in the literature. These studies are focused on the following aspects: 1) adopting different probability distributions for relevant time variables [8], [9]; 2) developing new modeling approaches [10]–[13]; 3) calculating an optimal movement threshold [13], [14]; 4) calculating a performance metric different from the signaling cost [15]; 5) designing an optimal paging scheme [16], [17]; 6) exploiting the history of UE movement to enhance the MBLU scheme [18], [19]; 7) applying the MBLU scheme to group location management [20]; and 8) using more realistic architectures for location management [5], [10], [11], [21]–[24].

In a cellular network, it may occur that a UE abnormally detaches (i.e., disconnects) from the network due to some unexpected reasons, such as battery removal and sudden loss of network coverage, so that the UE has no enough time to notify the network of its detachment. In this case, the network is unaware of the UE's detachment. When an incoming call to the UE arrives after the detachment, as usual the network follows the routine procedure to set up communication links and page the UE to deliver the call but only ends up in vain. To avoid wasting network resource to deliver calls to abnormally detached UEs, existing cellular networks use periodic LUs to detect the presence of a UE. A UE runs a periodic LU timer to control periodic LUs. After the expiry of this timer, the UE performs a periodic LU. Corresponding to the periodic LU timer, the network runs a mobile reachable time (sometimes also called an implicit detach timer) that is slightly larger than the periodic LU timer. If the network does not receive a periodic LU from the UE before the expiry of the mobile reachable timer, then the network believes that the UE has detached from the network and will not page for the subsequent incoming calls to the UE. Any events from which the network can infer that the UE is still attached to the network cause the reset of the mobile

reachable timer and the periodic LU timer. Such events include UE attachment, LU, call origination, call termination, etc. (See [25]–[30] for optimally setting the periodic LU timer.) Periodic LUs are intrinsically the previously introduced TBLUs. In the TBLU scheme, a UE needs to set up a counter to count the elapsed time since the last LU. When this counter reaches the time threshold, the UE performs an LU. The function of this counter is the same as that of the periodic LU timer. Therefore, in the following we call this counter the periodic LU timer.

B. Motivation

When the MBLU scheme is implemented in a cellular network, it must incorporate TBLUs to detect the UE presence. Thus, a hybrid-movement-based and time-based LU (HMTBLU) scheme arises. Under the HMTBLU scheme, a UE performs an LU either when reaching the movement threshold for MBLUs or when reaching the time threshold for TBLUs. In the HMTBLU scheme, there are two LU types, i.e., an MBLU due to the reaching of the movement threshold and a TBLU due to the reaching of the time threshold. The occurrence of either an MBLU or a TBLU resets both the movement counter and the periodic LU timer.

Most of relevant studies in the literature (see [4], [5], [7]–[24], and [31]–[33]) treated the MBLU scheme and the TBLU scheme separately so that the results obtained in these studies are unavailable to the HMTBLU scheme. In [34], Lee and Cho proposed an approximate approach for calculating an optimal time threshold when the TBLU scheme is incorporated into the previously introduced static LU schemes. In [35] and [36], a hybrid LU scheme is proposed that combines the TBLU scheme and the MBLU scheme. For convenience, we refer to this hybrid LU scheme as a sequentially HMTBLU (sHMTBLU) scheme. The sHMTBLU scheme and the HMTBLU scheme differ in the following aspects. First, the two differ in their criteria for performing an LU. Under the sHMTBLU scheme, a UE performs an LU when it reaches first the time threshold and then the movement threshold or, alternatively, the movement threshold first and then the time threshold. In other words, the sHMTBLU and HMTBLU schemes are the sequential combination and the parallel combination of the TBLU and MBLU schemes, respectively. Second, the two differ in the number of LU types. The sHMTBLU scheme has only one LU type, whereas the HMTBLU scheme has two LU types. Third, the two differ in their objectives to combine the TBLU and MBLU schemes. The sHMTBLU scheme does so for a better performance (i.e., to reduce the signaling cost due to the operations of LU and paging), whereas the HMTBLU scheme does so for the capability to detect the UE presence. It is observed in [35] that when the coefficient of variation (CoV) of the cell residence time is large, the sHMTBLU scheme outperforms the MBLU scheme and the TBLU scheme, whereas when the CoV is small, the MBLU scheme performs best. Fourth, the two differ in whether they have the capability to detect the UE presence. The sHMTBLU has no such a capability. Finally, the two differ in the number of applicable paging scheme types. In the TBLU scheme, when an incoming call arrives, the number of cells crossed since the

last LU before the call arrival is not bounded. Denote by M the movement threshold. In the MBLU scheme, this number is bounded by $M - 1$. Define a paging area as an area within which a UE can be located. The paging area of the sHMTBLU scheme is infinitely large such that the sHMTBLU scheme can use only a sequential paging scheme. The paging area of the HMTBLU scheme is made up of the cells that are less than M cells away from the cell where the last LU occurs. Thus, the HMTBLU scheme can use both a sequential paging scheme and the parallel paging scheme.

In summary, although the MBLU scheme has drawn extensive attention, none of the existing studies considered the issue of detecting the UE presence to avoid wasting network resource to deliver incoming calls to abnormally detached UEs. The HMTBLU scheme emerges naturally when TBLUs are incorporated into the MBLU scheme to endow the MBLU scheme with the capability of detecting the UE presence. The sHMTBLU scheme is similar to the HMTBLU scheme because they are both the combinations of the MBLU scheme and the TBLU scheme. However, they differ much from each other so that the results for the sHMTBLU scheme cannot be applied to the HMTBLU scheme. One remarkable difference is that the sHMTBLU scheme has no capability of detecting the UE presence.

C. Our Contributions

Cost analysis is necessary for minimizing the signaling cost of the HMTBLU scheme. In this paper, we develop an approach of embedded Markov chain to characterize the dependence between the interwound MBLUs and TBLUs in the HMTBLU scheme. We derive analytical formulas for the signaling cost of the HMTBLU scheme and formulas that can be exploited to design an optimal sequential paging scheme. We validate the accuracy of these formulas through simulation. Based on these formulas, we conduct a numerical study to investigate the influence of various parameters on the signaling cost. The contributions of this paper are as follows. First, without exception, all the existing studies on the MBLU scheme failed to emphasize the problem of UE abnormal detachment, which is a popular phenomenon in cellular communications, so that the results obtained in these studies are not ready for application in cellular networks. This paper for the first time identifies this problem when studying the MBLU scheme. This paper proposes to combine the TBLU scheme and the MBLU scheme so that the resulting HMTBLU scheme has the capability to detect the UE presence. First, the derived formulas can guide the implementation of the HMTBLU scheme in cellular networks. Third, we demonstrate that, although a long time has elapsed since the MBLU scheme was proposed in 1995 by Bar-Noy *et al.* [3], it still has vitality in fourth-generation (4G) cellular network Long-Term Evolution (LTE). When applied in LTE, the MBLU scheme can simplify the allocation of RAs for a UE while not severely impairing the performance of the LU scheme used in LTE. Fourth, the developed mathematical approach has been seldom applied in existing studies on LU schemes and can be extended to model other LU schemes, including the LU scheme used in LTE.

The remaining paper is organized as follows. Section II calculates the signaling cost of the HMTBLU scheme using an approach of embedded Markov chain. Section III validates the accuracy of the analytical formulas derived in this paper through simulation and conducts a numerical study to investigate the influence of various parameters on the signaling cost. Section IV discusses the potential value of the HMTBLU scheme in LTE and presents some existing applications of the embedded Markov chain approach in studying LU schemes. Finally, Section V concludes this paper.

II. MODELING AND COST ANALYSIS OF THE HYBRID-MOVEMENT-BASED AND TIME-BASED LOCATION UPDATE SCHEME

A. Preliminaries

As aforementioned in Section I-A, any events from which a network can infer the presence of a UE trigger the reset of the periodic LU timer running at the UE side and the mobile reachable timer running at the network side. LU and call arrival are two such events. To perform an MBLU or a TBLU, the UE and the network need to exchange dedicated messages, such as a random access request message and its corresponding acknowledgment message, an LU request message and its corresponding acceptance message, etc. We refer to an LU that necessitates the exchange of dedicated messages as an explicit LU. In the case of a call arrival event, after completing a call, the network knows the current location of the UE at the accuracy of a cell level so that, without explicitly exchanging any dedicated message between the UE and the network, an LU has been asynchronously completed at the UE and the network. We refer to this LU as an implicit LU. Apparently, an implicit LU incurs no signaling cost. Therefore, in this paper, we assume that after the completion of a call, an implicit LU occurs but has no signaling cost. Since an implicit LU occurs after a call and triggers the reset of the movement counter and the periodic LU timer, the duration of a call exerts no impact on the signaling cost of the HMTBLU scheme due to the operations of MBLU, TBLU, and paging. Therefore, in this paper, we assume that the call duration is zero.

Denote, respectively, by M and T the movement threshold for MBLUs and the time threshold for TBLUs. Fig. 1 shows a flowchart depicting the LU operations in the HMTBLU scheme. In Fig. 1, MC and PLT are two variables set up to store the number of crossed cells and the elapsed time since the last LU, respectively. The operation $PLT++$ means that a very small slot of time has elapsed and it is added to PLT. The operation $MC++$ is the abbreviation of $MC = MC + 1$. The HMTBLU scheme functions as per the following principles. First, after the UE enters a new cell, increase MC by one; afterward, if MC equals the movement threshold M , then perform an MBLU and reset MC and PLT. Second, after the completion of a call, reset MC and PLT. Finally, if PLT equals the time threshold T , then perform a TBLU and reset MC and PLT.

Denote by t_c the call interarrival time that is the time between two consecutive call arrivals. Assume that the call arrival process to a UE is a Poisson process with rate λ . That is, t_c follows an exponential distribution with rate λ . Denote by

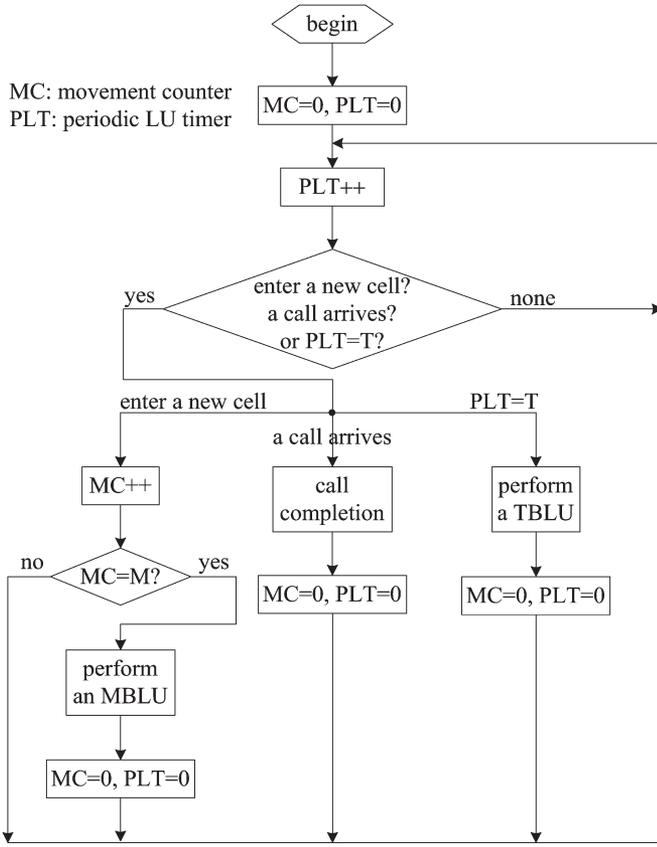


Fig. 1. Flowchart depicting the LU operations in the HMTBLU scheme.

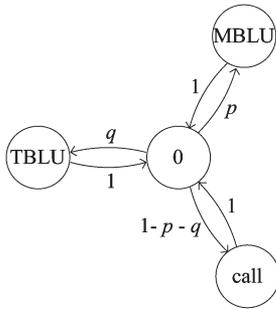


Fig. 2. Embedded Markov chain for the HMTBLU scheme.

$t_j, j = 1, 2, \dots$, the residence time in cell j . Assume that $t_j, j = 1, 2, \dots$, are independent and identically distributed (i.i.d.). That is, all the cells in a cellular network are homogeneous. Further assume that $t_j, j = 1, 2, \dots$ follows an exponential distribution with rate η . Note that the approach of embedded Markov chain developed in this paper can be easily extended to support t_j that follow a hyper-Erlang distribution. The exponential assumption for t_c may be inaccurate. However, it is necessary for the tractability of mathematical analyses to obtain some rough insights into the HMTBLU scheme. These insights are pivotal and sufficient at the early stage of planning a network.

B. Embedded Markov Chain Model

Fig. 2 shows an embedded Markov chain for the HMTBLU scheme. In Fig. 2, state MBLU/TBLU represents the state

at which an MBLU/TBLU occurs due to its reaching the movement/time threshold. State call represents the state that an incoming call arrives. State 0 represents the idle state or, equivalently, the state returned to after the UE completes an MBLU, a TBLU, or a call. For simplicity, assume that the time needed in a UE and the network to process an LU is negligible so that states MBLU and TBLU are both transient states with respect to their state residence time. Given that we have assumed that the call duration is zero, state call is also a transient state. Note that, in an embedded Markov chain, the equilibrium probability of a state is directly proportional to the number of times rather than the duration the state has been visited. Therefore, states MBLU, TBLU, and call should have nonzero equilibrium probabilities [cf. (2)]. Denote by p the transition probability from state 0 to state MBLU. This transition occurs when the movement counter reaches the movement threshold M before the next call arrives and the periodic LU timer reaches the time threshold T . It follows that

$$\begin{aligned}
 p &\triangleq \Pr(0 \rightarrow \text{MBLU}) \\
 &= \Pr(t_1 + \dots + t_M < t_c, t_1 + \dots + t_M < T) \\
 &= \int_0^T \frac{\eta^M x^{M-1}}{(M-1)!} e^{-\eta x} e^{-\lambda x} dx \\
 &= \alpha^M \int_0^T \frac{(\eta + \lambda)^M x^{M-1}}{(M-1)!} e^{-(\eta + \lambda)x} dx \\
 &= \alpha^M \left\{ 1 - e^{-(\eta + \lambda)T} \sum_{j=0}^{M-1} \frac{[(\eta + \lambda)T]^j}{j!} \right\}
 \end{aligned}$$

where

$$\alpha \triangleq \Pr(t_c > t_j) = \frac{\eta}{\eta + \lambda}. \tag{1}$$

Denote by q the transition probability from state 0 to state TBLU. This transition occurs when the periodic LU timer reaches T before the next call arrives and the movement counter reaches M . It follows that

$$\begin{aligned}
 q &\triangleq \Pr(0 \rightarrow \text{TBLU}) \\
 &= \Pr(T < t_c, T < t_1 + \dots + t_M) \\
 &= e^{-(\eta + \lambda)T} \sum_{j=0}^{M-1} \frac{(\eta T)^j}{j!}.
 \end{aligned}$$

Similarly, the transition probability from state 0 to state call can be expressed as

$$\begin{aligned}
 \Pr(0 \rightarrow \text{call}) &= \Pr(t_c < t_1 + \dots + t_M, t_c < T) \\
 &= (1 - \alpha) \sum_{j=0}^{M-1} \alpha^j \left\{ 1 - \frac{\sum_{k=0}^j \frac{[(\eta + \lambda)T]^k}{k!}}{e^{(\eta + \lambda)T}} \right\} \\
 &= 1 - p - q.
 \end{aligned}$$

After completing an MBLU or a TBLU, the UE returns to state 0. An implicit LU occurs after a call so that the UE returns to state 0 after a call. It follows that

$$\Pr(\text{MBLU} \rightarrow 0) = \Pr(\text{TBLU} \rightarrow 0) = \Pr(\text{call} \rightarrow 0) = 1.$$

Denote by Θ the state space of the Markov chain. That is, $\Theta = \{0, \text{MBLU}, \text{TBLU}, \text{call}\}$. Denote by Π_θ , $\theta \in \Theta$, the equilibrium probability of state θ . It follows from Fig. 2 that the equilibrium probabilities of the Markov chain can be expressed in terms of Π_0 as

$$\begin{cases} \Pi_0 = \Pi_0 \\ \Pi_{\text{MBLU}} = p\Pi_0 \\ \Pi_{\text{TBLU}} = q\Pi_0 \\ \Pi_{\text{call}} = (1 - p - q)\Pi_0. \end{cases} \quad (2)$$

Denote by r_θ , $\theta \in \Theta$, the residence time in state θ . It follows that

$$\begin{cases} r_{\text{MBLU}} = r_{\text{TBLU}} = r_{\text{call}} = 0 \\ r_0 = \min(t_1 + \dots + t_M, t_c, T). \end{cases}$$

It follows from Appendix A that

$$\bar{r}_0 \triangleq E(r_0) = (1 - p - q) \frac{1}{\lambda}. \quad (3)$$

Denote by r the time between two consecutive state transitions. It follows that

$$\begin{aligned} r &= \sum_{\theta \in \Theta} \Pi_\theta r_\theta = \Pi_0 r_0 \\ \bar{r} &\triangleq E(r) = \Pi_0 \bar{r}_0. \end{aligned}$$

C. Paging Scheme

Define the last LU cell as the cell where the last LU before the arrival of the next call occurs. Under the HMTBLU scheme, the paging area comprises the cells that are less than M cells away from the last LU cell. When all the cells in the network are regular hexagons of the same size, there are

$$N_{\text{PA}} = 3M^2 - 3M + 1, \quad M = 1, 2, \dots \quad (4)$$

cells in the paging area. There are two paging scheme types, namely, the parallel paging scheme and the sequential paging scheme. The parallel paging scheme pages all the cells in the paging area simultaneously, whereas a sequential paging scheme partitions the paging area into disjoint subareas and pages these subareas one by one until the UE is located. Compared with the parallel paging scheme, a sequential paging scheme has less paging cost but longer paging delay. The paging delay can be restricted by limiting the number of subareas.

Denote by $P(j)$, $j = 0, 1, \dots, M - 1$, the probability that, upon a call arrival, the UE has made j movements since the last

LU before the call arrival. It follows that

$$\begin{aligned} P(j) &\triangleq \Pr(t_1 + \dots + t_j < t_c < t_1 + \dots + t_{j+1} \mid 0 \rightarrow \text{call}) \\ &= \frac{\Pr(t_c > t_1 + \dots + t_j, t_c < t_1 + \dots + t_{j+1}, t_c < T)}{\Pr(0 \rightarrow \text{call})} \\ &= \frac{\int_{x=0}^T \lambda e^{-\lambda x} \int_{y=0}^x \frac{\eta^j y^{j-1}}{(j-1)!} e^{-\eta y} \int_{u=x-y}^{+\infty} \eta e^{-\eta u} du dy dx}{\Pr(0 \rightarrow \text{call})} \\ &= \frac{\alpha^j \left\{ 1 - e^{-(\eta+\lambda)T} \sum_{k=0}^j \frac{[(\eta+\lambda)T]^k}{k!} \right\}}{\sum_{i=0}^{M-1} \alpha^i \left\{ 1 - e^{-(\eta+\lambda)T} \sum_{k=0}^i \frac{[(\eta+\lambda)T]^k}{k!} \right\}}, \\ & \quad j = 0, 1, \dots, M - 1. \end{aligned} \quad (5)$$

The validity of (5) is checked in Appendix B. The $P(j)$ distribution can be further applied to designing an optimal sequential paging scheme that can restrict the paging delay and at the same time minimize the paging cost. Denote by $P_d(j)$, $j = 0, 1, \dots, M - 1$, the probability that, upon a call arrival, the UE is in a cell that is j cells away from the last LU cell. The $P_d(j)$ distribution is a prerequisite for designing an optimal sequential paging scheme. The $P_d(j)$ distribution depends on the $P(j)$ distribution and a mobility model describing the UE movements among cells. In [16], Akyildiz *et al.* gave a combinatorial approach for calculating the $P_d(j)$ distribution given the $P(j)$ distribution and the mobility model. In [17], Zhu and Leung proposed the necessary condition for a sequential paging scheme to be optimal and a genetic algorithm for searching for an optimal sequential paging scheme, given the $P_d(j)$ distribution. Therefore, it is straightforward to apply the $P(j)$ distribution derived in this paper to designing an optimal sequential paging scheme. For purposes of demonstration, in this paper, we consider only the parallel paging scheme.

D. Signaling Cost

Denote by $N_{\text{MBLU,ST}}$ the expected number of MBLUs triggered by a state transition in the embedded Markov chain. Similarly, we define $N_{\text{TBLU,ST}}$. The transitions from state 0 to states MBLU and TBLU trigger, respectively, an MBLU and a TBLU. It follows that

$$\begin{cases} N_{\text{MBLU,ST}} = \Pi_0 \times p \times 1 = \Pi_{\text{MBLU}} \\ N_{\text{TBLU,ST}} = \Pi_0 \times q \times 1 = \Pi_{\text{TBLU}}. \end{cases}$$

Denote by $N_{\text{MBLU,UT}}$ the expected number of MBLUs occurring per unit time. Similarly, we define $N_{\text{TBLU,UT}}$. Since both the $N_{\text{MBLU,ST}}$ MBLUs and the $N_{\text{TBLU,ST}}$ TBLUs occur during time r , it follows that

$$N_{\text{sub,UT}} = N_{\text{sub,ST}} / \bar{r}$$

where $\text{sub} \in \{\text{MBLU}, \text{TBLU}\}$. Denote by N_{MBLU} the expected number of MBLUs occurring during the call interarrival time. Similarly, we define N_{TBLU} . It follows that

$$N_{\text{sub}} = N_{\text{sub,UT}} \frac{1}{\lambda}, \quad \text{sub} \in \{\text{MBLU}, \text{TBLU}\}.$$

Specifically, we have

$$\begin{cases} N_{\text{MBLU}} = \frac{p}{1-p-q} \\ N_{\text{TBLU}} = \frac{q}{1-p-q}. \end{cases} \quad (6)$$

The validity of (6) is checked in Appendix C. Denote by N_{paging} the number of cells paged to locate the UE when an incoming call arrives. Since we use the parallel paging scheme, it follows that

$$N_{\text{paging}} = N_{\text{PA}} = 3M^2 - 3M + 1, \quad M = 1, 2, \dots$$

Denote by δ_{MBLU} the signaling cost of performing an MBLU. Similarly, we define δ_{TBLU} . Denote by δ_{paging} the signaling cost of performing a paging in a cell. Denote by C the expected overall signaling cost of the HMTBLU scheme caused by the operations of MBLU, TBLU, and paging during the call interarrival time. It follows that

$$\begin{aligned} C &= N_{\text{MBLU}}\delta_{\text{MBLU}} + N_{\text{TBLU}}\delta_{\text{TBLU}} + N_{\text{paging}}\delta_{\text{paging}} \\ &= \frac{p}{1-p-q}\delta_{\text{MBLU}} + \frac{q}{1-p-q}\delta_{\text{TBLU}} \\ &\quad + (3M^2 - 3M + 1)\delta_{\text{paging}}, \quad M = 1, 2, \dots \end{aligned}$$

III. SIMULATION AND NUMERICAL STUDY

Here, we first conduct simulations to validate the accuracy of the derived analytical formulas. Then, we carry out a numerical study to investigate the influence of various parameters on the signaling cost of the HMTBLU scheme to obtain more insights into the interwound MBLUs and TBLUs. The relevant parameters are set as follows. First, the time threshold T for TBLUs is directly proportional to the expected call interarrival time $1/\lambda$. That is, T can be expressed as $T = c/\lambda$, where c is a factor. Parameter c depends on many factors, such as the characteristics of incoming and outgoing traffic, RA crossing rate, network coverage, a user's habit of switching off and powering on his/her mobile terminal, etc. (For more details on determining c , see [25]–[30].) Hereafter, the value of c is deliberately selected for a clearer demonstration of the influence of T . Second, $\delta_{\text{MBLU}} = \delta_{\text{TBLU}} = \delta$, and $\delta_{\text{paging}} = 1$.

The simulations are implemented in MATLAB. In the simulations, the cell residence time and call interarrival time are both exponentially distributed to abide by the assumptions made in Section II-A for the analytical analyses. In each simulation, we simulate a certain number of calls, e.g., N_{call} calls, to obtain a performance metric. Table I compares the analytical and simulation results of N_{MBLU} and N_{TBLU} . Table II compares those of the $P(j)$ distribution given in (5). In Tables I and II, $1/\eta = 0.01/\lambda$, $T = 0.05/\lambda$, and $N_{\text{call}} = 10^5$. In Tables I and II, the entry *rel. diff.* represents the relative difference of the analytical result to the simulation result. Tables I and II manifest that the analytical and simulation results match closely.

Fig. 3 shows the total number of MBLUs and TBLUs, $N_{\text{MBLU}} + N_{\text{TBLU}}$, as a function of the time threshold T . In Fig. 3, $\lambda/\eta = 0.01$ or 100, and $M = 5$. In the literature, λ/η is called the call-to-mobility ratio. Given that η/λ is the expected number of cells crossed during the call interarrival time t_c (cf. (9) in Appendix C), the larger the λ/η , the smaller the

TABLE I
COMPARISON BETWEEN THE ANALYTICAL AND SIMULATION RESULTS OF N_{MBLU} AND N_{TBLU} . $1/\eta = 0.01/\lambda$, $T = 0.05/\lambda$, AND $N_{\text{call}} = 10^5$

M		2	4	6	8
N_{MBLU}	analytical	48.9234	20.4136	8.4038	2.6898
	simulation	48.7846	20.3828	8.4212	2.6974
	rel. diff. (%)	0.2847	0.1512	0.2070	0.2792
N_{TBLU}	analytical	1.9966	7.2180	13.3062	17.3184
	simulation	1.9910	7.2089	13.3426	17.3327
	rel. diff. (%)	0.2827	0.1274	0.2724	0.0828

TABLE II
COMPARISON BETWEEN THE ANALYTICAL AND SIMULATION RESULTS OF THE $P(j)$ GIVEN IN (5) WHEN $M = 6$. ALL THE OTHER PARAMETERS ARE SET AS IN TABLE I

j	0	1	2	3	4	5
analytical	0.2234	0.2140	0.1939	0.1619	0.1228	0.0840
simulation	0.2244	0.2132	0.1960	0.1624	0.1192	0.0848
rel. diff. (%)	0.4543	0.3671	1.0718	0.2733	2.9715	0.8990

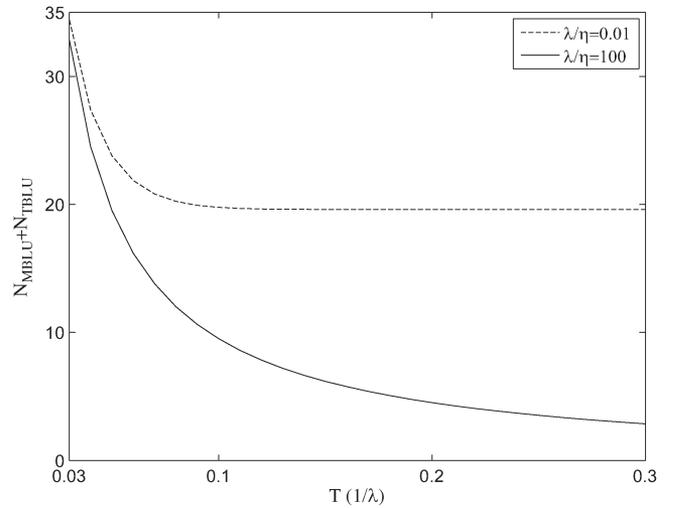


Fig. 3. Total number of MBLUs and TBLUs versus the time threshold $M = 5$.

mobility or, equivalently, the fewer the cells crossed during t_c . In Fig. 3, we observe the following. First, $N_{\text{MBLU}} + N_{\text{TBLU}}$ is a decreasing function of T . The current observation can be explained in two cases, i.e., $M = 1$ and $M \geq 2$. In the case of $M = 1$, it follows that

$$N_{\text{MBLU}} = \frac{E(t_c)}{E(t_j)} = \frac{\eta}{\lambda}$$

irrespective of T . On the other hand, N_{TBLU} is a decreasing function of T . Therefore, in the case of $M = 1$, $N_{\text{MBLU}} + N_{\text{TBLU}}$ is a decreasing function of T . In the case of $M \geq 2$, we consider two subcases, $T \rightarrow 0$ and $T \rightarrow \infty$. When $M \geq 2$ and $T \rightarrow 0$, $N_{\text{TBLU}} \rightarrow \infty$, and $N_{\text{MBLU}} \rightarrow 0$. When $M \geq 2$ and $T \rightarrow \infty$, $N_{\text{TBLU}} \rightarrow 0$ so that the HMTBLU scheme reduces to the MBLU scheme, and hence [cf. (10) in Appendix C]

$$N_{\text{MBLU}} = \frac{\alpha^M}{1 - \alpha^M}.$$

During the process of T changing from 0 to ∞ , N_{TBLU} changes from ∞ to 0, whereas N_{MBLU} changes from 0 to $\alpha^M/(1 - \alpha^M)$. Therefore, during this process, the decreasing rate in N_{TBLU} is larger than the increasing rate in N_{MBLU} . This

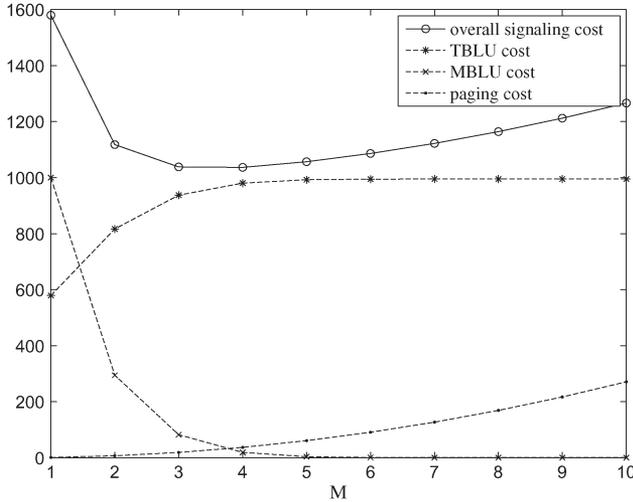


Fig. 4. Overall signaling cost, MBLU cost, TBLU cost, and paging cost versus the movement threshold. $\lambda/\eta = 0.01$, $T = 0.01/\lambda$, and $\delta = 10$.

statement becomes clearer when expressed in mathematics. Denote by $N_{TBLU}(t)$ the N_{TBLU} when $T = t$, $t > 0$. Similarly, we define $N_{MBLU}(t)$. Suppose that two numbers T_1 and T_2 satisfy $T_2 > T_1 > 0$. It follows that, when $M \geq 2$

$$\frac{N_{TBLU}(T_1) - N_{TBLU}(T_2)}{T_2 - T_1} > \frac{N_{MBLU}(T_2) - N_{MBLU}(T_1)}{T_2 - T_1}$$

so that

$$N_{MBLU}(T_1) + N_{TBLU}(T_1) > N_{MBLU}(T_2) + N_{TBLU}(T_2)$$

which completes the proof for the case of $M \geq 2$. Second, when $T \rightarrow \infty$, $N_{MBLU} + N_{TBLU} \rightarrow \alpha^M/(1 - \alpha^M)$. Finally, when T is fixed, the higher the mobility, the larger the $N_{MBLU} + N_{TBLU}$. The explanations for the last two observations are straightforward and thus are skipped.

Fig. 4 shows the overall signaling cost, MBLU cost (i.e., $N_{MBLU}\delta$), TBLU cost (i.e., $N_{TBLU}\delta$), and paging cost (i.e., $N_{\text{paging}}\delta_{\text{paging}}$) of the HMTBLU scheme as a function of the movement threshold M . In Fig. 4, $\lambda/\eta = 0.01$, $T = 0.01/\lambda$, and $\delta = 10$. In Fig. 4, we find that the overall signaling cost is a downward convex function of M . The explanation is as follows. As M increases, the MBLU cost decreases, and the TBLU cost increases, where the decreasing rate in the former is larger than the increasing rate in the latter. As a combined effect, $(N_{MBLU} + N_{TBLU})\delta$ is a decreasing function of M . When $M \geq 5$ or so, the MBLU cost approaches zero, and there is no discernible change with the TBLU cost. When $M = 5$, since $T = 1/\eta = 0.01/\lambda$, intuitively, there are five T 's between two consecutive MBLUs so that, before an MBLU occurs, most likely a TBLU has occurred. In other words, when $M \geq 5$, most likely, no MBLU occurs so that the HMTBLU scheme

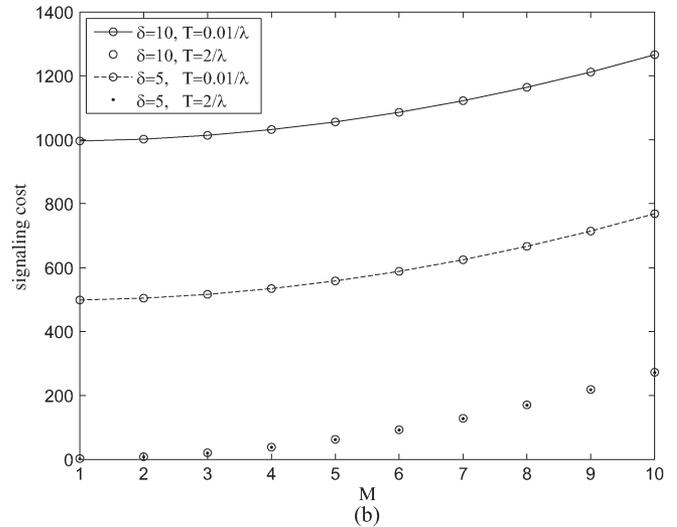
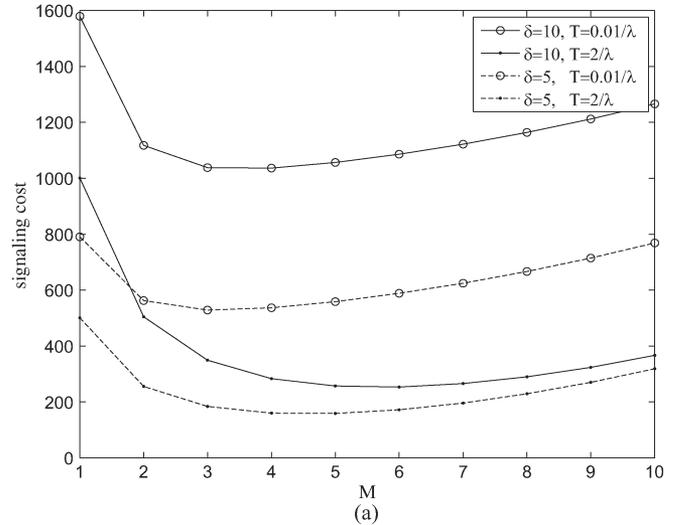


Fig. 5. Signaling cost of the HMTBLU scheme versus the movement threshold. (a) $\lambda/\eta = 0.01$. (b) $\lambda/\eta = 100$.

reduces to the TBLU scheme and accordingly (cf. Proposition 2 in Appendix C)

$$N_{MBLU}\delta \approx 0$$

$$N_{TBLU}\delta \approx \frac{e^{-\lambda T}}{1 - e^{-\lambda T}}\delta = 995$$

which are corroborated in Fig. 4. In summary, the decreasing rate of $(N_{MBLU} + N_{TBLU})\delta$ with respect to M becomes increasingly smaller such that $(N_{MBLU} + N_{TBLU})\delta$ converges to a constant. On the other hand, it follows from (4) that the paging cost is a quadratically increasing function of M . There is such a value of M , which is denoted by M_{opt} , that, before M_{opt} the decrease in $(N_{MBLU} + N_{TBLU})\delta$ is larger than the increase in $N_{\text{paging}}\delta_{\text{paging}}$, and after M_{opt} , the situation is reversed. As an overall effect, when $M \leq M_{\text{opt}}$, the overall signaling cost is a decreasing function of M , and when $M \geq M_{\text{opt}}$, it is an increasing function of M . Therefore, the overall signaling cost is a downward convex function of M , and M_{opt} is the optimal value of M that can minimize the overall signaling cost.

As a summary of Figs. 3 and 4, Fig. 5 shows the signaling cost of the HMTBLU scheme as a function of the movement threshold M when λ/η , T , and δ vary. In Fig. 5, we observe the following. First, in both the cases of high mobility (i.e., $\lambda/\eta = 0.01$) and low mobility (i.e., $\lambda/\eta = 100$), the signaling cost of the HMTBLU scheme is a downward convex function of M , suggesting the existence of M_{opt} that can minimize the signaling cost. The explanation for the case of high mobility can be found in Fig. 4. In the following, we explain for the case of low mobility. When $\lambda/\eta = 100$, the expected number of cells crossed during the call interarrival time t_c is 0.01 so that the HMTBLU scheme reduces to the TBLU scheme and accordingly (cf. Proposition 2 in Appendix C)

$$\begin{cases} N_{\text{MBLU}} \approx 0 \\ N_{\text{TBLU}} \approx \frac{e^{-\lambda T}}{1 - e^{-\lambda T}} \end{cases} \quad (7)$$

irrespective of M . Given that the number of paged cells N_{PA} [cf. (4)] is a quadratic function of M , when $\lambda/\eta = 100$, the signaling cost is an increasing function of M , which is also a downward convex function. Second, the M_{opt} of high mobility is larger than that of low mobility. The explanation is straightforward. When the mobility is high, the UE will cross more cells during t_c so that, in this case, the cost of MBLUs and TBLUs dominates the overall signaling cost. In this case, a larger M leads to the increase in the TBLU cost and the decrease in the MBLU cost. It follows from Fig. 4 that the increase is smaller than the decrease. Therefore, the overall effect is that a larger M helps reduce the cost of MBLUs and TBLUs and accordingly reduce the overall signaling cost. When the mobility is low, the call arrivals are frequent such that the paging cost dominates the overall signaling cost. In this case, a smaller M helps reduce the paging cost and accordingly reduce the overall signaling cost. Third, the signaling cost decreases with the increase in T . The explanation can be found in Fig. 3. The current observation reveals that the HMTBLU scheme always has higher signaling cost than the MBLU scheme because the latter is a special case of the former when $T \rightarrow \infty$. When TBLUs are incorporated into the MBLU scheme, the rise in the signaling cost is an exchange for the capability of the resulting HMTBLU scheme to detect the UE presence. It is difficult to compare the signaling costs of the TBLU scheme and the HMTBLU scheme. As shown previously in Section I-B, the TBLU scheme has an infinite paging area and thus can use only a sequential paging scheme. In some situations, e.g., if we want to locate a UE in two steps, the paging cost of the TBLU scheme can be infinite. Fortunately, the TBLU scheme is used mainly for detecting the UE presence rather than for regular LU so that it will not be used alone. Fourth, the parameter δ exerts conspicuous influence on the signaling cost of the HMTBLU scheme except in the case of low mobility and large T . In Fig. 5(b), when $T = 2/\lambda$, the signaling costs of $\delta = 10$ and $\delta = 5$ are almost identical. The explanation is as follows. When $\lambda/\eta = 100$ and $T = 2/\lambda$, it follows from (7) that $N_{\text{MBLU}} \approx 0$ and $N_{\text{TBLU}} \approx 0.16$, two very small numbers. Therefore, the signaling costs of $\delta = 10$ and $\delta = 5$ differ little.

IV. DISCUSSIONS

Here, we briefly discuss the potential application of the HMTBLU scheme in LTE. In addition, we present some existing applications of the mathematical modeling approach developed in this paper.

A. Potential Application of the HMTBLU Scheme in LTE

As introduced in Section I-A, existing 2G and 3G cellular networks use static LU schemes [37]–[40]. Under a static LU scheme, a UE performs an LU when moving into a new RA. In addition to the drawback of ineffectiveness, static LU schemes also have the following drawbacks, namely, uneven distribution of LU signaling and ping-pong LU effect. Under a static LU scheme, all LUs occur in the boundary cells of a RA, causing the uneven distribution of LU signaling among the cells in the RA, which in some extreme cases can lead to signaling storm in the boundary cells. Under a static LU scheme, when a UE moves back-and-forth between two adjacent RAs, the UE will perform repetitive LUs. This phenomenon is called the ping-pong LU effect. When occurring frequently and excessively, the ping-pong LU effect can incur high signaling cost. To cope with the above drawbacks of static LU schemes, LTE designs a new LU scheme, which is called a tracking area list (TAL)-based LU scheme [41]. In LTE, a RA is called a tracking area (TA). Under the TAL-based LU scheme, when performing an LU, a UE is allocated a group of TAs, which is called a TAL, instead of a single TA. Only when moving into a new TA that does not belong to the allocated TAL, does the UE need to perform an LU so that the UE can move freely within the allocated TAL without performing any LU.

The performance of the TAL-based LU scheme hinges on the allocation of TAL. If TALs are not properly allocated, the TAL-based LU scheme may not benefit the signaling cost. Except [42], most existing studies on TAL allocation assumed that a TA contains only one cell [43]–[47]. This assumption makes the concept of TA in LTE trivial. Moreover, some existing studies simply treated the TAL-based LU scheme as the MBLU scheme or the DBLU scheme [43], [44]. Based on its activity scope and the regularity in its mobility, a UE in a cellular network can be classified as a local UE or a global UE [42]. For a specific region, local UEs are local residents of the region, whereas global UEs are roamers to the region. A local UE often follows an ordered pattern to visit several fixed places so that its mobility exhibits strong regularity. In [42], Wang *et al.* exploited this regularity to allocate an optimal TAL for a local UE. The TAL allocation algorithms proposed by Razavi *et al.* in [45]–[47] can be applied to global UEs. However, these algorithms have the following drawbacks. First, each cell forms a tailored TAL by including the neighbor cells that bring the least signaling cost into the TAL without considering the impact on neighbor cells, so that it is impossible to achieve global optimization within the whole network. Second, a cell allocates a common TAL to all the UEs that perform LU in the cell so that this TAL may be cost ineffective for some UEs. Moreover, this TAL allocation approach makes the TAL-based LU scheme vulnerable to the uneven distribution

of LU signaling and the ping-pong LU effect. In summary, the allocation of TAL for a global UE remains a challenge.

In the following, we propose a method for allocating TALs for a global UE. First, allocate the UE a TAL that contains the maximum number of TALs allowed by Third Generation Partnership Project specification 3GPP TS 23.401 [41], which is 16. In this way, the signaling cost of LU is minimized, whereas the signaling cost of paging is maximized, causing unbalance between the two costs. Second, the allocated TAL can be regarded as a big RA that contains a large number of cells. Then, within the TAL, we can apply the HMTBLU scheme to balance the two costs and thus reduce the overall signaling cost. Specifically, within the TAL, a UE performs only MBLUs and TBLUs according to the HMTBLU scheme, and when moving out of the TAL, the UE performs a tracking area update, which is the terminology used in LTE for LU. Here, TBLUs are performed for detecting the UE presence. Upon a call arrival, instead of paging in all the cells within the TAL, the network pages only in the cells that are within the TAL and, at the same time, are less than M cells away from the last LU cell. Therefore, the paging cost decreases, and the LU cost increases so that, by adjusting relevant parameters, it is possible to balance the two costs and accordingly optimize the overall signaling cost. Consequently, when the HMTBLU scheme is incorporated into the TAL-based LU scheme, the HMTBLU scheme functions at cell levels, whereas the TAL-based LU scheme functions at TA levels. The above TAL allocation method can simplify the allocation of TAL for a global UE and at the same time does not severely compromise the performance of the TAL-based LU scheme.

B. Existing Applications of the Embedded Markov Chain Approach

The approach of embedded Markov chain developed in this paper can be extended to study other LU schemes. When the modeling of LU scheme is concerned, this approach has been rarely used in existing studies. In [31], Akyildiz and Ho used this approach to study a dynamic time-based LU scheme. In [5] and [48], Wang *et al.* employed this approach to study the MBLU scheme under more complicated situations. In [42], Wang *et al.* exploited this approach to optimally allocate a TAL for a local UE. Currently, we are conducting two studies using this approach and other techniques. In the first study, we analyze the signaling cost of the DBLU scheme and can, for the first time, obtain closed-form analytical results. Not all of the existing studies on the DBLU scheme obtained such results and consequently obtained the signaling cost only through numerical calculation or simulation. Although the HMTBLU scheme can be used to simplify the allocation of TAL for a global UE, the resulting performance is not optimal. The second study is focused on optimally allocating TALs for a global UE. We believe that this approach can find more application arenas.

V. CONCLUSION

Among the DBLU, MBLU, and TBLU schemes, the MBLU scheme is the most practical due to its effectiveness and easy-to-

implement nature. Cellular networks use periodic LUs to detect the UE presence to avoid wasting network resource to deliver incoming calls to a UE that has abnormally detached from the network before these calls arrive. Periodic LUs are TBLUs in nature. When implementing the MBLU scheme in a cellular network, we must incorporate TBLUs into the MBLU scheme to make the MBLU scheme has the capability to detect the UE presence. Therefore, the HMTBLU scheme comes into being. Under the HMTBLU scheme, an LU occurs when either the movement threshold or the time threshold is reached, and the movement counter and the periodic LU timer reset when an LU occurs.

In this paper, we have developed an approach of embedded Markov chain to analyze the signaling cost of the HMTBLU scheme. We derived analytical formulas for the signaling cost and formulas that can be further applied to designing an optimal sequential paging scheme. We validated the accuracy of these formulas through simulation. Based on these formulas, we conducted a numerical study to investigate the influence of various parameters on the signaling cost. In the numerical study, we observed the following. First, the signaling cost of the HMTBLU scheme is a downward convex function of the movement threshold. That is, there is a value of the movement threshold that can minimize the signaling cost. Second, the signaling cost of HMTBLU scheme is a decreasing function of the time threshold. That is, the HMTBLU scheme always has higher signaling cost than the MBLU scheme.

APPENDIX A PROOF OF (3)

Denote by $f_{r_0}(x)$ the probability density function of r_0 . Denote by $\overline{F}_c(x)$, $\overline{F}_M(x)$, and $\overline{F}_{r_0}(x)$ the survival functions of t_c , $t_1 + \dots + t_M$, and r_0 , respectively. Denote by T^- a number that sufficiently approaches T from the left-hand side. It follows that

$$\begin{aligned}
 E(r_0) &= T \Pr(r_0 = T) + \int_{x=0}^{T^-} x f_{r_0}(x) dx \\
 &= T \Pr(r_0 = T) - \int_{x=0}^{T^-} x [\overline{F}_{r_0}(x)]^{(1)} dx \\
 &= T \Pr(r_0 = T) - \int_{x=0}^{T^-} x [\overline{F}_c(x) \overline{F}_M(x)]^{(1)} dx \\
 &= T \overline{F}_c(T) \overline{F}_M(T) - x \overline{F}_c(x) \overline{F}_M(x) \Big|_{x=0}^{T^-} \\
 &\quad + \int_{x=0}^{T^-} \overline{F}_c(x) \overline{F}_M(x) dx \\
 &= \int_{x=0}^T \overline{F}_c(x) \overline{F}_M(x) dx \\
 &= (1 - p - q) \frac{1}{\lambda}
 \end{aligned}$$

where $[\cdot]^{(1)}$ denotes the first derivative of a function.

APPENDIX B
VALIDITY CHECK OF (5)

When $T \rightarrow +\infty$, in the HMBLU scheme, no TBLU occurs so that the HMBLU scheme reduces to the MBLU scheme. In [17, Eq. (20)] Zhu and Leung derived the $P(j)$ distribution for the MBLU scheme as

$$P(j) = \frac{\alpha^j}{\sum_{i=0}^{M-1} \alpha^i}, \quad j = 0, 1, \dots, M - 1.$$

It can be verified that when $T \rightarrow +\infty$, (5) reduces to the last equation.

APPENDIX C
VALIDITY CHECK OF (6)

Before checking the validity of (6), first we introduce some propositions that will be applied later.

Proposition 1: Denote by $X \sim \text{Exp}(\mu)$ that random variable X follows an exponential distribution with rate μ . Suppose that $X_1 \sim \text{Exp}(\mu_1)$ and $X_2 \sim \text{Exp}(\mu_2)$ and that X_1 and X_2 are independent. Define random variable X_3 as

$$X_3 \triangleq \{X_1 \mid X_1 < X_2\}.$$

It follows that

$$X_3 \sim \text{Exp}(\mu_1 + \mu_2).$$

Proposition 2: In the TBLU scheme, the number of TBLUs occurring during the call interarrival time is $e^{-\lambda T} / (1 - e^{-\lambda T})$.

We skip the proofs of propositions 1 and 2 due to their simplicity.

Proposition 3: In the HMTBLU scheme, when the movement threshold $M = 1$, the expected number of TBLUs occurring during the call interarrival time t_c is

$$N_{\text{TBLU}} = \frac{1}{1 - \alpha} \frac{e^{-(\eta+\lambda)T}}{1 - e^{-(\eta+\lambda)T}}.$$

Proof: Denote by E_k , $k = 0, 1, \dots$, the event that the UE crosses k cells during t_c . Fig. 6 shows a timing diagram for event E_k . In Fig. 6, the UE is in cell 1 when the previous call arrives and crosses k cells before receiving the next call in cell

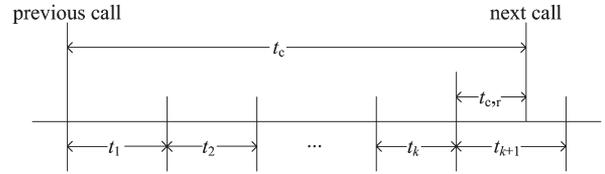


Fig. 6. Timing diagram depicting k cell boundary crossings during t_c .

$k + 1$. Denote by $\beta(k)$ the probability of event E_k . It follows that

$$\begin{aligned} \beta(k) &= \Pr(t_1 + \dots + t_k < t_c < t_1 + \dots + t_{k+1}) \\ &= \alpha^k (1 - \alpha), \quad k = 0, 1, \dots \end{aligned}$$

where α is given in (1). When event E_k occurs, denote by t'_j , $j = 1, \dots, k$, the residence time in cell j and, by $t_{c,r}$, the time from when the UE enters cell $k + 1$ until the arrival of the next call. It follows from Proposition 1 that

$$\begin{aligned} t'_j &= \{t_j \mid t_j < t_c\} \sim \text{Exp}(\eta + \lambda), \quad j = 1, \dots, k \\ t_{c,r} &= \{t_c \mid t_c < t_{k+1}\} \sim \text{Exp}(\eta + \lambda). \end{aligned}$$

That is, t'_j , $j = 1, \dots, k$, and $t_{c,r}$ are identically distributed. When event E_k , $k = 0, 1, \dots$, occurs, we can get (8), shown at the bottom of the page. Equation (8) suggests that t'_j , $j = 1, \dots, k$, and $t_{c,r}$ are independent. Denote by N_j , $j = 1, \dots, k$, the number of TBLUs occurring during t'_j and, by N_{k+1} , the number of TBLUs occurring during $t_{c,r}$. In the case of $M = 1$, whenever moving into a new cell, the UE performs an MBLU, which in turn resets the time counter. Therefore, all N_j , $j = 1, \dots, k + 1$, are i.i.d. It follows from Proposition 2 that

$$E(N_j) = \frac{e^{-(\eta+\lambda)T}}{1 - e^{-(\eta+\lambda)T}}, \quad j = 1, \dots, k + 1.$$

It follows that

$$\begin{aligned} N_{\text{TBLU}} &= \sum_{k=0}^{+\infty} \beta(k) E(N_1 + \dots + N_{k+1}) \\ &= \sum_{k=0}^{+\infty} \beta(k) (k + 1) E(N_j) \\ &= \frac{1}{1 - \alpha} \frac{e^{-(\eta+\lambda)T}}{1 - e^{-(\eta+\lambda)T}} \end{aligned}$$

which completes the proof. ■

$$\begin{aligned} &\Pr(t'_1 > x_1, \dots, t'_k > x_k, t_{c,r} > x_{k+1}) \\ &= \Pr(t_1 > x_1, \dots, t_k > x_k, t_c - (t_1 + \dots + t_k) > x_{k+1} \mid t_1 + \dots + t_k < t_c < t_1 + \dots + t_{k+1}) \\ &= \frac{1}{\beta(k)} \int_{y_1=x_1}^{+\infty} \eta e^{-\eta y_1} \dots \int_{y_k=x_k}^{+\infty} \eta e^{-\eta y_k} \int_{u=\sum_{j=1}^k y_j + x_{k+1}}^{+\infty} \lambda e^{-\lambda u} \int_{y_{k+1}=u-\sum_{j=1}^k y_j}^{+\infty} \eta e^{-\eta y_{k+1}} dy_{k+1} dudy_k \dots dy_1 \\ &= e^{-(\eta+\lambda)x_1} \dots e^{-(\eta+\lambda)x_{k+1}} \\ &= \Pr(t'_1 > x_1) \dots \Pr(t'_k > x_k) \Pr(t_{c,r} > x_{k+1}), \quad k = 0, 1, \dots \end{aligned} \tag{8}$$

We use the following special cases to check the validity of the N_{MBLU} and N_{TBLU} given in (6).

- Case I: $M = 1$. In this case, the UE performs an MBLU whenever moving into a new cell. That is, N_{MBLU} equals the expected number of cells crossed during t_c , which is

$$\frac{E(t_c)}{E(t_j)} = \frac{\eta}{\lambda} \quad (9)$$

because the cell residence time t_j , $j = 1, 2, \dots$ is exponential, and hence, the cell boundary crossings during t_c form an equilibrium renewal process [49], [50]. It follows from Proposition 3 that, in this case

$$N_{\text{TBLU}} = \frac{1}{1 - \alpha} \frac{e^{-(\eta+\lambda)T}}{1 - e^{-(\eta+\lambda)T}}.$$

- Case II: $T \rightarrow +\infty$. In this case, in the HMTBLU scheme no TBLU occurs so that the HMTBLU scheme reduces to the MBLU scheme. The N_{MBLU} for the MBLU scheme was derived in [8, Eqs. (12) and (14)], [12, Th. 6], [13], [14, Eq. (17)], [15], and [16, Eqs. (2) and (9)] as

$$N_{\text{MBLU}} = \frac{\alpha^M}{1 - \alpha^M}. \quad (10)$$

- Case III: $T \rightarrow 0$. In this case, N_{TBLU} is infinite, and N_{MBLU} depends on M . When $M = 1$, it follows from Case I that $N_{\text{MBLU}} = \eta/\lambda$, and when $M \geq 2$, $M = 0$.
- Case IV: $M \rightarrow +\infty$. In this case, in the HMTBLU scheme, no MBLU occurs so that the HMTBLU scheme reduces to the TBLU scheme.
- Case V: $\lambda/\eta \rightarrow +\infty$. In this case, the expected number of cells crossed during t_c is

$$\frac{E(t_c)}{E(t_j)} = \frac{\eta}{\lambda} \rightarrow 0.$$

That is, in this case, the movement threshold can never be reached so that no MBLU occurs. Therefore, in this case, the HMTBLU scheme reduces to the TBLU scheme. It follows from Proposition 2 that, in Cases IV and V

$$E(N_{\text{TBLU}}) = \frac{e^{-\lambda T}}{1 - e^{-\lambda T}}.$$

It can be verified that (6) meets all the above checking criteria.

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