A Lightweight Multimode Medical Image Fusion Method Using Similarity Measure Between Intuitionistic Fuzzy Sets Joint Laplacian Pyramid

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Abstract—Medical image fusion combines the multiple features of human tissue from different source images, which will be conducive to clinical diagnosis. Because of the human visual perception and mechanisms of medical imaging, abundant fuzzy features exist in medical image fusion with remarkable influence. To address this issue, we introduce the similarity measure of fuzzy set theory into medical image fusion to abstract and measure these fuzzy features. In this study, we present a new similarity measure of intuitionistic fuzzy set theory to represent image features and present a lightweight medical image fusion technique on the basis of this new measure. First, the new similarity measure is proposed according to geometric modeling technique and verified by mathematical justification. Second, a set of high frequency sub-images and a low frequency sub-image are produced by Laplacian pyramid decomposition to separate different image features. Third, our proposed similarity measure is utilized to describe and extract the detailed image features. Finally, two fusion rules and inverse Laplacian pyramid decomposition are sequentially used to get the fused image. This research shows that the similarity measure of fuzzy set theory can get excellent performance in medical image fusion. Experiments reveal that our similarity measure and medical image fusion method both have the superior performance to most of the existing methods.

Index Terms—Feature extraction, fuzzy sets, Laplacian pyramid, image processing, information fusion, medical image fusion.

I. INTRODUCTION

A. Motivation

The cognition and judgment of human for the real world are often characterized by non-quantitative fuzziness, that is, the description of physical objects in the real world is usually imprecise and uncertain [1], [2], [3]. Therefore, a mass of vague and uncertain information (or features) exists in the field of computer science when it is used for practical applications. These fuzzy features are usually difficult to be processed with conventional techniques, but they are helpful for pattern recognition or classification [1], [4], [6]. To effectively address this problem, Zadeh [1] established fuzzy set (FS) theory in 1965. Because of the excellent capacity, FS theory provides a powerful mathematical tool for addressing the fuzzy and uncertain problems in various fields, such as information fusion, image processing, and automatic control [2], [3], [5], [6].

Considering the limited descriptive power of conventional FS theory [1], scholars have introduced some enhanced theory based on conventional FS to further improve the performance for dealing with fuzzy features, such as type-2 fuzzy set (T2FS) [7], [8] and intuitionistic fuzzy set (IFS) [9]. Moreover, the information measurement of fuzzy sets is one of the most significant techniques, including entropy, similarity, and distance measure [10], [11]. Among them, similarity and distance measures are two relative concepts in fuzzy theory, and they can effectively measure or quantify the similarity or difference between two different fuzzy sets and are widely used in pattern recognition and classification, clustering analysis, decision making, and feature extraction [11], [12], [13]. Many scholars have devoted themselves in this field and proposed a series of fuzzy measurement models [12], [13], [14]. However, most of the existing measures still have shortcomings that may lead to some unreasonable or incorrect decisions in their applications. Thus, more explorations are wanted to improve the performances of fuzzy measures, and the extensions of these measures for the real applications are also expected.
Medical image fusion is a challenging problem with abundant fuzzy features in information fusion [15], and it is usually used to fuse the complementary information of computed tomography (CT) and magnetic resonance imaging (MRI), which are obtained by the corresponding sensors [16], [17]. Therefore, medical image fusion can be employed to overcome the limitation of information description in single sensor images that are related to the acquisition equipment or physical conditions. Generally, there exists a mass of fuzzy features in image fusion, and the fuzziness mainly comes from two points: first, the information loss of the imaging process will generate some fuzzy and uncertainty features; second, the fuzzy and uncertainty of human visual perception for the visual information [17]. Besides, it is a significant factor for medical image fusion to get the pixel weight that should be related to the features of pixel value and its neighboring pixels, which can be regarded as a fuzzy and uncertainty problem. Thus, the feature presentation of FS theory is expected to enhance the fusion performance of medical images [21], [22]. Besides, it is necessary and reasonable to introduce some novel techniques based on fuzzy theory to measure the features of medical images because of the ability for addressing the imprecise and uncertain information.

Generally, medical image fusion methods are considered as two strategies: transform domain and spatial domain methods [15], [16], [17], [18]. Conventional spatial domain-based methods usually have the advantage of fast speed and smaller memory, but the performance is usually inferior to the transform domain-based methods due to the limited feature representation ability. Besides, the popular deep learning method is generally treated as the spatial domain-based technique, and most deep learning models require a mass of training samples with excessive running time and memory space [18], [19]. However, it is a difficult problem to get enough training samples in this field. Transform domain-based image fusion methods usually include three steps: image transform or image decomposition, sub-band images or coefficients fusion, and inverse transform or image reconstruction. Transform domain-based methods are the most popular techniques in medical image fusion, which are benefit by the stable feature representation ability, but the fuzziness or uncertainty of medical image features is not often well-handled in conventional methods. In addition, most medical image fusion methods tend to be more complex for producing competitive fused images, which leads to an increase in running time and computing resources. Therefore, it is necessary to explore some lightweight and effective medical image fusion methods, as well as the feature representation method based on the measure of FS theory.

B. Our Contributions

According to the above analysis, the performance of similarity measure between FSs should be improved for its real applications, and the lightweight or easily implemented method of medical image fusion is significant for clinical diagnosis. Therefore, this work combines the theoretical result with the real application to propose a lightweight medical image fusion method via the feature measurement of IFSs. The contributions of our work are summarized as follows:

1) We propose a new similarity measure of IFS theory based on geometric modeling technique that is accessible and superior compared with the most of existing methods.
2) We introduce the new proposed similarity measure between IFSs into the feature presentation of medical images, which is a combination of fuzzy theoretical study and practical application.
3) We design an effective and lightweight medical image fusion technique by introducing the new proposed similarity measure between IFSs, which has outstanding performance in at medical image fusion, as well as the simple structure and fast processing speed.

The remainder contents are presented as follows. Section II reports the basic knowledge. Section III presents the new similarity measure and its corresponding theoretical proof, as well as the our new method. Section IV shows the numerical experiments and the applications of the reported measure theory for medical image fusion. Section V concludes this work.

II. PRELIMINARIES

To effectively deal with the vague or imprecise information in the real world, Zadeh proposed FS theory [1], which utilizes membership function to present the fuzzy features [6]. FS theory makes up the deficiency of conventional set theory and lays the foundation for the development of fuzzy theory. In 1986, IFS theory [9], [20] was proposed by reintroducing hesitation in the conventional FS theory, which further improves the ability to describe uncertain information. Besides, the similarity measure is also a hot topic in FS to measure the similarity between two FS [21], [22].

A. Intuitionistic Fuzzy Sets

There are three feature parameters as membership, non-membership, and hesitancy in IFS theory, and the definitions and properties are introduced as below.

Definition 1: Given a domain of discourse $X = \{x_1, x_2, \ldots, x_n\}$, the definition of IFS $A$ is:

$$A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\},$$

where $\mu_A(x)$ is the membership, and $\nu_A(x)$ is the non-membership. The hesitancy function $\pi_A(x)$ satisfies the constraint conditions with $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$; and $\mu_A(x)$, $\nu_A(x)$, and $\pi_A(x)$ are all fall into the value ranges of [0, 1]. What’s more, the IFS can be translated into the conventional FS, when $\pi_A(x) = 0$.

Atanassov and Gargov [23] revealed that the IFS has an equipollent extension of FS as interval-valued fuzzy set (IVFS) which has a mapping relation with IFS. For example, given a IVFS $B = \{(x, \mu_{BL}(x), \mu_{BT}(x)) \mid x \in X\}$ with $0 \leq \mu_{BL}(x) \leq \mu_{BT}(x) \leq 1$, and the map $f$ which assigns to every IVFS $B$ with an IFS $A$: $B = f(A)$, given by $\mu_{BL}(x) = \mu_A(x)$

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and $\mu_B(x) = 1 - \nu_A(x)$. The mapping is the same in reverse, the properties in IFSs are therefore adapted to IVFSs as well.

**Definition 2:** The fuzzy operations between two IFSs $A$ and $B$ are described as follows [24]:
1) If $A \subseteq B$, then $\mu_A(x) \leq \mu_B(x), \nu_A(x) \geq \nu_B(x)$;
2) If $A = B$, then $\mu_A(x) = \mu_B(x), \nu_A(x) = \nu_B(x)$.

### B. Similarity Measure Between Intuitionistic Fuzzy Sets

Similarity measure describes the similarity (or difference) degree between two IFSs, which is an important indicators in IFS theory [10], [11]. The definition of similarity measure between IFSs can be presented by Definition 3.

**Definition 3:** The similarity measure between two different IFSs $A$ and $B$ must satisfy five established Properties [25], [26], [27], as follows:
1) $0 \leq s(A,B) \leq 1$;
2) $s(A,B) = 1$ if and only if $A = B$;
3) $s(A,B) = s(B,A)$;
4) if $A \subseteq B \subseteq C$, then $s(A,C) \leq s(A,B)$ and $s(A,C) \leq s(B,C)$;
5) if $A$ is a crisp set, then $s(A, A^C) = 0$.

### III. OUR PROPOSED METHOD

This section reports our proposed similarity measure and lightweight medical image fusion method. First, the new similarity measure of IFSs is introduced via geometric modeling method which is built by the fuzzy numbers of transformed symmetrical triangle from the feature parameters of IFSs, and the new measure is verified by Definition 3 to prove its rationality. Second, an effective and lightweight medical image fusion scheme is designed with the combination of Laplacian pyramid decomposition (LPD) and the proposed similarity theory which is applied to measure the vague and imprecise features in the source medical images.

### A. Our Proposed Similarity Measure Between IFSs

In IFS theory, similarity measure is a significant theoretical model because of its excellent ability for presenting the vague or uncertain features in information science. And the effectiveness of similarity measure will have seriously influence on the reasonability of decisions. The shortcomings of the existing measures can be classified into two situations: one is that some measures are unable to make decisions in some specific cases, and other measures may give unreasonable decisions. Therefore, a more effective similarity measure is proposed to improve the performance of IFSs. First, the different forms of IFS are given as Definition 4.

**Definition 4:** A given IFS $A$ in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$ can be converted to IVFS, which is expressed as follows:

$$A = \{(x, \mu_A(x), 1-\nu_A(x)) | x \in X \}.$$ 

For simplicity, the fuzzy numbers of transformed IVFS $A$ are described as below:

$$A_T = \{(x, T_{\mu_A}(x), T_{\nu_A}(x)) | x \in X \},$$

where the fuzzy number $T_{\mu_A}(x)$ is another form of feature parameter $\mu_A(x)$, and the fuzzy number $T_{\nu_A}(x)$ is another form of $1-\nu_A(x)$.

When calculate the distance or similarity between two IFSs $A$ and $B$, we put the transformed fuzzy numbers of IFSs $A$ and $B$ on different coordinate axis respectively, and the corresponding transformed isosceles triangles are built according to transformed fuzzy numbers, and then the measure can be constructed based on these transformed isosceles triangles by the distances between some specific points. Fig. 1 shows the transformed geometric model of two different IFSs which are set up on the corresponding axes ($x$ or $y$). Geometric model-based modeling method is intuitive and easy to understand. Fig. 1(a) is the situation that IFS $A$ is different from IFS $B$, and the situation of IFS $A = B$ is shown as Fig. 1(b). $O_A$ and $O_B$ are respectively the transformed triangle vertex of IFSs $A$ and $B$. $N_1$ and $N_2$ are respectively the intersection of $T_{\mu_A}T_{\mu_B} = T_{\mu_A}x/T_{\mu_B} + T_{\mu_A}$ and $T_{\nu_A}T_{\nu_B} = T_{\nu_A}x/T_{\nu_B} + T_{\nu_A}$ with the diagonal line. Our proposed similarity measure is related to the sum of two specific line segments: one is the length difference between $T_{\mu_A}N_1$ and $T_{\mu_B}N_1$, and another is the length difference value between $T_{\nu_A}N_2$ and $T_{\nu_B}N_2$. Therefore, the coordinates of $N_1$ and $N_2$ should be calculated firstly, and the operation is shown as follows.

The $X$-axes of $N_1$ is presented as:

$$N_1/T_{\mu_A}T_{\mu_B} = N_1/T_{\mu_B} + T_{\mu_A},$$

$$\Rightarrow -T_{\mu_A}N_1x/T_{\mu_B} + T_{\mu_A} = N_1x,$$

$$\Rightarrow N_1x = T_{\mu_A}T_{\mu_B}/(T_{\mu_B} + T_{\mu_A}),$$

then, we can get the $Y$-axes of $N_1$ as:

$$N_{1y} = N_1x = T_{\mu_A}T_{\mu_B}/(T_{\mu_B} + T_{\mu_A}).$$

Besides, the $X$-axes of $N_2$ is presented as:

$$N_{2x}/T_{\nu_A}T_{\nu_B} = N_{2x}/T_{\nu_B} + T_{\nu_A},$$

$$\Rightarrow -T_{\nu_A}N_{2x}/T_{\nu_B} + T_{\nu_A} = N_{2x},$$

$$\Rightarrow N_{2x} = T_{\nu_A}T_{\nu_B}/(T_{\nu_B} + T_{\nu_A}),$$

then, the $Y$-axes of $N_2$ is presented as:

$$N_{2y} = N_{2x} = T_{\nu_A}T_{\nu_B}/(T_{\nu_B} + T_{\nu_A}).$$

Fig. 1. Transformed equilateral triangles of IFSs $A$ and $B$. (a) Example 1; (b) Example 2.
As a result, the $T_{μA}N_1$ and $T_{μB}N_1$ can be calculated according to the coordinate of $N_1$ as:

$$T_{μA}N_1 = \sqrt{(T_{μA} - N_{1y})^2 + (N_1 - N_{1y})^2}$$

$$= \sqrt{(T_{μA} - N_{1y})^2 + (N_1 - N_{1y})^2}$$

$$= \sqrt{\left(\frac{T_{μA} - T_{μA}T_{μB}}{T_{μB} + T_{μA}}\right)^2 + \left(\frac{T_{μA} - T_{μB}}{T_{μB} + T_{μA}}\right)^2}$$

$$= \sqrt{\left(\frac{T_{μA}T_{μB} + T_{μB}^2}{T_{μB} + T_{μA}} - T_{μA}T_{μB}\right)^2 + \left(\frac{T_{μA}T_{μB}}{T_{μB} + T_{μA}}\right)^2}$$

$$= \sqrt{\left(\frac{T_{μA}^2 + T_{μB}^2}{T_{μB} + T_{μA}}\right)^2 + \left(\frac{T_{μA}T_{μB}}{T_{μB} + T_{μA}}\right)^2}$$

$$= \frac{T_{μA}N_1}{T_{μB} + T_{μA}}$$

$$T_{μB}N_1 = \sqrt{(T_{μB} - N_{1x})^2 + (N_1 - N_{1x})^2}$$

$$= \sqrt{(T_{μB} - N_{1x})^2 + (N_1 - N_{1x})^2}$$

$$= \sqrt{\left(\frac{T_{μB} - T_{μA}T_{μB}}{T_{μB} + T_{μA}}\right)^2 + \left(\frac{T_{μB} - T_{μA}}{T_{μB} + T_{μA}}\right)^2}$$

$$= \sqrt{\left(\frac{T_{μB}T_{μA} + T_{μA}^2}{T_{μB} + T_{μA}} - T_{μB}T_{μA}\right)^2 + \left(\frac{T_{μB}T_{μA}}{T_{μB} + T_{μA}}\right)^2}$$

$$= \sqrt{\left(\frac{T_{μB}^2 + T_{μA}^2}{T_{μB} + T_{μA}}\right)^2 + \left(\frac{T_{μB}T_{μA}}{T_{μB} + T_{μA}}\right)^2}$$

$$= \frac{T_{μB}N_1}{T_{μB} + T_{μA}}$$

Then, the length difference between $T_{μA}N_1$ and $T_{μB}N_1$ can be get as:

$$d_1 = \frac{T_{μB}N_1 - T_{μA}N_1}{T_{μB} + T_{μA}}$$

$$= \frac{T_{μB}N_1}{T_{μB} + T_{μA}} - \frac{T_{μA}N_1}{T_{μB} + T_{μA}}$$

$$= \left(\frac{T_{μB} - T_{μA}}{T_{μB} + T_{μA}}\right)\sqrt{T_{μA}^2 + T_{μB}^2}$$

By the same derivation, the length difference value between $T_{μA}N_2$ and $T_{μB}N_2$ can be got as:

$$d_2 = \frac{T_{μB}N_2 - T_{μA}N_2}{T_{μB} + T_{μA}}$$

$$= \frac{T_{μB}N_2}{T_{μB} + T_{μA}} - \frac{T_{μA}N_2}{T_{μB} + T_{μA}}$$

$$= \left(\frac{T_{μB} - T_{μA}}{T_{μB} + T_{μA}}\right)\sqrt{T_{μA}^2 + T_{μB}^2}$$

Therefore, the distance between IFSs $A$ and $B$ is presented as (1):

$$d = \frac{1}{2}(d_1 + d_2)$$

$$= \frac{1}{2}\left[\frac{(T_{μB} - T_{μA})\sqrt{T_{μA}^2 + T_{μB}^2}}{T_{μB} + T_{μA}} + \frac{(T_{μB} - T_{μA})\sqrt{T_{μA}^2 + T_{μB}^2}}{T_{μB} + T_{μA}}\right]$$

$$= \frac{1}{2}\left[\frac{(μ_μB - μ_μA)\sqrt{μ_μA^2 + μ_μB^2}}{μ_μB + μ_μA} + \frac{(μ_μB - μ_μA)\sqrt{μ_μA^2 + μ_μB^2}}{μ_μB + μ_μA}\right]$$

Similarly, the similarity measure $s(A, B)$ is obtained as (3):

$$s(A, B) = 1 - \frac{1}{2n}\sum_{i=1}^{n}\left(\frac{(μ_μB(x_i) - μ_μA(x_i))\sqrt{μ_μA(x_i)^2 + μ_μB(x_i)^2}}{μ_μB(x_i) + μ_μA(x_i)}\right)$$

$$= 1 - \frac{1}{2n}\sum_{i=1}^{n}\left(\frac{(μ_μB(x_i) - μ_μA(x_i))\sqrt{μ_μA(x_i)^2 + μ_μB(x_i)^2}}{μ_μB(x_i) + μ_μA(x_i)}\right)$$

$$= 1 - \frac{1}{2n}\sum_{i=1}^{n}\left(\frac{(μ_μB(x_i) - μ_μA(x_i))\sqrt{μ_μA(x_i)^2 + μ_μB(x_i)^2}}{μ_μB(x_i) + μ_μA(x_i)}\right)$$

The proposed measures in (3) should satisfy the Definition 4, and the proof and conclusion are shown as follows.

Property 1: $0 \leq s(A, B) \leq 1$.

Proof: Because of $\min s(A, B) \leq s(A, B) \leq \max s(A, B)$, it can be proved by calculating the $\min s(A, B)$ and $\max s(A, B)$.

Based on (1), when $T_{μB}N_1 = T_{μA}N_1$ and $T_{μB}N_2 = T_{μA}N_2$, we can get $d_1 = 0$ and $d_2 = 0$. According to (3), we can get $\max s(A, B)$ as:

$$\max s(A, B) = 1 - \frac{1}{2}\left(\min d_1 + \min d_2\right) = 1 - \frac{1}{2}(0 + 0) = 1.$$

From Fig. 1, we can see that $0 \leq T_{μB}N_1 \leq 1$ and $0 \leq T_{μA}N_1 \leq 1$, then we can get $0 \leq T_{μA}N_1 - T_{μB}N_1 \leq 1$ and $0 \leq T_{μB}N_2 - T_{μA}N_2 \leq 1$, that is $\max d_1 = 1$ (where $1 \leq i \leq 2$). Thus, $\min s(A, B)$ can be presented as:

$$\min s(A, B) = 1 - \frac{1}{2}\left(\max d_1 + \max d_2\right) = 1 - \frac{1}{2}(1 + 1) = 0.$$

Based on above analysis, we can get $0 \leq s(A, B) \leq 1$.

The proof of Property 1 is completed.

Property 2: $s(A, B) = 1$ if and only if $A = B$.

Proof:

1) We should firstly proof a situation that: if $s(A, B) = 1$, then $A = B$. 

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Based on (1) and (3), the \( d_1 = 0 \) and \( d_2 = 0 \) must be satisfied simultaneously due to \( d_1 \geq 0 \) and \( d_2 \geq 0 \). We can get:

\[
d_1 = \frac{(\mu_B - \mu_A) \sqrt{\mu_A^2 + \mu_B^2}}{\mu_B + \mu_A} = 0,
\]

and

\[
d_2 = \frac{(\nu_A - \nu_B) \sqrt{(1 - \nu_A)^2 + (1 - \nu_B)^2}}{2 - \nu_A - \nu_B} = 0.
\]

Because of \( \mu_B + \mu_A \geq 0 \) and \( 2 - \nu_A - \nu_B \geq 0 \) in the denominators of \( d_1 \) and \( d_2 \), the numerators must be 0 if \( d = \frac{1}{2}(d_1 + d_2) = 0 \) is satisfied. Let the numerators of \( d_1 \) and \( d_2 \) be a simultaneous system of equations as (4):

\[
\begin{cases}
(\mu_B - \mu_A) \sqrt{\mu_A^2 + \mu_B^2} = 0 \\
(\nu_A - \nu_B) \sqrt{(1 - \nu_A)^2 + (1 - \nu_B)^2} = 0
\end{cases} \tag{4}
\]

In (4), each equation can be divided into two cases, and the following two equation sets can be derived:

\[
\begin{align*}
\text{Property 3: } & s(A, B) = s(B, A). \\
\text{Proof: } & \text{The following equation transformation can be obtained due to the mathematical operation of absolute value:}
\end{align*}
\]

\[
s(A, B) = 1 - \frac{1}{2} \left( \frac{(\mu_B - \mu_A) \sqrt{\mu_A^2 + \mu_B^2}}{\mu_B + \mu_A} \right)
+ \frac{(\nu_A - \nu_B) \sqrt{(1 - \nu_A)^2 + (1 - \nu_B)^2}}{2 - \nu_A - \nu_B}
= 1 - \frac{1}{2} \left( \frac{(\mu_B - \mu_A) \sqrt{\mu_A^2 + \mu_B^2}}{\mu_B + \mu_A} \right)
+ \frac{(\nu_A - \nu_B) \sqrt{(1 - \nu_A)^2 + (1 - \nu_B)^2}}{2 - \nu_A - \nu_B}
= s(B, A).
\]

Then, the \( s(A, B) = s(B, A) \) is obtained.

The proof of Property 3 is completed.

Property 4: if \( A \subseteq B \subseteq C \), then \( s(A, C) \leq s(A, B) \) and \( s(A, C) \leq s(B, C) \).

Proof: If \( A \subseteq B \subseteq C \), then \( 0 \leq \mu_A \leq \mu_B \leq \mu_C \leq 1 \) and \( 1 \geq \nu_A \geq \nu_B \geq \nu_C \geq 0 \), that means \( \mu_B - \mu_A \geq 0 \) and \( \nu_A - \nu_B \geq 0 \). Therefore, we can get:

\[
\frac{(\mu_B - \mu_A) \sqrt{\mu_A^2 + \mu_B^2}}{\mu_B + \mu_A} \geq 0,
\]

and

\[
\frac{(\nu_A - \nu_B) \sqrt{(1 - \nu_A)^2 + (1 - \nu_B)^2}}{2 - \nu_A - \nu_B} \geq 0.
\]

Then, the (3) is simplified as follows:

\[
s(A, B)
= 1 - \frac{1}{2} \left( \frac{(\mu_B - \mu_A) \sqrt{\mu_A^2 + \mu_B^2}}{\mu_B + \mu_A} \right)
+ \frac{(\nu_A - \nu_B) \sqrt{(1 - \nu_A)^2 + (1 - \nu_B)^2}}{2 - \nu_A - \nu_B}
= 1 - \frac{1}{2} \left( \frac{(\mu_B - \mu_A) \sqrt{\mu_A^2 + \mu_B^2}}{\mu_B + \mu_A} \right)
+ \frac{(\nu_A - \nu_B) \sqrt{(1 - \nu_A)^2 + (1 - \nu_B)^2}}{2 - \nu_A - \nu_B}
= s(B, A).
\]

The proof of Property 2 is completed.
\[ s(A, C) = 1 - \frac{1}{2} \left( \frac{\sqrt{\mu_C^2 + \mu_C^2}}{\mu_C + \mu_A} \right) + \left( \frac{\nu_A - \nu_C}{2 - \nu_A - \nu_C} \right) \sqrt{1 - (\nu_A^2 + (1 - \nu_C)^2)} \]

\[ s(A, B) = s(A, C) \]

\[ s(A, B) - s(A, C) = \left( \frac{\sqrt{\mu_C^2 + \mu_C^2}}{\mu_C + \mu_A} \right) + \left( \frac{\nu_A - \nu_C}{2 - \nu_A - \nu_C} \right) \sqrt{1 - (\nu_A^2 + (1 - \nu_C)^2)} \]

In (5), we first derive the equation \( s(A, C)_1 - s(A, B)_1 \), and then we can obtain the following results:

\[ s(A, C)_1 = \frac{1}{\mu_C + \mu_A} \left( \frac{\mu_C \mu_B + \mu_C \mu_B - \mu_A^2}{2 - \nu_A - \nu_C} \right) \]

From the above equation, we can get:

\[ \begin{align*}
\mu_C \mu_B - \mu_A^2 &\geq 0 \quad \text{and} \quad \sqrt{\mu_A^2 + \mu_C^2} - \sqrt{\mu_A^2 + \mu_B^2} \\
\mu_C \mu_A - \mu_B \mu_B &\geq 0 \quad \text{and} \quad \sqrt{\mu_A^2 + \mu_C^2} + \sqrt{\mu_A^2 + \mu_B^2} \\
(\mu_C + \mu_A)(\mu_B + \mu_A) &\geq 0
\end{align*} \]

\[ \]
Because of \( \nu_B - \nu_C \geq 0, 1 - \nu_A \geq 0, \) and \((1 - \nu_A)^2 + (1 - \nu_B)^2 \geq 0\), we can get:
\[
\begin{align*}
(\nu_B - \nu_C)(1 - \nu_A) & \sqrt{(1 - \nu_A)^2 + (1 - \nu_B)^2} \geq 0, \\
(2 - \nu_A - \nu_C)(2 - \nu_A - \nu_B) & \geq 0.
\end{align*}
\]

Thus, \( s(A, C) + s(A, B) \geq 0 \) is obtained, and then we can get the final result as follows:
\[
s(A, B) - s(A, C)
= \frac{1}{2} [s(A, C)_1 - s(A, B)_1] + \frac{1}{2} [s(A, C) - s(A, B)]
\geq 0.
\]

Therefore, \( s(B, C) \geq s(A, C) \) is also obtained according to the same derivation.

**Property 4:** If \( A \) is a crisp set, then \( s(A, A^C) = 0 \).

**Proof:** If \( A \) is a crisp set, then \( \mu_A = 1, \mu_{A^C} = 0 \) and \( \nu_A = 0, \nu_{A^C} = 1 \). Therefore, we can get:
\[
s(A, A^C)
= 1 - \frac{1}{2} \left[ \left( \frac{\mu_{A^C} - \mu_A}{\mu_{A^C} + \mu_A} \right)^2 + \right. \\
\left. \frac{(\nu_A - \nu_{A^C})}{2 - \nu_A - \nu_{A^C}} \right]^{1/2}
= 1 - \frac{1}{2} \left( \frac{0 - 1}{0 + 1} \right)^2 + \frac{0 - 1}{2 - 1 - 0}
= 1 - \frac{1}{2} [1] = 0.
\]

The proof of Property 5 is completed. □

**Corollary 1:** Given two IFSs \( A = \{ (x_i, \mu_A(x_i), \nu_A(x_i)) \} \)
\( x_i \in X \) and \( B = \{ (x_i, \mu_B(x_i), \nu_B(x_i)) \} \) \( x_i \in X \) in the domain of \( X = \{ x_1, x_2, \ldots, x_n \} \), and the similarity between IFSs \( A \) and \( B \) is represented as follows:
\[
s_{L,J}(A, B) = \sum_{i=1}^{n} w_i s_i,
\]
where \( w_i \) is the weight of \( s_i \) with the constraint \( \sum_{i=1}^{n} w_i = 1 \).

**B. Our Proposed Medical Image Fusion Method via the Similarity Measure of IFSs**

A lightweight medical image fusion method based on the proposed similarity measure of IFSs is introduced in this subsection. Because of the strong power of IFS theory for analyzing the imprecise and vague features in an image, this work introduces the new proposed similarity measure of IFSs into medical image fusion. In medical image fusion, the pixel fusion weight of each source image is the key point, and the weights are usually relate to the features of the pixel and its neighboring pixels [16]. Therefore, the source medical images are first decomposed by LPD to obtain a set of sub-band coefficients (or sub-band images), and the sub-band coefficients are considered as the components of the source image pixels. Then, our fusion rule based on the proposed similarity measure is employed to confirm which sub-band coefficient should be kept. At last, the fused medical image is recovered according to the fused sub-band coefficients by inverse LPD. Fig. 2 shows the proposed fusion method.

1) **Laplacian Pyramid Decomposition of Medical Image:**

The LPD [16] is a classical analysis method for presenting the features of a source image. A LPD is performed on the basis of Gaussian pyramid decomposition (GPD), and the GPD is therefore conducted firstly. Given a source image \( G_0 \), Gaussian low-pass filter and down-sampling operations are conducted on \( G_0 \) to get next layer. Thus, the next layer \( l \)-th is got by (6):
\[
G_l(i, j) = \sum_{m=-2}^{2} \sum_{n=-2}^{2} \omega(m, n)G_{l-1}(2i + m, 2j + n),
\]
where \( 1 \leq l \leq N, 0 \leq i < R_l, 0 \leq j < C_l \) \( (N \) is the layer number of GPD, and \( R_l \) and \( C_l \) respectively present the row and column of the \( l \)-th layer in GPD; \( G_l(i, j) \) is the GPD pixel of the position \((i, j)\) at the \( l \)-th layer; \( \omega(m, n) \) denotes a given Gaussian matrix with the size of \( 5 \times 5 \), as (7):
\[
\omega = \frac{1}{256} \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix}.
\]

Because of the down sampling operation in GPD, the size of \( G_l \) is a quarter to that of the \( G_{l-1} \). If the interpolation is performed on \( G_l \) to get \( G_l' \), the size of \( G_l' \) will be the same as that of \( G_{l-1} \), as (8):
\[
G_l'(i, j) = \sum_{m=-2}^{2} \sum_{n=-2}^{2} \omega(m, n)G_l \left( \frac{i + m}{2}, \frac{j + n}{2} \right),
\]
where \( 0 < l \leq N, 0 \leq i < R_l, 0 \leq j < C_l \), and
\[
G_l \left( \frac{i + m}{2}, \frac{j + n}{2} \right) = \begin{cases} G_l(i + m, j + n) & \text{if } i + m \text{ and } j + n \text{ are integer} \\ 0 & \text{other} \end{cases}.
\]

Thus, the LPD can be presented as (10):
\[
\begin{align*}
LP_l & = G_l - G_l', \quad 0 \leq l < N \\
LP_N & = G_N,
\end{align*}
\]
where \( N \) denotes the total number of LPD layers; \( G_l \) presents the \( l \)-th layer of GPD, and \( G_{l+1} \) is the \( l \)-th dilated layer; \( LP_l \) is the \( l \)-th layer of LPD.

According to (10), each layer of LPD is enlarged by the interpolation operation on GPD; thus, the size of the current layer is the same as its next layer. Therefore, the final medical image can be obtained by adding all these layers one after the other, and the inverse of LPD can be presented as (11):
\[
\begin{align*}
G_N & = LP_N, \\
G_l & = LP_l + G_{l+1}, \quad 0 \leq l < N.
\end{align*}
\]

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Two examples of three-layer LPD are given in Fig. 3, and Fig. 3(b) and (d) show that LPD can describe the significant features of the medical images \( A \) and \( B \). Specifically, the edges and detailed information of the medical images can be clearly presented in these sub-images. In this method, a three-layer of LPD is employed based on our numerous attempts, which has acceptable computation with competitive performance.

2) Feature Extraction of Sub-Band Coefficients Based on IFSs: The similarity of FS has outstanding capacity to measure and address the imprecise information in various fields, such as image processing [17]. This work proposes a novel lightweight fusion method based on fuzzy similarity measure that is utilized to represent medical image features.

First, the fuzzification of sub-band coefficients are performed by a given membership function that is triangular MF [28], which is expressed as (12):

\[
\mu_{FS}(im(i,j)) = \begin{cases} 
0, & \text{if } im(i,j) \leq a \text{ or } im(i,j) \geq c \\
(\text{im}(i,j) - a) / (b - a), & \text{if } a < im(i,j) \text{ and } im(i,j) < b \\
(\text{c} - \text{im}(i,j)) / (c - b), & \text{if } b < im(i,j) \text{ and } im(i,j) < c
\end{cases}
\]  

where \( \text{im}(i,j) \) denotes the pixel value, and \( \mu_{FS}(im(i,j)) \) is corresponding to MF; | and & are respectively logical OR and AND operation. Moreover, \( a, b, \) and \( c \) are respectively the minimum, mean, and maximum value in \( \text{im}(i,j) \).

From (12), we can know that the parameters of membership function need to be calculated firstly. To keep the consistency of the fuzzification of sub-band coefficients obtained by different source images, the corresponding sub-band coefficients are concatenated, and then the parameters \( a, b, \) and \( c \) can be obtained according to the concatenated coefficient matrix, as shown in (13)–(15):

\[
a = \min \text{ (concatenated } \langle \text{im}^A, \text{im}^B \rangle \text{ )}, \\
b = \text{mean } \text{ (concatenated } \langle \text{im}^A, \text{im}^B \rangle \text{ )}, \\
c = \max \text{ (concatenated } \langle \text{im}^A, \text{im}^B \rangle \text{ )},
\]

where \( \text{im}^A \) and \( \text{im}^B \) are the sub-band coefficients from medical images \( A \) and \( B \), respectively.

Fig. 4 shows the membership matrices of sub-images \( A \) and \( B \). We can get that the unique features of each sub-image are effectively extracted. Besides, the membership matrices of FSs of different sub-images can be further transformed into the membership matrices of IFSs, which are expressed as (16)–(18):

\[
\mu_{IFS}(im(i,j)) = [\mu_{FS}(im(i,j))]^\alpha, \\
\nu_{IFS}(im(i,j)) = 1 - [\mu_{FS}(im(i,j))]^{1/\alpha},
\]
where $\alpha = 2$.

Moreover, the membership matrices of IFSs of the sub-images $A$ and $B$ are respectively shown in Fig. 5, which shows that the key features are better enhanced compared with the membership matrices of FSs in Fig. 4.

To confirm which sub-band coefficients from the two different source images should be kept, a targeted scheme is designed based on our novel similarity measure of IFSs. First, we transform the image pixel values into IFSs according to (12)–(18). The idea is shown as: if we have a crisp intuitionistic fuzzy set $C$ (which means $\mu_C = 1$, $\nu_C = 0$ and $\pi_C = 0$), we can respectively measure the similarity between two unknown elements (coefficients) with the given crisp intuitionistic fuzzy set (IFS $C$). Thus, we can determine the two unknown elements (IFSs $A$ and $B$). The similarity between the IFSs $A$ and $C$ is shown as (19), and the similarity between the IFSs $B$ and $C$ is shown as (20):

$$s_{LJ\_AC}(i,j) = \frac{1}{2h} \sum_{i=1}^{n} \left( \frac{\mu_{C} - \mu_{AIFS}(im(i,j))}{\mu_{C} + \mu_{AIFS}(im(i,j))} \sqrt{\frac{1 - \nu_{AIFS}(im(i,j))}{2 - \nu_{AIFS}(im(i,j))}} \right) \left( \frac{1 - \nu_{AIFS}(im(i,j))}{2 - \nu_{AIFS}(im(i,j))} \right),$$

(19)

$$s_{LJ\_BC}(i,j) = \frac{1}{2h} \sum_{i=1}^{n} \left( \frac{\mu_{C} - \mu_{BIFS}(im(i,j))}{\mu_{C} + \mu_{BIFS}(im(i,j))} \sqrt{\frac{1 - \nu_{BIFS}(im(i,j))}{2 - \nu_{BIFS}(im(i,j))}} \right) \left( \frac{1 - \nu_{BIFS}(im(i,j))}{2 - \nu_{BIFS}(im(i,j))} \right),$$

(20)

where $s_{LJ\_AC}$ is the similarity between IFSs $A$ and $C$, $s_{LJ\_BC}$ is the similarity between IFSs $B$ and $C$.

Thus, the similarity of IFS is used to measure the features of an image. For nonlinear human vision, the importance of a single pixel is closely related to its adjacent pixels. To combine the information of pixel and its neighboring pixels, a convolution operation is respectively used for the similarity matrix of medical images $A$ as (21) and $B$ as (22):

$$CS_{AC}(i,j) = \sum_{m=1}^{2} \sum_{n=1}^{2} \omega(m,n)s_{LJ\_AC}(i,j),$$

(21)

$$CS_{BC}(i,j) = \sum_{m=1}^{2} \sum_{n=1}^{2} \omega(m,n)s_{LJ\_BC}(i,j),$$

(22)

where $\omega$ is a given convolution kernel; $CS$ is the convolutional similarity matrix.

Fig. 6 shows that the similarity matrices and their corresponding convolution matrices of sub-images $A$ and $B$. From Fig. 6(a) and (b), we can find that the similarity matrices are effective to describe and extract the feature information of these medical images. But the features can obviously present the discrete distribution of similarity matrices of sub-images $A$ and $B$, which can be found clearly from the 3D plots of the similarity matrices of these sub-band images, as Fig. 6(e) and (f). However, the fused image generated by the similarity matrices of sub-images $A$ and $B$ may be not satisfactory because the regional information is ignored. The similarity matrices after convolution operation are displayed in Fig. 6(c) and (d), which indicate that the relationship among the surrounding pixels is established compared with the
discrete similarity matrices in Fig. 6(a) and (b), and the regional features of the images are therefore effectively presented. This experiment reveals that the proposed similarity between IFSs is beneficial to present medical image features.

3) The Fusion Rules of Sub-Images:

The high frequency sub-band images (or sub-band coefficients) present the details of medical images and are important for human visual system. In general, the details in high frequency sub-image are difficult to fuse them directly because they are fine features in small scale. The quantification of the importance of these features is the key issue to fuse them. In this work, we employ IFSs-based measure to quantize the features of high frequency sub-images, so that a simple rule is then used for fusing them, and the operation is defined as (23):

\[
FHC = \begin{cases} 
HC_A, & (HCS_{AC} > HCS_{AB}) \\
HC_B, & (HCS_{AC} < HCS_{AB}) \\
(HC_A + HC_B)/2, & (HCS_{AC} = HCS_{AB})
\end{cases}
\]

(23)

where \(FHC\) is the fused high frequency sub-band coefficients; \(HC_A\) and \(HC_B\) are the high frequency sub-band coefficients from medical images \(A\) and \(B\), respectively; \(HCS\) is the high frequency convolutional similarity matrix.

The low frequency sub-image presents the approximate features of the medical images, and the amplitude of the coefficient can provide enough information to fuse them. Therefore, we employ a simple fusion rule to deal with them, the operation is defined as (24):

\[
FLC = \begin{cases} 
LC_A, & \text{abs}(LC_A) \geq \text{abs}(LC_B) \\
LC_B, & \text{abs}(LC_A) < \text{abs}(LC_B)
\end{cases}
\]

(24)

where \(FLC\) represents the fused low frequency sub-band coefficients; \(LC_A\) and \(LC_B\) are respectively the low frequency sub-band coefficients from medical images \(A\) and \(B\).

The fused low and high frequency sub-images are presented in Fig. 7. Compared with Fig. 3, the details of sub-images of different source medical images are fused effectively, such as the features of skeleton and the textures of brain in Fig. 7.

IV. EXPERIMENTS AND ANALYSIS

The effectiveness of our method is verified by numerical experiments and medical image fusion experiments.

A. Numerical Experiments of Similarity Measures

The numerical experiments for fuzzy similarity measure usually have two forms: fuzzy set experiment and pattern recognition experiment. In this paper, eighteen classic similarity or distance measures are selected to contrast with the proposed measure: \(S_C(A,B)[29]\), \(S_{HK}(A,B)[30]\), \(S_{DC}(A,B)[25]\), \(S_{FS}(A,B)[31]\), \(S_{FS}(A,B)[31]\), \(S_{F}(A,B)[12]\), \(S_{HY1}(A,B)[12]\), \(S_{HY1}(A,B)[12]\), \(d_W(X,A,B)[33]\), \(S_Y(A,B)[13]\), \(S_{F}(A,B)\)
According to similarity theory, the sample $A$ should be classified into pattern $A_1$ in both two cases. We set the weight $w_i$ to 1/3 in all measures, and the results of pattern recognition are shown in Table IV. In this experiment, the similarity measure is utilized to classify sample $B$ into one of the three given patterns. According to the references [11], [14], the sample $B$ should be classified into pattern $A_1$ in both two cases. We set the weight $w_1$ to 1/3 in all measures, and the results of pattern recognition are shown in Table V. According to similarity theory, the sample $B$ should be classified into one of the given pattern that has the maximal similarity value. From Table V, we can see that most of the previous measures do not make a right decision for recognizing the pattern of IFS $B$, and there are two reasons: the first one is that some measures have the same maximal similarity value in different patterns, for example, the $S_C$ in Cases 1 and 2, and $S_{HK}$ in Case 2; and the second reason is that some measures do not get the measurement result, for example, $S_Y$ has the problem that the divisor is 0. In addition, some measures have the different

\[ S_{CR} \]

\[ S_{HK} \]

\[ S_{DC} \]

\[ S_{BC} \]

\[ S_{HC} \]

\[ S_{ED} \]

\[ S_{EF} \]

\[ S_{BF} \]

\[ S_{AB} \]

\[ S_{BC} \]

\[ S_{DC} \]

\[ S_{BC} \]

\[ S_{HC} \]

\[ S_{ED} \]

\[ S_{EF} \]

\[ S_{BF} \]

\[ S_{AB} \]

\[ S_{BC} \]

\[ S_{DC} \]

\[ S_{BC} \]

\[ S_{HC} \]

\[ S_{ED} \]

\[ S_{EF} \]

\[ S_{BF} \]

\[ S_{AB} \]

\[ S_{BC} \]

\[ S_{DC} \]

\[ S_{BC} \]

\[ S_{HC} \]

\[ S_{ED} \]

\[ S_{EF} \]

\[ S_{BF} \]

\[ S_{AB} \]

\[ S_{BC} \]

\[ S_{DC} \]

\[ S_{BC} \]

\[ S_{HC} \]

\[ S_{ED} \]

\[ S_{EF} \]

\[ S_{BF} \]

\[ S_{AB} \]

\[ S_{BC} \]

\[ S_{DC} \]

\[ S_{BC} \]

\[ S_{HC} \]

\[ S_{ED} \]

\[ S_{EF} \]

\[ S_{BF} \]

\[ S_{AB} \]

\[ S_{BC} \]

\[ S_{DC} \]

\[ S_{BC} \]

\[ S_{HC} \]

\[ S_{ED} \]

\[ S_{EF} \]

\[ S_{BF} \]

\[ S_{AB} \]

\[ S_{BC} \]

\[ S_{DC} \]

\[ S_{BC} \]

\[ S_{HC} \]

\[ S_{ED} \]

\[ S_{EF} \]

\[ S_{BF} \]

\[ S_{AB} \]

\[ S_{BC} \]

\[ S_{DC} \]

\[ S_{BC} \]

\[ S_{HC} \]

\[ S_{ED} \]

\[ S_{EF} \]

\[ S_{BF} \]

\[ S_{AB} \]

\[ S_{BC} \]

\[ S_{DC} \]

\[ S_{BC} \]

\[ S_{HC} \]

\[ S_{ED} \]

\[ S_{EF} \]

\[ S_{BF} \]

\[ S_{AB} \]

\[ S_{BC} \]

\[ S_{DC} \]

\[ S_{BC} \]

\[ S_{HC} \]

\[ S_{ED} \]

\[ S_{EF} \]

\[ S_{BF} \]

\[ S_{AB} \]

\[ S_{BC} \]

\[ S_{DC} \]

\[ S_{BC} \]

\[ S_{HC} \]

\[ S_{ED} \]

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\[ S_{BF} \]

\[ S_{AB} \]

\[ S_{BC} \]

\[ S_{DC} \]

\[ S_{BC} \]

\[ S_{HC} \]

\[ S_{ED} \]

\[ S_{EF} \]

\[ S_{BF} \]

\[ S_{AB} \]

\[ S_{BC} \]

\[ S_{DC} \]

\[ S_{BC} \]

\[ S_{HC} \]

\[ S_{ED} \]

\[ S_{EF} \]

\[ S_{BF} \]

\[ S_{AB} \]
TABLE V
THE PATTERN RECOGNITION RESULTS OF DIFFERENT MEASURES

<table>
<thead>
<tr>
<th>measures</th>
<th>case 1</th>
<th>case 2</th>
<th>classification result</th>
<th>classification result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(S_2)</td>
<td>0.8000</td>
<td>0.7667</td>
<td>0.8000</td>
<td>Undetermined</td>
</tr>
<tr>
<td>(S_{HK})</td>
<td>0.5667</td>
<td>0.5333</td>
<td>0.5333</td>
<td>AI</td>
</tr>
<tr>
<td>(S_{DC})</td>
<td>0.8000</td>
<td>0.7667</td>
<td>0.8000</td>
<td>Undetermined</td>
</tr>
<tr>
<td>(S_{PB})</td>
<td>0.5667</td>
<td>0.5333</td>
<td>0.5333</td>
<td>AI</td>
</tr>
<tr>
<td>(S_{PG})</td>
<td>0.7167</td>
<td>0.7167</td>
<td>0.7167</td>
<td>Undetermined</td>
</tr>
<tr>
<td>(S_{ML})</td>
<td>0.6444</td>
<td>0.6111</td>
<td>0.6222</td>
<td>AI</td>
</tr>
<tr>
<td>(S_{M})</td>
<td>0.5667</td>
<td>0.5333</td>
<td>0.5333</td>
<td>AI</td>
</tr>
<tr>
<td>(S_{WS})</td>
<td>0.5667</td>
<td>0.5333</td>
<td>0.5333</td>
<td>AI</td>
</tr>
<tr>
<td>(S_{WS})</td>
<td>0.3302</td>
<td>0.2988</td>
<td>0.2988</td>
<td>A3</td>
</tr>
<tr>
<td>(S_{WS})</td>
<td>0.3041</td>
<td>0.2763</td>
<td>0.2763</td>
<td>A3</td>
</tr>
<tr>
<td>(S_{WS})</td>
<td>0.4667</td>
<td>0.4167</td>
<td>0.4333</td>
<td>A1</td>
</tr>
<tr>
<td>(S_{S})</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>Undetermined</td>
</tr>
<tr>
<td>(D_{SA})</td>
<td>0.8000</td>
<td>0.7667</td>
<td>0.8000</td>
<td>Undetermined</td>
</tr>
<tr>
<td>(S_{F})</td>
<td>0.1929</td>
<td>0.0231</td>
<td>0.0232</td>
<td>A1</td>
</tr>
<tr>
<td>(S_{GL}^{L})</td>
<td>0.7822</td>
<td>0.7667</td>
<td>0.7970</td>
<td>A1</td>
</tr>
<tr>
<td>(S_{GL}^{H})</td>
<td>0.6100</td>
<td>0.6044</td>
<td>0.6156</td>
<td>A3</td>
</tr>
<tr>
<td>(D_{B})</td>
<td>0.5667</td>
<td>0.5444</td>
<td>0.5667</td>
<td>Undetermined</td>
</tr>
<tr>
<td>(S_{L})</td>
<td>0.6853</td>
<td>0.7554</td>
<td>0.7404</td>
<td>A2</td>
</tr>
<tr>
<td>(D_{L}^{W})</td>
<td>0.0998</td>
<td>0.1390</td>
<td>0.1266</td>
<td>A3</td>
</tr>
<tr>
<td>(S_{L})</td>
<td>0.6170</td>
<td>0.5705</td>
<td>0.5761</td>
<td>A1</td>
</tr>
</tbody>
</table>

\(p = 1 \text{ in } S_{DC}, S_{LS}^{R}, S_{LS}^{H}, S_{M} \text{ and } D_{BA}; t = 2 \text{ in } D_{BA}; \omega_i = 1/3 \text{ in } S_{LS}^{H}\)

Fig. 9. The first set of fused medical images S1. (a) Image A; (b) Image B; (c) MSTBM; (d) MGIVF; (e) GFS; (f) CBF; (g) DCHWT; (h) ASR; (i) CSR; (j) MSVD; (k) CVT; (l) LP-SR; (m) RP-SR; (n) DWT-SR; (o) DTCWT-SR; (p) CVT-SR; (q) IFCNN; (r) SwinFusion; (s) EMFusion; (t) Our.

Fig. 10. The second set of fused medical images S2. (a) Image A; (b) Image B; (c) MSTBM; (d) MGIVF; (e) GFS; (f) CBF; (g) DCHWT; (h) ASR; (i) CSR; (j) MSVD; (k) CVT; (l) LP-SR; (m) RP-SR; (n) DWT-SR; (o) DTCWT-SR; (p) CVT-SR; (q) IFCNN; (r) SwinFusion; (s) EMFusion; (t) Our.

decision results with other similarities, such as \(S_{GK}, S_{JJ}\) and \(D_{LS}^{R}\) in Case 1. From the Table V, we can see that most of the existing measures do not make effective decision in these two cases, and our proposed measure \(S_{JJ}\) can make a reasonable decision according to the largest similarity value, and it can produce satisfactory performance.

From above experiments, it can be considered that our measure can make the rational decisions and achieve better effect compared with the contrastive measures in pattern recognition experiments. Thus, we can employ the proposed measure of IFSs for medical image fusion, and the following sub-section introduces the experiments on medical image fusion to verify the performance.

3) Example of Medical Image Fusion: We copy some real pixel values from the medical images to show the processes of our image fusion method, which are correspondence to the scheme in Fig. 2. In this example of medical image fusion, Fig. 8(a) presents the source images and the approximate position of the selected pixels, Fig. 8(b) presents the selected coefficients in the first layer in LPD, Fig. 8(c) presents membership degrees of the selected LPD coefficients, Fig. 8(d) presents the
Fig. 11. The third set of fused medical images S3. (a) Image A; (b) Image B; (c) MSTBM; (d) MGVIF; (e) GFS; (f) CBF; (g) DCHWT; (h) ASR; (i) CSR; (j) MSVD; (k) CVT; (l) LP-SR; (m) RP-SR; (n) DWT-SR; (o) DTCWT-SR; (p) CVT-SR; (q) IFCNN; (r) SwinFusion; (s) EMFusion; (t) Our.

Fig. 12. The fourth set of fused medical images S4. (a) Image A; (b) Image B; (c) MSTBM; (d) MGVIF; (e) GFS; (f) CBF; (g) DCHWT; (h) ASR; (i) CSR; (j) MSVD; (k) CVT; (l) LP-SR; (m) RP-SR; (n) DWT-SR; (o) DTCWT-SR; (p) CVT-SR; (q) IFCNN; (r) SwinFusion; (s) EMFusion; (t) Our.

Fig. 13. The fifth set of fused medical images S5. (a) Image A; (b) Image B; (c) MSTBM; (d) MGVIF; (e) GFS; (f) CBF; (g) DCHWT; (h) ASR; (i) CSR; (j) MSVD; (k) CVT; (l) LP-SR; (m) RP-SR; (n) DWT-SR; (o) DTCWT-SR; (p) CVT-SR; (q) IFCNN; (r) SwinFusion; (s) EMFusion; (t) Our.

Fig. 14. The sixth set of fused medical images S6. (a) Image A; (b) Image B; (c) MSTBM; (d) MGVIF; (e) GFS; (f) CBF; (g) DCHWT; (h) ASR; (i) CSR; (j) MSVD; (k) CVT; (l) LP-SR; (m) RP-SR; (n) DWT-SR; (o) DTCWT-SR; (p) CVT-SR; (q) IFCNN; (r) SwinFusion; (s) EMFusion; (t) Our.
depend on it. Thus, we can infer that other useful information is taken into account, which indirectly verifies the effect of the proposed method.

B. Experiments of Medical Image Fusion

To learn the effectiveness of our proposed medical fusion image scheme on the basis of similar measure of IFSs, several popular fusion methods are performed to contrast with our new method, including MSTBM [6], MGIVF [39], GFS [40], CBF [41], DCHWT [42], ASR [43], CSR [44], MSVD [45], CVT [46], [47], LP-SR [47], RP-SR [47], DWT-SR [47], DTCWT-SR [47], CVT-SR [47], IFCNN [48], SwinFusion [49], and EMFusion [50].

In medical image fusion, edge based on similarity measure ($Q_{abf}$), mutual information (MI), standard deviation (STD), and feature mutual information (FMI) are the widely used evaluation metrics, which can be employed to verify the performance of fusion methods. $Q_{abf}$ is used to evaluate edge information kept by the fused image. MI calculates the information quantity kept by the fused image. STD presents contrasting features of the final images with the statistical distribution. FMI shows the feature quantity of fused image obtaining from the source images.

Fig. 9 shows the first set of fused images generated by our method and other contrast fusion methods. Fig. 9 shows that most image fusion methods cannot effectively fuse the source images, but MSTBM, SwinFusion and our method can achieve outstanding performance as shown in Fig. 9(c), (r), and (t). Meanwhile, the brightness of some fused images, such as those in Fig. 9(i) and (j), are visibly inferior to that of other methods. In addition, the clarity of Fig. 9(m) and (n) suffers drops at different degrees. Thus, the effect of our method is superior to most of the contrast methods visually.

Fig. 10 presents the second set of fused medical images produced by different fusion methods. From Fig. 10, we can see that only four methods can effectively accomplish the fusion task of different medical images, such as MSTBM, RP-SR, SwinFusion, and the proposed method, and the corresponding fused images are shown in Fig. 10(c), (m), (r), and (t). Moreover, the brightness of some fused images, such as those in Fig. 10(i) and (j), are darker than other fused images. Besides, the clarity of Fig. 10(n) and (s) has the problem of the visibility declines.

Fig. 11 presents the third set of fused medical images produced via our method and the contrast fusion methods. The fused images in Fig. 11(c), (q), (r), and (t), which are generated by MSTBM, IFCNN, SwinFusion, and our proposed method, can accurately present the significant medical images features, but other methods cannot achieve good performance for the third pair of images. Meanwhile, the brightness of some fused images, such as those in Fig. 11(i), (j), and (s), are visibly inferior to that of other methods. In addition, the clarity of Fig. 11(n) is decreased.

The fourth set of fused medical images produced via our method and the contrast image fusion methods are shown in
Fig. 12. We can see that most fusion methods do not get satisfying performance, and the fused images do not simultaneously present the features of the two medical images, except the fused images in Fig. 12(c), (r), and (t), which are generated by MSTBM, SwinFusion, and our method. Meanwhile, the brightness of some fused images, such as those in Fig. 12(b), (i), (j), and (k) generated by ASR, CSR, MSVD, and CVT, are lower than that of other methods. In addition, the clarity of Fig. 12(n) is decreased.

Fig. 13 presents the fifth set of fused images produced by different image fusion methods. From Fig. 12, it can be seen that only the generated images in Fig. 13(c), (l), (m), (r) and (t) have the common characteristic of the medical images, and other methods cannot effectively fuse them. Furthermore, the brightness of some fused images are lower than that of other methods, such as the fused medical image in Fig. 13(b), (i), (j), and (k) which are respectively generated by ASR, CSR, MSVD, and CVT. In addition, the clarity of Fig. 13(m) and (n) suffers decrease to different degrees.

Fig. 14 is the fused results produced by different methods. We can see that the fusion performance of DCHWT, ASR, MSVD and RP-SR methods, which are shown in Fig. 14(g), (h), (j), (m) and (s), are poorer than other methods. Besides, the brightness of the generated images in Fig. 14(g), (h), (i), (j), and (k) generated by DCHWT, ASR, CSR, MSVD, and CVT are lower than that of other methods. Furthermore, the clarity of Fig. 14(j) and (n) suffers decrease to some degrees. Therefore, our proposed method is relatively superior to other image fusion methods, and it can be regarded as an excellent medical image fusion method subjectively.

Table VI shows the runtime of different fusion methods, which are calculated by averaging the time of ten times experiments, and we do not provide the running time of deep learning-based methods because the difference in the operating environment. In Table VI, we can find that the runtime of our method is the least. The runtime of our new method is similar to that of MGIVF. Besides, the runtime of MSVD, CVT, LP-SR, RP-SR, and DWT-SR are several times of the proposed method, and the runtime of MSTBM, GFS, CBF, DCHWT, and DTCWT-SR are dozens of times of our proposed method. In addition, the runtime of ASR and CSR are much more than that of the proposed method.

Table VII presents the objective indicators of different fusion methods. The scores of MI, STD, and FMI values are used to indicate the differences between the fused images and the source medical images. The smaller the STD score is, the better the fusion performance is. The higher the MI and FMI values are, the better the fusion performance is. In addition, we can find that the three scores of No. 1 combination are the best, and the visual effect of No. 1 is also competitive, such as the position marked by the red arrow. The experiments show that our new fusion method can present and fuse more features from the source medical images compared with these contrast methods. Therefore, it can be concluded that the complexity of our new method is much less than that of other methods.

V. Conclusion

This work proposes an effective similarity measure between IFSs by introducing geometric modeling technique, and a lightweight medical image fusion method is also designed according to the new measure. First, a new similarity measure of IFS theory is conducted to improve the performance. Second, our new measure is applied to represent the medical image features of the obtained sub-band images which are decomposed by LPD. Third, two fusion rules are used to integrate the decomposed sub-band images. At last, the final result is recovered by reverse

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<td><strong>The Runtime of Different Methods</strong> (The Best Result are Highlighted by Bold Type)</td>
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Fig. 16. Ablation studies for fusion rules. (a) first combination of fusion rules; (b) second combination of fusion rules; (c) third combination of fusion rules; (d) fourth combination of fusion rules.
The experiments show that our similarity measure is reasonable and effective, and the performance of our proposed method is more effective than that of the contrast fusion methods with smaller cost.

This study combines the fuzzy measure of IFSs with multi-scale image analysis technique to describe and extract the precise features of medical images, so that the fused pixels can be determined based on the measured similarity information, which reveals the effectiveness of the similarity measure between IFSs in medical image fusion. In this work, our proposed similarity measure between IFSs can overcome the shortcomings of most existing theoretical models to provide reasonable decisions. Besides, this work also further expands the real application of similarity measure between IFSs and provides an effectively theoretical tools for medical image fusion.

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