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## Analysis of cost and quality of service of time-based dynamic mobility management in wireless networks

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Abstract A key observation of a time-based location management scheme (TBLMS) is that the simple paging method (i.e., the fastest paging method) does not guarantee to find a mobile terminal, no matter how small the location update cycle is and how big the radius of a paging area (PA) is. Therefore, in addition to cost analysis and optimization, there is one extra issue to deal with in a TBLMS, i.e., the quality of service (QoS), which is the probability that a mobile terminal can be found in the current PA. The main contributions of the paper are as follows. First, based on our previous results on random walks among rings of cell structures, we analyze the location distribution of a mobile terminal in a PA and the reachability of a mobile terminal in a PA when a phone call arrives, where the intercall time and the cell residence time can have an arbitrary probability distribution. Second, using results from renewal processes, we analyze the cost of dynamic mobility management in a TBLMS, where the inter-call time and the cell residence time can have an arbitrary probability distribution. Third, we develop a method to find a TBLMS which has the best combination of the location update cycle and the radius of a PA with the minimum cost of mobility management, while still satisfying the required QoS.

Keywords Cost analysis  $\cdot$  Location distribution  $\cdot$  Quality of service  $\cdot$  Reachability  $\cdot$  Renewal process  $\cdot$  Time-based location management scheme  $\cdot$  Wireless communication network

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## **1** Introduction

A dynamic location management scheme is an important component of a cellular wireless communication network that effectively and efficiently provides personal communication service (PCS) to mobile users. There are two essential tasks in mobility management, namely, location update (location registration) and terminal paging (call delivery). Location update is the process for a mobile terminal to periodically notify its current location to a network, so that the network can revise the mobile terminal's location profile in a location database. Terminal paging is the process for a network to search a mobile terminal by sending polling signals based on the information of its last reported location, so that an incoming phone call can be routed to the mobile terminal. The location database entry of a mobile terminal is updated when the mobile terminal performs a location update and/or when a network performs a terminal paging during the call delivery to the mobile terminal. Both location update and terminal paging consume significant communication bandwidth of a wireless network, battery power of mobile terminals, memory space in location registers and databases, and computing time at base stations. Therefore, both location update cost and terminal paging cost should be minimized.

There are three location update methods, namely, the distance-based method, the movement-based method, and the time-based method [11]. Accordingly, there are three types of dynamic location management schemes, namely, *distance-based location management schemes* (DBLMS), *movement-based location management schemes* (MBLMS), and *time-based location management schemes* (TBLMS). A TBLMS (a DBLMS and an MBLMS, respectively) employs the time-based (the distance-based and the

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movement-based, respectively) location update method. Furthermore, a DBLMS or an MBLMS or a TBLMS can use various terminal paging methods. It is well known that compared with a DBLMS and an MBLMS, a TBLMS is the easiest to implement in real wireless communication systems. Among the three location update methods, it is also the easiest to analyze the dynamic mobility management cost for a TBLMS.

The design and analysis of any dynamic location management scheme depend on a mobility model of mobile terminals. Various mobility models have been proposed in the literature, including the shortest distance mobility model [2, 3], the fluid flow model [7, 12], the big move and the random walk models [14], the user mobility pattern scheme [16], the cell coordinates system [38], the isotropic diffusive motion model [40], one-dimensional Markov chains [4, 11, 15, 37, 44], and two-dimensional Markov chains [5, 7, 19, 26]. Recently, we developed a ring level random walk model to accurately represent the movement of a mobile terminal in two-dimensional cellular structures [35]. This Markov chain model has been used to analyze the paging area (PA) residence time and the cost of dynamic mobility management in a DBLMS. It will continue to be used in this paper to study location distribution and reachability of a mobile terminal in a PA, and to analyze the quality of service (QoS) in a TBLMS. It can be further employed to investigate paging cost reduction methods.

Dynamic mobility management is an important and fundamental research issue in wireless communication, and significant effort has been devoted by many researchers. The performance of movement-based location management schemes has been investigated in [5, 11, 21, 31–34, 36, 39, 55, 58]. The performance of distance-based location management schemes has been studied in [3, 9, 11, 15, 26, 35, 37, 38, 56, 59, 60]. The performance of time-based location management schemes has been considered in [4, 11, 12, 40]. Terminal paging methods with low cost and time delay have been studied by several researchers [2, 5, 5]10, 26, 42, 52, 54]. Other studies were reported in [7, 14, 16, 19, 20, 22, 41, 44, 53, 57], and some comparative studies were in [13, 29, 30, 43, 45]. Dynamic location management in a wireless communication network with a finite number of cells has been treated as an optimization problem which is solved by using bio-inspired methods such as simulated annealing, neural networks, and genetic algorithms [8, 46, 48–51]. The reader is also referred to the surveys in [6, 28, 47], [24, Ch. 15], and [25, Ch. 11].

The time-based location management schemes have been much less investigated than the movement-based location management schemes and the distance-based location management schemes. In [11], the time-based location update method was analyzed for a ring cellular topology. In [12], the cost of the progressive paging method was analyzed for a TBLMS in a hexagonal cell structure; however, there is no analysis of the location update cost. In [4], a location update method was proposed to determine the time of the next location update based on the inter-call time distribution, the distance traveled since the last location update, and the time elapsed since the last call arrival time. Unfortunately, the study was conducted for a one-dimensional PCS coverage area. In [40], attempt was also made to determine the time of the next location update, and such determination is based on known information of location distribution of a mobile terminal relative to the location when its location was last updated. However, the knowledge of location distribution is exactly what needs to be found.

To summarize the existing studies on TBLMS, we find that there is no comprehensive study on the location update cost, the location distribution of a mobile terminal in a PA, and the probability to find a mobile terminal in a PA when the next phone arrives, for a two-dimensional cell structure. Furthermore, there is no effective strategy to determine the best combination of the location update cycle and the size of the PA, such that the total cost of dynamic mobility management in a TBLMS is minimized while the required QoS is achieved.

In addition to cost analysis and optimization, there is one extra issue to deal with in a TBLMS, i.e., the QoS, which is the probability that a mobile terminal can be found in the current PA. A key observation of a TBLMS is that the simple paging method (i.e., the fastest paging method) does not guarantee to find a mobile terminal, no matter how small the location update cycle  $\tau$  is and how big the radius d of a PA is. It is a critical issue to study the probability that a mobile terminal can be found in the current PA, since this is directly related to the QoS in a wireless communication network. To this end, we need to know how a mobile terminal is distributed in a PA. This is the most challenging part in analyzing and optimizing the performance of a TBLMS, and is also the main topic of the present paper. Without such information, it is impossible to determine the appropriate values of  $\tau$  and d to satisfy the required QoS and to minimize the cost of dynamic mobility management in a TBLMS.

The main contributions of the paper are as follows. First, based on our previous results on random walks among rings of cell structures, we analyze the location distribution of a mobile terminal in a PA and the reachability of a mobile terminal in a PA when a phone call arrives, where the inter-call time and the cell residence time can have an arbitrary probability distribution. Second, using results from renewal processes, we analyze the cost of dynamic mobility management in a TBLMS, where the inter-call time and the cell residence time can have an arbitrary probability distribution. Third, we develop a method to find a TBLMS which has the best combination of the location update cycle and the radius of a PA with the minimum cost of mobility management, while still satisfying the required QoS. Furthermore, our results on the location distribution of a mobile terminal in a PA can be used for further investigation on the cost of various paging methods and paging cost reduction. To the best of the author's knowledge, there has been no similar result in the existing literature. Therefore, the present paper has made significant contribution to cost and QoS analysis of time-based location management schemes.

The rest of the paper is organized as follows. In Sect. 2, we provide background information of our study. In Sect. 3, we develop results from renewal processes to be used in this paper. In Sect. 4, we present known results on random walks among rings. In Sect. 5, we analyze the location distribution of a mobile terminal in a PA. In Sect. 6, we derive the reachability of a mobile terminal in a PA. In Sect. 7, we analyze the cost of dynamic mobility management in a TBLMS. In Sect. 8, we present simulation results and numerical data, and discuss cost minimization with guaranteed QoS. In Sect. 9, we conclude the paper.

#### 2 Background information

## 2.1 Wireless communication networks

A wireless communication network has the common hexagonal cell configuration or mesh cell configuration. In the *hexagonal cell structure* (see Fig. 1), cells are hexagons of identical size and each cell has six neighbors. In the *mesh cell structure* (see Fig. 2), cells are squares of identical size and each cell has eight neighbors. Throughout the paper, we let q be a constant such that q = 3 for the hexagonal cell configuration and q = 4 for the mesh cell configuration. By using the constant q, the hexagonal cell configuration and the mesh cell configuration can be treated in a unified way. For instance, we can say that each cell has 2q neighbors without mentioning the particular cell structure. The network is homogeneous in the sense that the behavior of a mobile terminal is statistically the same in all the cells.

Let *s* be the cell registered by a mobile terminal in the last location update. The cells in a wireless networks can be divided into rings, where *s* is the center of the network and called ring 0. The 2q neighbors of *s* constitute ring 1. In general, the neighbors of all the cells in ring *r*, except those neighbors in rings r - 1 and *r*, constitute ring r + 1. For all  $r \ge 0$ , the cells in ring *r* have distance r-*s*. For all  $r \ge 1$ , the number of cells in ring *r* is 2qr. Notice that the rings are defined with respect to *s*. When a mobile terminal updates



Fig. 1 The hexagonal cell configuration

					ring					
					d-1					
	(5,1)	(3,2)	(3,3)	(3,3)	(3,3)	(3,3)	(3,3)	(3,2)	(5,1)	
	(3,2)								(3,2)	
	(3 3)		_		ring 2				(3 3)	
	(0,0)								(0,0)	
	(3,3)				ring 1				(3,3)	
	(3,3)				ring 0				(3,3)	
	(3,3)								(3,3)	
	(3,3)								(3,3)	
	(3,2)								(3,2)	
	(5,1)	(3,2)	(3,3)	(3,3)	(3,3)	(3,3)	(3,3)	(3,2)	(5,1)	

Fig. 2 The mesh cell configuration

its location to another cell s', s' becomes the center of the network, and ring *r* consists of the 2qr cells whose distance to s' is *r*.

When a mobile terminal u moves out of a cell s, it is normally assumed that u moves into one of s's 2q neighbors with equal probability [21, 31], although this assumption is irrelevant in analyzing location update cost of movementbased schemes. However, how u moves into the neighboring cells is very important in analyzing location update cost of distance-based schemes; and in analyzing location distribution, reachability, and QoS of time-base schemes; and in reducing paging cost of all schemes. The reason is that the way u moves into the neighboring cells determines how fast or slow u reaches the boundary of a PA.

## 2.2 Location update methods

A mobile terminal *u* constantly moves from cell to cell. Such movement also results in movement from ring to ring. Let the sequence of cells visited by *u* before the next phone call be denoted as  $s_0, s_1, s_2, \ldots, s_d, \ldots$ , where  $s_0 = s$  is u's last registered cell (not the cell in which u received the previous phone call) and considered as *u*'s current location. There are three location update methods proposed in the current literature, namely, the distance-based method, the movement-based method, and the time-based method. In the distance-based location update method, location update is performed as soon as u moves into a cell  $s_i$  in ring d, where d is a distance threshold, i.e., the distance of u from the last registered cell s is d, such that  $s_i$  is registered as u's current location. It is clear that  $j \ge d$ , i.e., it takes at least d steps for *u* to reach ring *d*. In the movement-based location update method, location update is performed as soon as u has crossed cell boundaries for d times since the last location update, where d is a movement threshold. It is clear that the sequence of registered cells for u is  $s_d, s_{2d}, s_{3d}, \ldots$  In the time-based location update method, location update is performed every  $\tau$  units of time, where  $\tau$  is a time threshold, regardless of the current location of u.

### 2.3 Terminal paging methods

In all dynamic location management schemes, a current PA consists of rings 0, 1, 2, ..., d - 1, where d is some value appropriately chosen. We say that such a PA has radius d. Since the number of cells in ring r is 2qr, for all  $r \ge 1$ , the total number of cells in a PA is  $qd^2 - qd + 1$ . It should be noticed that a PA is defined with respect to the current location of a mobile terminal, and is changed whenever a mobile terminal updates its location. The radius d of a PA can be adjusted in accordance with various cost and performance considerations. On the other hand, the location and size of a cell are fixed in a wireless network.

Two terminal paging methods have been proposed in the literature. In the *simple paging method*, the radius of a PA is fixed at *d*, where *d* is the distance threshold used by a distance-based location update method, or the movement threshold used by a movement-based location update method, or appropriately chosen in accordance with the time threshold used by a time-based location update method. In the *selective paging method*, cells in a PA or the entire wireless communication network are divided into disjoint areas, such that these areas are paged one after another successively, until a mobile terminal is found. The

simple paging method is the fastest method, since it only sends polling signals once. In this paper, we will only consider the simple paging method.

## 2.4 Call handling models

We will consider two different call handling models [33]. In the *call plus location update* (CPLU) model, the location of a mobile terminal is updated each time a phone call arrives. That is, in addition to distance-based or movement-based or time-based location updates, the arrival of a phone call also initiates location update and defines a new PA. This causes the original location update cycle of a mobile terminal being interrupted. In the *call without location update* (CWLU) model, the arrival of a phone call has nothing to do with location update, that is, a mobile terminal still keeps its original location update cycles.

## 2.5 Notations

Throughout the paper, we use P[E] to denote the probability of an event *E*. For a random variable *T*, we use E(T) to represent the expectation of *T* and  $\lambda_T = E(T)^{-1}$ . The probability density function (pdf) of *T* is  $f_T(t)$ , and the cumulative distribution function (cdf) of *T* is  $F_T(t)$ . The Laplace transform of  $f_T(t)$  and  $F_T(t)$  for a nonnegative random variable *T* are defined as

$$f_T^*(s) = \boldsymbol{E}(e^{-sT}) = \int_0^\infty e^{-st} f_T(t) dt,$$

and

$$F_T^*(s) = \int_0^\infty e^{-st} F_T(t) dt.$$

There are several important random variables in the study of dynamic location management. The *inter-call time*  $T_c$  is defined as the length of the time interval between two consecutive phone calls. The *cell residence time*  $T_s$  is defined as the time a mobile terminal stays in a cell before it moves into a neighboring cell. The *paging area residence time*  $T_m$  is defined as the time a mobile terminal stays in the current PA before it moves out of the PA. The *location update time*  $T_u$  is defined as the time between two consecutive location updates, which is actually the time for a mobile terminal to across *d* cell boundaries in an MBLMS, or the PA residence time in a DBLMS, or the time threshold  $\tau$  in a TBLMS. The quantity  $\rho = \lambda_{T_c}/\lambda_{T_s}$  is the *call-to-mobility ratio*.

Throughout the paper, we use the notation TBLMS  $(\tau, d)$  to represent a dynamic location management scheme

using the time-based location update method with time threshold  $\tau$  and the simple paging method with paging radius *d*. We use the notations TBLMS ( $\tau$ , *d*)–CPLU and TBLMS ( $\tau$ , *d*)–CWLU to represent a TBLMS ( $\tau$ , *d*) under the CPLU model and the CWLU model, respectively.

The cost of dynamic location management contains two components, i.e., the cost of location update and the cost of terminal paging. The cost of location update is proportional to the number of location updates. If there are  $X_u$  location update between two consecutive phone calls, the cost of location update is  $\Delta_u X_u$ , where  $\Delta_u$  is a constant. Since  $X_u$  is a random variable, the location update cost is actually calculated as  $\Delta_u E(X_u)$ . The cost of terminal paging is proportional to the number of cells paged. If a PA has radius *d*, the cost of paging is  $\Delta_p(qd^2 - qd + 1)$ , where  $\Delta_p$ is a constant.

Dynamic location management is per-terminal based. A mobile terminal is specified by  $f_{T_c}(t)$  and  $f_{T_s}(t)$ , where  $f_{T_c(t)}$  is the call pattern and  $f_{T_s(t)}$  is the mobility pattern. Since a location update method determines the location update cost and a terminal paging method determinal, we need to find a balanced combination of a location update method and a terminal paging method such that the total location management cost for the mobile terminal is minimized.

## **3** Renewal processes

A *renewal process* is defined by a sequence of independent random variables  $T_1$ ,  $T_2$ ,  $T_3$ , ..., where  $T_2$ ,  $T_3$ , ... are a sequence of independent and identically distributed (i.i.d.) random variables with a common pdf, but  $T_1$  may have a different pdf. (The reader is referred to [1] for a general introduction to the renewal theory.) A renewal process has many associated random variables and properties. The most interesting property related to our study is the number of renewals in a random period of time. We use X(t) to denote the number of renewals in a time interval of length t. Let  $S_j = T_1 + T_2 + \cdots + T_j$ . If  $S_j \le t < S_{j+1}$ , we say that the number of renewals X(t) in a time interval of length t is j.

Let X be the number of renewals in a random time interval of length  $T_c$ . The following theorem (a folklore in renewal theory [33, 35]) gives the probability distribution of X for an arbitrary renewal process. The theorem will be used in Theorems 7 and 9.

**Theorem 1** For an arbitrary renewal process, we have

$$\boldsymbol{P}[X=j] = \int_{0}^{\infty} (F_{S_{j}}(t) - F_{S_{j+1}}(t)) f_{T_{c}}(t) dt,$$

for all  $j \ge 0$  and for arbitrary  $T_1, T_2, T_3, \dots$  and  $f_{T_c}(t)$ .

### 3.1 Ordinary renewal processes

An ordinary renewal process is defined by a sequence of i.i.d. random variables  $T_1, T_2, T_3, \dots$  with a common pdf  $f_T(t)$  [18]. In modeling dynamic location management schemes, the  $T_i$ 's can be cell residence times, or PA residence times, or location update times. When the  $T_i$ 's are cell residence times, each renewal stands for a cell boundary crossing, and X(t) is the number of cell boundary crossings in a time interval of length t. When the  $T_i$ 's are PA residence times, each renewal stands for a PA boundary crossing, and X(t) is the number of PA boundary crossings in a time interval of length t. When the  $T_i$ 's are location update times, each renewal stands for a location update, and X(t) is the number of location updates in a time interval of length t. When t is randomized according to inter-call time distribution  $f_{T_c}(t)$ , we are essentially considering the number of cell boundary crossings, or the number of PA boundary crossings, or the number of location updates between two consecutive phone calls.

The following theorem gives the probability distribution of X in an ordinary renewal process, where  $T_c$  has a hyper-Erlang distribution with pdf

$$f_{T_c}(t) = \sum_{i=1}^{k_c} w_{c,i} \left( \frac{\lambda_{c,i} (\lambda_{c,i} t)^{\gamma_{c,i}-1} e^{-\lambda_{c,i} t}}{(\gamma_{c,i}-1)!} \right),$$

where  $w_{c,1} + w_{c,2} + \cdots + w_{c,k_c} = 1$ . The theorem generalizes Equation (14) in [17]. A similar result has been proven for equilibrium renewal processes [33] and modified renewal processes [35]. The theorem will be used in Theorem 3.

**Theorem 2** For an ordinary renewal process, if  $T_c$  has a hyper-Erlang distribution, we have

$$\begin{aligned} \boldsymbol{P}[X=j] &= \sum_{i=1}^{k_c} w_{c,i} \left( \frac{\lambda_{c,i}^{\gamma_{c,i}}}{(\gamma_{c,i}-1)!} \right) \left( -\frac{\partial}{\partial s} \right)^{\gamma_{c,i}-1} \\ &\times \left[ \left( \frac{1-f_T^*(s)}{s} \right) (f_T^*(s))^j \right]_{s=\lambda_{c,i}}, \end{aligned}$$

for all  $j \geq 0$ .

The proof of the theorem is given in the "Appendix".

The following theorem gives E(X) for an ordinary renewal process based on Theorem 2.

**Theorem 3** If  $T_c$  has a hyper-Erlang distribution, the expected number of renewals in a random time interval of length  $T_c$  is

$$\boldsymbol{E}(X) = \sum_{i=1}^{k_c} w_{c,i} \left( \frac{\lambda_{c,i}^{\gamma_{c,i}}}{(\gamma_{c,i}-1)!} \right) \left( -\frac{\partial}{\partial s} \right)^{\gamma_{c,i}-1} \left[ \frac{f_T^*(s)}{s(1-f_T^*(s))} \right]_{s=\lambda_{c,i}}$$

#### for an ordinary renewal process with arbitrary $f_T(t)$ .

The above theorem can be proved by straightforward manipulation. It will be used in Theorem 14.

#### 3.2 Equilibrium renewal processes

Consider a mobile terminal *u* moving through a sequence of cells  $s_1, s_2, s_3, \ldots$ , with a sequence of cell residence times  $T_{s,1}$ ,  $T_{s,2}$ ,  $T_{s,3}$ , ... in an MBLMS. Assume that u is in  $s_1$  when a phone call arrives, i.e., u has been in  $s_1$  for a while. Clearly,  $T_{s,1}$  is the *residual time* of u in  $s_1$  and does not have the same pdf as the other  $T_{s,i}$ 's. Similarly, consider a mobile terminal u moving through a sequence of PAs PA<sub>1</sub>, PA<sub>2</sub>, PA<sub>3</sub>, ..., with a sequence of PA residence times  $T_{m,1}, T_{m,2}, T_{m,3}, \dots$  in a DBLMS. Assume that u is in PA<sub>1</sub> when a phone call arrives, i.e., u has been in PA<sub>1</sub> for a while. Clearly,  $T_{m,1}$  is the residual time of u in PA<sub>1</sub> and does not have the same pdf as the other  $T_{m,i}$ 's. Again, consider a sequence of location update times  $T_{u,1}, T_{u,2}, T_{u,3}, \dots$  in a TBLMS. Assume that u is in  $T_{u,1}$ when a phone call arrives, i.e., u has been in  $T_{u,1}$  for a while. Clearly,  $T_{u,1}$  is the residual time of u in  $T_{u,1}$  and does not have the same pdf as the other  $T_{u,i}$ 's.

An ordinary renewal process  $T_1$ ,  $T_2$ ,  $T_3$ , ..., where the pdf of  $T_1$  is the residual time of an ordinary  $T_i$ , is called an *equilibrium renewal process*, which can be regarded as an ordinary renewal process that has been running for a long time before it is first observed [18]. Notice that if the  $T_i$ 's have an exponential distribution, an equilibrium renewal process becomes an ordinary renewal process.

The expected number of renewals in a random time interval of an equilibrium renewal process is surprisingly easy to obtain. The following theorem is a simple and strong result in renewal theory [18]. The theorem will be used in Theorem 15.

**Theorem 4** For any probability distributions of  $T_c$  and T, the expected number of renewals in a random time interval of length  $T_c$  is  $\mathbf{E}(X) = \mathbf{E}(T_c)/\mathbf{E}(T) = \lambda_T/\lambda_{T_c}$  for an equilibrium renewal process.

#### 4 Random walks among rings

Consider a mobile terminal u in a cell s. After staying in s for  $T_s$  amount of time, u moves out of s and enters into one of the 2q neighbors of s. It is clear that the movement of a mobile terminal can be described by a random walk among the cells. Such a random walk can be characterized by a two-dimensional Markov chain, where for each cell, we have a state in the Markov chain associated with the cell. The transition probability from a cell to a neighboring

cell is 1/(2q). While the random walk among the cells and the cell level Markov chain accurately describe the movement of a mobile terminal in any cell structure, the number of states is exactly the same as the number of cells, which causes excessive computation cost in obtaining numerical data [35].

To reduce the number of states, we consider a random walk among the rings and construct a ring level Markov chain which contains states  $K_0, K_1, K_2, ..., K_d, ...,$  where state  $K_r$  means that a mobile terminal u is in ring  $r, r \ge 0$ . Initially, u is in state  $K_0$ . Instead of the probabilities of moving into neighboring cells, we are interested in the probabilities of moving into adjacent rings, i.e., the probability  $a_r$  of moving into ring r + 1 and the probability  $b_r$ of moving into ring r - 1. Let  $p_{ij}$  denote the transition probability from  $K_i$  to  $K_j$ , where  $i, j \ge 0$ . Then, we have  $p_{01} = 1, p_{r, r+1} = a_r, p_{r, r-1} = b_r, p_{r, r} = 1 - a_r - b_r$ , for all  $r \ge 1$ . All other  $p_{ij}$ 's not specified above are zeros. Using the  $a_r$ 's and the  $b_r$ 's, the matrix of transition probabilities  $P = [p_{ij}]$  is

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \cdots \\ b_1 & 1 - a_1 - b_1 & a_1 & 0 & 0 & \cdots \\ 0 & b_2 & 1 - a_2 - b_2 & a_2 & 0 & \cdots \\ 0 & 0 & b_3 & 1 - a_3 - b_3 & a_3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

We use  $p_{ij}^{(n)}$  to denote the *n*-step transition probability from  $K_i$  to  $K_j$ , where  $i, j \ge 0$ . The following result is well known ([23], p. 383), which will be used in Theorem 7 and Theorem 9.

**Theorem 5** If the nth power of P is  $P^n = [g_{ij}]$ , we have  $p_{ij}^{(n)} = g_{ij}$ , for all  $i, j \ge 0$ .

The exact values of the  $a_r$ 's and the  $b_r$ 's are extremely difficult to obtain. Fortunately, approximate values can be derived as follows, namely,  $a_1 = 1/2$ ,  $b_1 = 1/(2q)$ ,

$$a_r = \frac{(q-1)r + 1 - \delta_q}{2qr},$$

and

$$b_r = \frac{(q-1)r - q + 2 + \delta_q}{2qr},$$

for all r > 1, where  $\delta_q$  is an appropriately chosen value to adjust  $a_r$  and  $b_r$ . In our study, we set  $\delta_3 = 0.15$  and  $\delta_4 = 0.42$  empirically, which have been verified to be very accurate in modeling the movement of a mobile terminal [35].

Let N(d) denote the expected number of steps for a mobile terminal to move out of a PA of radius *d*. N(d) is actually the expected number of steps for a random walk starting from  $K_0$  to reach  $K_d$ . It was shown in [35] that

$$N(d) pprox lpha_q \cdot rac{d(qd-1)}{q-1},$$

where  $\alpha_3$  is roughly 0.55 and  $\alpha_4$  is roughly 0.60.

## 5 Location distribution in a PA

Let  $\xi_r, r \ge 0$ , denote the probability that a mobile terminal u is in ring r when a phone call arrives. The sequence  $(\xi_0, \xi_1, \xi_2, ...)$  is called a *location distribution* of u. In this section, we study location distribution of a mobile terminal in a PA when a phone call arrives. The result is critical in analyzing the QoS and in reducing paging cost in a TBLMS.

Let the pdf of an Erlang distribution be represented as

$$f_{\mathrm{Erlang}}(\lambda,\gamma,t) = rac{\lambda e^{-\lambda t} (\lambda t)^{\gamma-1}}{(\gamma-1)!},$$

and the cdf of an Erlang distribution be represented as

$$F_{\mathrm{Erlang}}(\lambda,\gamma,t) = 1 - e^{-\lambda t} \sum_{j=0}^{\gamma-1} \frac{(\lambda t)^j}{j!}.$$

The class of hyper-Erlang distributions are used extensively in this study. A random variable X with a hyper-Erlang distribution has a pdf

$$f_X(t) = \sum_{i=1}^k w_i \left( \frac{\lambda_i e^{-\lambda_i t} (\lambda_i t)^{\gamma_i - 1}}{(\gamma_i - 1)!} \right) = \sum_{i=1}^k w_i f_{\text{Erlang}}(\lambda_i, \gamma_i, t),$$

where  $w_1 + w_2 + \cdots + w_k = 1$ . Special forms of hyper-Erlang distributions include hyperexponential distributions  $(\gamma_i = 1 \text{ for all } 1 \le i \le k)$ ; exponential distributions  $(k = 1 \text{ and } \gamma_1 = 1)$ ; chi-square distributions  $(k = 1 \text{ and } \lambda_1 = 1/2)$ ; Erlang distributions (k = 1).

Let T' be the residual time of T. The following theorem will be used in our discussion.

**Theorem 6** If T has a hyper-Erlang distribution, T' also has a hyper-Erlang distribution.

The proof of the theorem is given in the "Appendix".

### 5.1 The CPLU model

In the CPLU model, there can be at most one phone call between two successive location updates, because each arriving phone call initiates a location update immediately. Consider any phone call C. Let  $T_c'$  be the time between the moment of the last location update before C arrives (when the current PA is established) and the moment when C arrives. Let  $X_s'$  denote the number of cell boundary crossings during the time interval of length  $T_c'$ . Let  $T_s'$  be the residual cell residence time of the cell where u resides when the last location update is performed.

**Theorem 7** In a TBLMS–CPLU, the probability that a mobile terminal is in ring r of a PA of radius d when a phone call arrives is

$$\xi_r = \sum_{j=0}^{\infty} \boldsymbol{P}[X'_s = j] p_{0r}^{(j)},$$

for all  $r \ge 0$ , where

$$\mathbf{P}[X'_{s}=j] = \int_{0}^{\infty} \left(F_{T'_{s}+(j-1)T_{s}}(t) - F_{T'_{s}+jT_{s}}(t)\right) f_{T'_{c}}(t) dt,$$

for all  $j \ge 0$ , and any probability distributions of  $T_c$  and  $T_s$ .

The proof of the theorem is given in the "Appendix".

It remains to find  $F_{T'_s+(j-1)T_s}(t)$ ,  $F_{T'_s+jT_s}(t)$ , and  $f_{T'_c}(t)$ . If  $T_s$  has an exponential distribution with

 $f_{T_s}(t) = \lambda_s e^{-\lambda_s t},$ 

then,  $T'_s$  and  $T_s$  have the same pdf, and both  $T'_s + (j-1)T_s$ and  $T'_s + jT_s$  have Erlang distributions, and we have

$$F_{T'_s+(j-1)T_s}(t) = F_{\text{Erlang}}(\lambda_s, j, t)$$

and

$$F_{T'_s+jT_s}(t) = F_{\text{Erlang}}(\lambda_s, j+1, t),$$

for all  $j \ge 0$ . (Notice that  $F_{\text{Erlang}}(\lambda, 0, t) = 1$ .)

There are two types of location update in a TBLMS. A *regular location update* (RLU) is performed every  $\tau$  units of time in a TBLMS–CPLU or a TBLMS–CWLU. An *irregular location update* (ILU) is performed by an arriving phone call in a TBLMS–CPLU.

As for  $f_{T'_{t}}(t)$ , we consider two cases.

• *Case 1*. If the last location update is irregular,  $f_{T'_c}(t)$  is simply  $f_{T_c}(t)$ . In Fig. 3, the time between ILU<sub>2</sub> and  $C_3$  is actually the time between  $C_2$  and  $C_3$ , i.e.,  $T_c$ . We will consider a hyper-Erlang distribution of  $T_c$  with pdf

$$f_{T_c}(t) = \sum_{i=1}^{k_c} w_{c,i} \left( \frac{\lambda_{c,i} e^{-\lambda_{c,i} t} (\lambda_{c,i} t)^{\gamma_{c,i}-1}}{(\gamma_{c,i}-1)!} \right)$$
$$= \sum_{i=1}^{k_c} w_{c,i} f_{\text{Erlang}}(\lambda_{c,i}, \gamma_{c,i}, t),$$

where  $w_{c,1} + w_{c,2} + \cdots + w_{c,k_c} = 1$ . Since we have the constraint  $0 \le T_c < \tau$ , the actual pdf of  $T_c$  is a truncated hyper-Erlang distribution, that is,

$$f_{T_c}(t) = \frac{1}{F_{T_c}(\tau)} \sum_{i=1}^{k_c} w_{c,i} f_{\text{Erlang}}(\lambda_{c,i}, \gamma_{c,i}, t), \quad 0 \le t < \tau,$$

and the cdf is

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$$F_{T_c}(t) = \frac{1}{F_{T_c}(\tau)} \sum_{i=1}^{k_c} w_{c,i} F_{\text{Erlang}}(\lambda_{c,i}, \gamma_{c,i}, t), \quad 0 \le t < \tau$$

where

$$F_{T_c}(\tau) = \sum_{i=1}^{k_c} w_{c,i} F_{\text{Erlang}}(\lambda_{c,i}, \gamma_{c,i}, \tau).$$

• *Case 2.* If the last location update is regular,  $T'_c$  is the residual time of  $T_c$ . In Fig. 3, the time between RLU<sub>1</sub> and  $C_1$  and the time between RLU<sub>3</sub> and  $C_2$  are residual times of  $T_c$ . By Theorem 6,  $T'_c$  also has a hyper-Erlang distribution, i.e.,

$$f_{T_c'}(t) = \left(\sum_{i=1}^{k_c} w_{c,i}\left(\frac{\gamma_{c,i}}{\lambda_{c,i}}\right)\right)^{-1} \sum_{i=1}^{k_c} \sum_{j=0}^{\gamma_{c,i}-1} \frac{w_{c,i}}{\lambda_{c,i}} \left(\frac{\lambda_{c,i}e^{-\lambda_{c,i}t}(\lambda_{c,i}t)^j}{j!}\right)$$

or,

$$f_{T_c'}(t) = \sum_{i=1}^{k_c} \sum_{j=1}^{\gamma_{c,i}} w_{c,i,j} f_{\mathrm{Erlang}}(\lambda_{c,i},j,t),$$

where

$$w_{c,i,j} = \frac{w_{c,i}}{\lambda_{c,i}} \left( \sum_{i=1}^{k_c} w_{c,i} \left( \frac{\gamma_{c,i}}{\lambda_{c,i}} \right) \right)^{-1},$$

for all  $1 \le i \le k_c$  and  $1 \le j \le \gamma_{c,i}$ . Since we have the constraint  $0 \le T'_c < \tau$ , the actual pdf of  $T'_c$  is a truncated hyper-Erlang distribution, that is,

$$f_{T_c'}(t) = \frac{1}{F_{T_c'}(\tau)} \sum_{i=1}^{k_c} \sum_{j=1}^{\gamma_{c,i}} w_{c,i,j} f_{\text{Erlang}}(\lambda_{c,i}, j, t), \quad 0 \le t < \tau,$$

**Fig. 4** Illustration of various time variables in a TBLMS–CWLU

and the cdf is

$$F_{T_c'}(t) = \frac{1}{F_{T_c'}(\tau)} \sum_{i=1}^{k_c} \sum_{j=1}^{\gamma_{c,i}} w_{c,i,j} F_{\text{Erlang}}(\lambda_{c,i}, j, t), \quad 0 \le t < \tau,$$

where

$$F_{T_c'}(\tau) = \sum_{i=1}^{k_c} \sum_{j=1}^{\gamma_{c,i}} w_{c,i,j} F_{\text{Erlang}}(\lambda_{c,i,j},\tau).$$

Notice that if  $T_c$  has an exponential distribution, the above two cases are identical.

By applying Theorem 7, we obtain the following result.

**Theorem 8** In a TBLMS–CPLU where  $T_c$  has a hyper-Erlang distribution and  $T_s$  has an exponential distribution, if the last location update is irregular, we have

$$\begin{split} \boldsymbol{P}[X'_{s} = j] &= \frac{1}{F_{T_{c}}(\tau)} \sum_{i=1}^{k_{c}} w_{c,i} \binom{j + \gamma_{c,i} - 1}{j} \\ &\times \frac{\lambda_{s}^{i} \lambda_{c,i}^{\gamma_{c,i}}}{(\lambda_{s} + \lambda_{c,i})^{j + \gamma_{c,i}}} F_{\text{Erlang}}(\lambda_{s} + \lambda_{c,i}, j + \gamma_{c,i}, \tau), \end{split}$$

for all  $j \ge 0$ ; if the last location update is regular, we have

$$\begin{split} \boldsymbol{P}[X'_{s} = j] &= \frac{1}{F_{T'_{c}}(\tau)} \sum_{i=1}^{k_{c}} \sum_{j'=1}^{\gamma_{c,i}} w_{c,i,j'} \binom{j+j'-1}{j} \\ &\times \frac{\lambda_{s}^{j} \lambda_{c,i}^{j'}}{(\lambda_{s} + \lambda_{c,i})^{j+j'}} F_{\mathrm{Erlang}}(\lambda_{s} + \lambda_{c,i}, j+j', \tau), \end{split}$$



## **Table 1** Location distribution in a TBLMS ( $q = 3, d = 20, \tau = N(d)E(T_s)$ )

r	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 4$	$\gamma = 5$
0	0.1269669526	0.0316208009	0.0152012603	0.0100878250	0.0076476716
1	0.2379818870	0.1325171146	0.0813558376	0.0574588885	0.0444219374
2	0.2113745807	0.1759141521	0.1292184269	0.0983939270	0.0787171743
3	0.1505111128	0.1659198101	0.1425953938	0.1180801994	0.0989107767
4	0.1005693766	0.1378084699	0.1359620925	0.1225210674	0.1081131781
5	0.0650011430	0.1064082527	0.1186599606	0.1160673221	0.1081757954
6	0.0411635041	0.0783368810	0.0974931321	0.1031224271	0.1015791171
7	0.0257108343	0.0557581362	0.0766402398	0.0873010171	0.0908397754
8	0.0159003739	0.0387006939	0.0582402271	0.0711533572	0.0781168304
9	0.0097598947	0.0263419581	0.0430834114	0.0562347471	0.0650389388
10	0.0059557979	0.0176520803	0.0311801771	0.0433227318	0.0526917474
11	0.0036173165	0.0116783712	0.0221569411	0.0326598094	0.0416949177
12	0.0021884150	0.0076434714	0.0155011164	0.0241622788	0.0323136637
13	0.0013194128	0.0049558984	0.0106962053	0.0175758354	0.0245701398
14	0.0007927950	0.0031852603	0.0072855570	0.0125794085	0.0183371279
15	0.0004743201	0.0020277962	0.0048939768	0.0088472135	0.0134080728
16	0.0002815529	0.0012735071	0.0032267937	0.0060806314	0.0095441288
17	0.0001639104	0.0007788205	0.0020583939	0.0040200303	0.0065018859
18	0.0000900893	0.0004452933	0.0012168864	0.0024441115	0.0040462196
19	0.0000400941	0.0002034067	0.0005681045	0.0011618483	0.0019523086
	0.9998633636	0.9991701749	0.9972341344	0.9932746767	0.9866214074

## **Table 2** Location distribution in a TBLMS $(q = 4, d = 20, \tau = N(d)E(T_s))$

r	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 4$	$\gamma = 5$
0	0.1188130166	0.0266219071	0.0123435445	0.0082726612	0.0063732545
1	0.2455545457	0.1392223690	0.0873256664	0.0629245830	0.0494665111
2	0.2143494265	0.1801225552	0.1341007300	0.1035840536	0.0839728785
3	0.1488611337	0.1643139140	0.1420891736	0.1187064509	0.1003891567
4	0.0988198534	0.1346914385	0.1328985758	0.1202196445	0.1066976132
5	0.0640487405	0.1037496052	0.1151245998	0.1125374607	0.1051108832
6	0.0409013088	0.0767005462	0.0945883023	0.0995944380	0.0979797323
7	0.0258575169	0.0550683991	0.0747388950	0.0844562386	0.0874932099
8	0.0162281764	0.0386783805	0.0573003885	0.0692365221	0.0754609226
9	0.0101287341	0.0267053017	0.0428856204	0.0552133203	0.0632277878
10	0.0062944774	0.0181866590	0.0314699033	0.0430253533	0.0516887076
11	0.0038979827	0.0122456816	0.0227136429	0.0328724925	0.0413588851
12	0.0024067664	0.0081664852	0.0161614359	0.0246840543	0.0324643087
13	0.0014820310	0.0053999053	0.0113530205	0.0182440032	0.0250297944
14	0.0009099593	0.0035412812	0.0078769437	0.0132754758	0.0189529924
15	0.0005563721	0.0023004933	0.0053899031	0.0094924434	0.0140603704
16	0.0003373244	0.0014732742	0.0036172193	0.0066272992	0.0101456386
17	0.0002002347	0.0009170414	0.0023440452	0.0044420434	0.0069931924
18	0.0001118102	0.0005317318	0.0014027799	0.0027289213	0.0043897799
19	0.0000502467	0.0002449142	0.0006594381	0.0013045451	0.0021274108
_	0.9998096574	0.9988818837	0.9963838281	0.9914420044	0.9833830301

## **Table 3** Location distribution in a TBLMS ( $q = 3, d = 20, \tau = dE(T_s)$ )

r	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 4$	$\gamma = 5$
0	0.1451408693	0.0463654054	0.0279904973	0.0224449421	0.0200510791
1	0.2654106639	0.1832957693	0.1410717850	0.1213600906	0.1110280720
2	0.2272954821	0.2262169726	0.2040920475	0.1880410197	0.1776706033
3	0.1531044358	0.1931059486	0.1971660548	0.1938331757	0.1898245179
4	0.0944343143	0.1408298243	0.1584456469	0.1648736949	0.1671472173
5	0.0547048517	0.0923265218	0.1121612310	0.1224924073	0.1281519421
6	0.0300172794	0.0556436644	0.0718637631	0.0817408210	0.0879307391
7	0.0156498890	0.0311909288	0.0423149468	0.0498002843	0.0548871177
8	0.0077616952	0.0163738010	0.0231176821	0.0279967644	0.0315118297
9	0.0036639436	0.0080861432	0.0117952367	0.0146328033	0.0167719115
10	0.0016469645	0.0037690408	0.0056478228	0.0071505803	0.0083256638
11	0.0007053410	0.0016624567	0.0025475122	0.0032816077	0.0038732613
12	0.0002880008	0.0006954448	0.0010858574	0.0014195789	0.0016954679
13	0.0001122100	0.0002764561	0.0004385480	0.0005806539	0.0007007002
14	0.0000417568	0.0001046236	0.0001682204	0.0002251872	0.0002742130
15	0.0000148571	0.0000377582	0.0000614162	0.0000830022	0.0001018805
16	0.0000050596	0.0000130153	0.0000213828	0.0000291400	0.0000360202
17	0.0000016505	0.0000042901	0.0000071094	0.0000097597	0.0000121394
18	0.0000005139	0.0000013477	0.0000022501	0.0000031085	0.0000038877
19	0.0000001435	0.000003788	0.000006362	0.000008834	0.0000011097
	0.9999999220	0.9999997916	0.9999996465	0.9999995050	0.9999993736

## **Table 4** Location distribution in a TBLMS $(q = 4, d = 20, \tau = dE(T_s))$

r	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 4$	$\gamma = 5$
0	0.1360188863	0.0391722458	0.0224505220	0.0178040069	0.0159059895
1	0.2732451807	0.1906628476	0.1479783148	0.1280795456	0.1176694267
2	0.2298971745	0.2296178636	0.2078880677	0.1920980556	0.1819099980
3	0.1512096441	0.1901994194	0.1940298392	0.1907530428	0.1868718771
4	0.0929658723	0.1377095488	0.1543313600	0.1602394495	0.1622463891
5	0.0543337175	0.0908779605	0.1097635047	0.1194278854	0.1246405386
6	0.0303469821	0.0556906676	0.0714337902	0.0808718200	0.0867113848
7	0.0162237301	0.0320009349	0.0430974250	0.0504570071	0.0553998582
8	0.0083044473	0.0173412153	0.0243030147	0.0292721682	0.0328126863
9	0.0040698362	0.0088950130	0.0128814706	0.0158930167	0.0181396600
10	0.0019095705	0.0043303445	0.0064440169	0.0081149271	0.0094086846
11	0.0008579019	0.0020050131	0.0030523296	0.0039115958	0.0045977488
12	0.0003691579	0.0008845049	0.0013725812	0.0017855935	0.0021240821
13	0.0001522171	0.0003723554	0.0005872985	0.0007739860	0.0009304184
14	0.0000601808	0.0001498038	0.0002395853	0.0003193149	0.0003874120
15	0.0000228302	0.0000576755	0.0000933516	0.0001256437	0.0001536860
16	0.0000083166	0.0000212770	0.0000347965	0.0000472376	0.0000581993
17	0.0000029101	0.0000075261	0.0000124194	0.0000169877	0.0000210644
18	0.0000009729	0.0000025393	0.0000042229	0.0000058142	0.0000072500
19	0.0000002892	0.0000007603	0.0000012720	0.0000017603	0.0000022047
	0.9999998184	0.9999995167	0.9999991828	0.9999988585	0.9999985587

for all  $j \geq 0$ .

The proof of the theorem is given in the "Appendix".

## 5.2 The CWLU model

In the CWLU model, there can be many phone calls between two successive location updates. Let  $C_1, C_2, ..., C_{\gamma}, ...$  be a sequence of phone calls between two successive RLUs *U* and *U'* (see Fig. 4).

Let  $T_{c, \gamma}$  be the inter-call time between  $C_{\gamma-1}$  and  $C_{\gamma}$ , where  $\gamma \geq 2$ , and

$$T_c(\gamma) = T_{c,1} + T_{c,2} + \dots + T_{c,\gamma}$$

denote the time between the moment of the last location update before  $C_{\gamma}$  arrives (when the current PA is established) and the moment when  $C_{\gamma}$  arrives, where  $\gamma \ge 1$ , and  $T_{c,1} = T'_c$ . Let  $X_s(\gamma)$  denote the number of cell boundary crossings during the time interval of length  $T_c(\gamma)$ .

Again, by a proof similar to that of Theorem 7, we have the following result.

**Theorem 9** In a TBLMS–CWLU, the probability that a mobile terminal is in ring r of a PA of radius d when the yth phone call arrives is

$$\xi_r(\gamma) = \sum_{j=0}^{\infty} \boldsymbol{P}[X_s(\gamma) = j] p_{0r}^{(j)},$$

**Table 5** Reachability  $I(\tau, d, \gamma)$  in a TBLMS  $(q = 3, \tau = N(d)E(T_s))$ 

for all  $r \ge 0$  and  $\gamma \ge 1$ , where

$$\boldsymbol{P}[X_{s}(\gamma)=j] = \int_{0}^{\infty} \left(F_{T'_{s}+(j-1)T_{s}}(t) - F_{T'_{s}+jT_{s}}(t)\right) f_{T_{c}(\gamma)}(t) dt,$$

for all  $j \ge 0$ , and any probability distributions of  $T_c$  and  $T_s$ .

If  $T_c$  has a hyper-Erlang distribution, we have  $T_c(1) = T_{c,1} = T'_c$  and we can get the same result as Case 2 of the CPLU model for  $\gamma = 1$ , i.e., the first phone call. However, for  $\gamma \ge 2$ ,  $T_c(\gamma)$  has a complicated distribution which is not hyper-Erlang anymore. To have unified discussion for all  $\gamma \ge 1$ , we consider an exponential distribution of  $T_c$  with pdf

$$f_{T_c}(t) = \lambda_c e^{-\lambda_c t}.$$

Because all the  $T_{c, \gamma}$ 's have the same pdf, where  $\gamma \ge 1$ ,  $T_c(\gamma)$  has an Erlang distribution,

$$f_{T_c(\gamma)}(t) = f_{\text{Erlang}}(\lambda_c, \gamma, t).$$

Since we have the constraint  $0 \le T_c(\gamma) < \tau$ , the actual pdf of  $T_c(\gamma)$  is a truncated Erlang distribution. that is,

$$f_{T_c(\gamma)}(t) = \frac{1}{F_{T_c(\gamma)}(\tau)} f_{\text{Erlang}}(\lambda_c, \gamma, t), \quad 0 \le t < \tau,$$

and the cdf is

$$F_{T_c(\boldsymbol{\gamma})}(t) = \frac{1}{F_{T_c(\boldsymbol{\gamma})}(\tau)} F_{\text{Erlang}}(\lambda_c, \boldsymbol{\gamma}, t), \quad 0 \leq t < \tau,$$

d	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 4$	$\gamma = 5$
1	0.6517126530	0.5524613717	0.5081425054	0.4835902754	0.4681495833
2	0.7580215514	0.6582760034	0.6086962580	0.5801767140	0.5620016526
3	0.8103915353	0.7117367466	0.6592007765	0.6283810136	0.6086904610
4	0.8495806141	0.7511260248	0.6944374990	0.6603590315	0.6384936004
5	0.8843756544	0.7877709218	0.7260701363	0.6874858548	0.6624424463
6	0.9155096608	0.8248597266	0.7586404676	0.7145581660	0.6852256537
7	0.9416347124	0.8620945937	0.7939329391	0.7442486745	0.7096617466
8	0.9616781634	0.8971058038	0.8315204447	0.7777912458	0.7376590079
9	0.9757845863	0.9270826587	0.8689632557	0.8146796369	0.7701214554
10	0.9850689702	0.9504153223	0.9029059485	0.8524741172	0.8063698629
11	0.9909191253	0.9672204310	0.9308359373	0.8876798908	0.8438596665
12	0.9945168632	0.9787048143	0.9520683513	0.9174894651	0.8790743859
13	0.9967025373	0.9863140214	0.9673664077	0.9408799431	0.9092470995
14	0.9980222183	0.9912691561	0.9780426063	0.9583326400	0.9333521576
15	0.9988161809	0.9944623845	0.9853557776	0.9709730229	0.9517706748
16	0.9992926428	0.9965049738	0.9903046209	0.9799649592	0.9654810322
17	0.9995779808	0.9978037113	0.9936224170	0.9862818336	0.9755202197
18	0.9997485571	0.9986252106	0.9958292005	0.9906746790	0.9827818504
19	0.9998503689	0.9991424568	0.9972867724	0.9937025341	0.9879802267
20	0.9999110526	0.9994667971	0.9982434539	0.9957729257	0.9916671578

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		-			
d	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 4$	$\gamma = 5$
1	0.6482069710	0.5473990406	0.5022343759	0.4771594572	0.4613666918
2	0.7616692478	0.6629015490	0.6135804994	0.5851152325	0.5669319264
3	0.8128179149	0.7156847270	0.6639874591	0.6336643199	0.6142927949
4	0.8503013080	0.7533033450	0.6977600759	0.6644788228	0.6431685452
5	0.8838182428	0.7883266456	0.7279436134	0.6904085618	0.6661337187
6	0.9140917196	0.8239116701	0.7589954783	0.7161726264	0.6878333949
7	0.9397881360	0.8598562309	0.7926808304	0.7443502472	0.7109791181
8	0.9597892886	0.8939902782	0.8287260230	0.7761870313	0.7374067905
9	0.9741087943	0.9236320333	0.8649794634	0.8113633534	0.7680566088
10	0.9837130672	0.9471107404	0.8983266068	0.8477781010	0.8024736580
11	0.9898860241	0.9643506349	0.9262683970	0.8822318780	0.8384915883
12	0.9937604597	0.9763693069	0.9479316964	0.9119611122	0.8729025134
13	0.9961638911	0.9844944275	0.9638530685	0.9357564159	0.9029698006
14	0.9976464899	0.9898946991	0.9751855472	0.9538612317	0.9274701442
15	0.9985583273	0.9934483496	0.9831041728	0.9672275705	0.9465491100
16	0.9991180232	0.9957709116	0.9885729732	0.9769211064	0.9610162273
17	0.9994610365	0.9972806577	0.9923170825	0.9838670239	0.9718092826
18	0.9996709754	0.9982574866	0.9948617696	0.9887970674	0.9797679212
19	0.9997993173	0.9988869117	0.9965801627	0.9922675585	0.9855798587
20	0.9998776948	0.9992909900	0.9977338601	0.9946925248	0.9897875252

## **Table 6** Reachability $I(\tau, d, \gamma)$ in a TBLMS $(q = 4, \tau = N(d)E(T_s))$

## **Table 7** Reachability $I(\tau, d, \gamma)$ in a TBLMS $(q = 3, \tau = dE(T_s))$

d	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 4$	$\gamma = 5$
1	0.6517126530	0.5524613717	0.5081425054	0.4835902754	0.4681495833
2	0.8617217259	0.8004687967	0.7677351011	0.7477982610	0.7345213627
3	0.9453282184	0.9147213397	0.8963020937	0.8842582982	0.8758555161
4	0.9775711077	0.9628895008	0.9532646666	0.9466201439	0.9418084382
5	0.9905526960	0.9835958298	0.9787222349	0.9752081578	0.9725839992
6	0.9959435809	0.9926577081	0.9902276581	0.9884110093	0.9870186621
7	0.9982329419	0.9966808287	0.9954789762	0.9945523138	0.9938259233
8	0.9992215644	0.9984874181	0.9978956422	0.9974268708	0.9970520587
9	0.9996540145	0.9993061420	0.9990154696	0.9987796020	0.9985876420
10	0.9998451104	0.9996799630	0.9995373738	0.9994191163	0.9993213173
11	0.9999302473	0.9998517048	0.9997818015	0.9997226529	0.9996730134
12	0.9999684320	0.9999310161	0.9998967544	0.9998672187	0.9998420923
13	0.9999856537	0.9999678030	0.9999510102	0.9999362782	0.9999235856
14	0.9999934571	0.9999849295	0.9999766977	0.9999693552	0.9999629533
15	0.9999970069	0.9999929284	0.9999888923	0.9999852348	0.9999820095
16	0.9999986272	0.9999966746	0.9999946953	0.9999928740	0.9999912505
17	0.9999993689	0.9999984333	0.9999974624	0.9999965557	0.9999957390
18	0.9999997093	0.9999992607	0.9999987843	0.9999983330	0.9999979224
19	0.9999998659	0.9999996506	0.9999994168	0.9999991922	0.9999989859
20	0.9999999380	0.9999998347	0.9999997199	0.9999996081	0.9999995045

where

$$F_{T_c(\gamma)}(\tau) = F_{\text{Erlang}}(\lambda_c, \gamma, \tau).$$

By applying Theorem 9, we obtain the following result, which can be proven in a way similar to that of Theorem 8.

**Theorem 10** In a TBLMS–CWLU where both  $T_c$  and  $T_s$  have exponential distributions, we have

$$\begin{split} \boldsymbol{P}[X_s(\gamma) = j] \\ &= \binom{j+\gamma-1}{j} \frac{\lambda_s^j \lambda_c^\gamma}{(\lambda_s + \lambda_c)^{j+\gamma}} \\ &\cdot \frac{F_{\text{Erlang}}(\lambda_s + \lambda_c, j+\gamma, \tau)}{F_{\text{Erlang}}(\lambda_c, \gamma, \tau)}, \end{split}$$

for all  $j \ge 0$ .

## 5.3 Numerical Data

In this section, we present numerical data.

We consider an exponential distribution of  $T_c$  with

$$f_{T_c}(t) = \lambda_c e^{-\lambda_c t},$$

and an exponential distribution of  $T_s$  with

 $f_{T_s}(t) = \lambda_s e^{-\lambda_s t}.$ 

In Tables 1, 2, 3 and 4, we show  $\xi_r$ , the probability that a mobile terminal is in ring *r* of a PA of radius *d* when the  $\gamma$ th phone call arrives in a TBLMS–CWLU, where d = 20,

**Table 8** Reachability  $I(\tau, d, \gamma)$  in a TBLMS  $(q = 4, \tau = dE(T_s))$ 

 $\lambda_c = 1$ , and  $\lambda_s = 10$ . The data are calculated by using Theorem 10. In Tables 1 and 2, we set  $\tau = N(d)E(T_s)$ , i.e., the location update time is the expected PA residence time and the expected location update time in a DBLMS. In Tables 3 and 4, we set  $\tau = dE(T_s)$ , i.e., the location update time is the expected location update time in an MBLMS.

It is noticed that when  $\gamma = 1$ , we have  $I(\tau, r, 1) = I(\tau, r)$ , i.e., the probability that a mobile terminal is in a PA of radius *r* when a phone call arrives in a TBLMS–CPLU obtained by Theorem 8. Therefore, the column for  $\gamma = 1$  in each table also gives the location distribution when a phone call arrives in a TBLMS–CPLU.

It is observed that in the case where  $\tau = dE(T_s)$ , the probability that a mobile terminal is far away from the center of a PA is less than the case where  $\tau = N(d)E(T_s)$ . Also, in the case where q = 4, a mobile terminal moves more quickly away from the center of a PA than the case where q = 3.

We emphasize that there is a major difference between a location distribution in a TBLMS and a location distribution in a DBLMS or an MBLMS. In a TBLMS, when a phone call arrives, a mobile terminal can reside in any ring  $r \ge 0$ , that is, there is no guarantee to find a mobile terminal no matter how large a PA is. However, in a DBLMS or an MBLMS, when a phone call arrives, a mobile terminal can only reside in rings  $0 \le r \le d - 1$ , where *d* is the radius of a PA, that is, it is guaranteed that a mobile terminal can be found in a PA of radius *d*. This implies that

$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 4$	$\gamma = 5$
0.6482069710	0.5473990406	0.5022343759	0.4771594572	0.4613666918
0.8606245304	0.7987926622	0.7657208460	0.7455675237	0.7321420673
0.9446148467	0.9136432530	0.8950214762	0.8828542221	0.8743703986
0.9767097876	0.9615126311	0.9515742408	0.9447264179	0.9397750511
0.9898488840	0.9824076088	0.9772132836	0.9734785633	0.9706959926
0.9954651211	0.9918115472	0.9891213414	0.9871172834	0.9855857048
0.9979373158	0.9961363369	0.9947486162	0.9936829501	0.9928503752
0.9990488615	0.9981574835	0.9974427565	0.9968790783	0.9964300395
0.9995566800	0.9991138291	0.9987458380	0.9984486222	0.9982076889
0.9997915816	0.9995708179	0.9993812908	0.9992248683	0.9990960439
0.9999013222	0.9997909398	0.9996932650	0.9996110296	0.9995423122
0.9999530054	0.9998976688	0.9998472912	0.9998040829	0.9997674876
0.9999775085	0.9999497038	0.9999236987	0.9999010018	0.9998815350
0.9999891905	0.9999751917	0.9999617566	0.9999498347	0.9999394873
0.9999947862	0.9999877261	0.9999807795	0.9999745168	0.9999690196
0.9999974774	0.9999939114	0.9999903170	0.9999870266	0.9999841070
0.9999987762	0.9999969727	0.9999951115	0.9999933824	0.9999918322
0.9999994049	0.9999984918	0.9999975275	0.9999966186	0.9999957956
0.9999997100	0.9999992473	0.9999987474	0.9999982695	0.9999978326
0.9999998584	0.9999996237	0.9999993645	0.9999991132	0.9999988812
	$\begin{split} \gamma &= 1 \\ \hline 0.6482069710 \\ \hline 0.8606245304 \\ \hline 0.9446148467 \\ \hline 0.9767097876 \\ \hline 0.9898488840 \\ \hline 0.9954651211 \\ \hline 0.9979373158 \\ \hline 0.99990488615 \\ \hline 0.99990488615 \\ \hline 0.9999013222 \\ \hline 0.9999913222 \\ \hline 0.9999913054 \\ \hline 0.99999715085 \\ \hline 0.99999775085 \\ \hline 0.999999775085 \\ \hline 0.99999775085 \\ \hline 0.999999775085 \\ \hline 0.99999977508 \\ \hline 0.999999977508 \\ \hline 0.99999999977508 \\ \hline 0.9999999977508 \\ \hline 0.999999977508 \\ \hline 0.9999999977508 \\ \hline 0.9999999999977508 \\ \hline 0.9999999977508 \\ \hline 0.9999999999977508 \\ \hline 0.99999999977508 \\ \hline 0.9999999999977508 \\ \hline 0.9999999999999977508 \\ \hline 0.99999999977508 \\ \hline 0.9999999999977508 \\ \hline 0.9999999977508 \\ \hline 0.999999999999977508 \\ \hline 0.999999999999977508 \\ \hline 0.99999999999999999999999999999999999$	$\gamma = 1$ $\gamma = 2$ 0.64820697100.54739904060.86062453040.79879266220.94461484670.91364325300.97670978760.96151263110.98984888400.98240760880.99546512110.99181154720.99793731580.99613633690.99904886150.99815748350.99955668000.99911382910.99997158160.99957081790.99990132220.99979093980.99997750850.9999766880.99997750850.99997519170.99999478620.99998772610.9999977620.99999849180.9999971000.9999924730.9999985840.999996237	$\gamma = 1$ $\gamma = 2$ $\gamma = 3$ 0.64820697100.54739904060.50223437590.86062453040.79879266220.76572084600.94461484670.91364325300.89502147620.97670978760.96151263110.95157424080.98984888400.98240760880.97721328360.99546512110.99181154720.98912134140.99793731580.99613633690.99474861620.99904886150.99911382910.99874583800.99979158160.99957081790.99938129080.99990132220.99979093980.99969326500.99997750850.99997750850.99998766880.99999715820.9999713810.99992369870.999997750850.99997519170.99998077550.999997750850.99998772610.9999031700.99999775020.99999697270.99999511150.9999971000.9999924730.99999752750.9999971000.999992370.999993645	$\gamma = 1$ $\gamma = 2$ $\gamma = 3$ $\gamma = 4$ 0.64820697100.54739904060.50223437590.47715945720.86062453040.79879266220.76572084600.74556752370.94461484670.91364325300.89502147620.88285422210.97670978760.96151263110.95157424080.94472641790.98984888400.98240760880.97721328360.97347856330.99546512110.99181154720.98912134140.98711728340.99793731580.99613633690.99474861620.99368295010.99904886150.99815748350.99744275650.99687907830.99955668000.99911382910.99874583800.99844862220.9997158160.99957081790.99938129080.99922486830.99990132220.9997093980.9996326500.99961102960.99995300540.9999470380.99992369870.99990100180.99999750850.9999775060.99999100180.99999775660.99999747740.9999931140.99999031700.99998702660.99999747740.99999617270.9999901750.9999938240.99999971000.9999924730.9999974740.99999826950.9999971000.9999924730.9999974740.99999926550.9999985840.9999924730.99999364550.9999991132

for a PA with radius *d*, we have  $\xi_0 + \xi_1 + \xi_2 + \cdots + \xi_{d-1} = I(\tau, d, \gamma) < 1$  in a TBLMS, but  $\xi_0 + \xi_1 + \xi_2 + \cdots + \xi_{d-1} = 1$  in a DBLMS or an MBLMS. The last line in each table gives  $I(\tau, d, \gamma) = \xi_0 + \xi_1 + \xi_2 + \cdots + \xi_{d-1}$ .

## 6 Reachability in a PA

In a time-based location management scheme TBLMS  $(\tau, d)$ , the location of a mobile terminal is updated every  $\tau$  units of time, and the PA has radius *d*. Since it is not guaranteed that a mobile terminal can be found in the current PA no matter what the values of  $\tau$  and *d* are, QoS becomes an important issue for a TBLMS in addition to cost concern.

The following two theorems give the probability that a mobile terminal can be reached, i.e., it is still in a PA, when the next phone call arrives.

**Theorem 11** In a TBLMS–CPLU, the probability  $I(\tau, d)$  that a mobile terminal is in a PA of radius d when a phone call arrives is

$$I(\tau,d) = \sum_{r=0}^{d-1} \xi_r,$$

for all  $d \geq 1$ .

The proof of the above theorem follows the definition of  $\xi_r$ .

**Theorem 12** In a TBLMS–CWLU, the probability  $I(\tau, d, \gamma)$  that a mobile terminal is in a PA of radius d when the  $\gamma$ th phone call arrives is

$$I(\tau, d, \gamma) = \sum_{r=0}^{d-1} \xi_r(\gamma),$$
  
for all  $d \ge 1$  and  $\gamma \ge 1$ .

The proof of the above theorem follows the definition of  $\xi_r(\gamma)$ .

A time-based location management scheme is acceptable to a mobile terminal u if the probability  $I(\tau, d)$  (given by Theorem 11) or  $I(\tau, d, \gamma)$  (given by Theorem 12) that u is found in the current PA when a phone call arrives is above certain level Q. We define  $I(\tau, d)$  or  $I(\tau, d, \gamma)$  as a measure of QoS.

In Tables 5, 6, 7 and 8, we show reachability  $I(\tau, d, \gamma)$  calculated by using Theorem 12 with the same parameter setting as Tables 1, 2, 3 and 4. It is observed that as *d* increases, the chance to find a mobile terminal in a PA of radius *d* increases significantly.

d CPLU CWLU Analytical Relative differ (%) Analytical Relative differ (%) Simulation Simulation 1 20.50417 20.50680 0.01284 20.00000 19.95018 -0.249102 10.50833 10.47401 -0.3266210.00000 9.99221 -0.077903 7.17916 7.22726 0.66997 6.66667 6.68684 0.30260 4 0.40449 5.00000 5.00125 0.02500 5.51666 5.53897 5 4.52081 0.23266 4.00000 3.99060 4.53133 -0.23500-0.296146 3.85830 3.84687 3.33333 3.30958 -0.712607 3.38625 3.38546 -0.023332.85714 2.86595 0.30825 8 3.03324 3.03520 0.06446 2.50000 2.50828 0.33120 9 2.75960 2.76805 0.30634 2.22222 2.22693 0.21185 0.03525 2.00000 1.99204 10 2.54149 2.54239 -0.398000.26205 0.37830 11 2.36379 2.36998 1.81818 1.82506 12 -0.004480.08360 2.21637 2.21627 1.66667 1.66806 13 2.09225 2.09041 -0.087971.53846 1.53471 -0.243851.98643 -0.020331.42857 -0.3046014 1.98603 1.42422 15 1.89526 1.88918 -0.320541.33333 1.33263 -0.0527516 1.81597 1.81323 -0.150681.25000 1.24798 -0.1616017 -0.425901.74647 1.74981 0.19151 1.17647 1.17146 18 1.68512 1.68632 0.07135 1.11111 1.10430 -0.6130019 1.63063 1.62654 -0.250971.05263 1.04490 -0.7345020 1.58198 1.57963 -0.148341.00000 1.00116 0.11600

**Table 9** Comparison of analytical data and simulation results of  $E(X_u)$ 

## 7 Cost of location management

## 7.1 The CPLU model

It is a straightforward observation that in the CPLU model, since the arrival of a phone call always starts a new rhythm of location update, the number of location update in a time interval of length  $T_c$  is

$$X_u = \left\lfloor \frac{T_c}{\tau} \right\rfloor + 1$$

for any interval  $\tau$  of location update. The expected number of location update is

$$\begin{split} \boldsymbol{E}(X_u) &= \int_0^\infty \Big( \Big| \frac{t}{\tau} \Big| + 1 \Big) f_{T_c}(t) dt \leq \int_0^\infty \Big( \frac{t}{\tau} + 1 \Big) f_{T_c}(t) dt \\ &= \Big( \frac{\boldsymbol{E}(T_c)}{\tau} + 1 \Big). \end{split}$$

A more accurate analysis is the following.

**Theorem 13** The total cost of location management in a  $TBLMS(\tau, d)$  under the CPLU model is

$$C_{CPLU}(\tau, d) = \Delta_u \left( \sum_{h=0}^{\infty} (1 - F_{T_c}(h\tau)) \right) + \Delta_p (qd^2 - qd + 1),$$

for arbitrary  $T_c$ , with QoS  $I(\tau, d)$  given by Theorem 11.

The proof of the theorem is given in the "Appendix".

For example, let us consider an exponential distribution of  $T_c$  with

$$f_{T_c}(t) = \lambda_c e^{-\lambda_c t}.$$

Since  $R_{T_c}(h\tau) = e^{-\lambda_c h\tau}$ , we have

$$\sum_{h=0}^{\infty} R_{T_c}(h au) = \sum_{h=0}^{\infty} e^{-\lambda_c h au} = rac{1}{1-e^{-\lambda_c au}}$$

This result is a special case of the next theorem.

To have an analysis using a renewal process, we notice that the sequence of location update times is a sequence of deterministic values of  $\tau$ , which can be viewed as i.i.d. random variables whose pdf is a unit impulse function with

$$f_{T_{u}}^{*}(s) = e^{-\tau s}.$$

A sequence of location update times between two consecutive phone calls is an ordinary renewal process. The expected number of location update  $E(X_u) = E(X) + 1$ , where E(X) can be obtained by using Theorem 3. Thus, we have the following theorem.

**Theorem 14** If  $T_c$  has a hyper-Erlang distribution, the total cost of location management in a TBLMS( $\tau$ , d) under the CPLU model is

$$C_{CPLU}(\tau, d) = \Delta_u(\boldsymbol{E}(X) + 1) + \Delta_p(qd^2 - qd + 1),$$

where

$$\boldsymbol{E}(\boldsymbol{X}) = \sum_{i=1}^{k_c} w_{c,i} \left( \frac{\lambda_{c,i}^{\gamma_{c,i}}}{(\gamma_{c,i}-1)!} \right) \left( -\frac{\partial}{\partial s} \right)^{\gamma_{c,i}-1} \left[ \frac{e^{-\tau s}}{s(1-e^{-\tau s})} \right]_{s=\lambda_{c,i}},$$

with QoS  $I(\tau, d)$  given by Theorem 11.





## 7.2 The CWLU model

In the CWLU model, a phone call arrives into the midst of a time interval  $\tau$ . A sequence of location update times between two consecutive phone calls is an equilibrium renewal process. By using Theorem 4, we get the following result.

**Theorem 15** For any probability distribution of  $T_c$ , the total cost of location management in a  $TBLMS(\tau, d)$  under the CWLU model is

$$C_{CWLU}(\tau, d) = rac{\Delta_u}{\lambda_{T_c} \tau} + \Delta_p (qd^2 - qd + 1),$$

with QoS  $I(\tau, d, \gamma)$  given by Theorem 12.

Fig. 6 Location management

 $\tau = N(d)\boldsymbol{E}(T_s))$ 

cost in TBLMS–CWLU (q = 3,

## 8 Simulation results and numerical data

We consider an exponential distribution of  $T_c$  with

$$f_{T_c}(t) = \lambda_c e^{-\lambda_c t},$$

and an exponential distribution of  $T_s$  with

$$f_{T_s}(t) = \lambda_s e^{-\lambda_s t}$$

It is clear that  $\rho = \lambda_c / \lambda_s$ . By Theorems 14 and 15, the total cost of location management in a TBLMS  $(\tau, d)$  is

$$C_{\text{CPLU}}(\tau, d) = \frac{\Delta_u}{1 - e^{-\lambda_c \tau}} + \Delta_p (qd^2 - qd + 1),$$

under the CPLU model, and



**Fig. 7** Location management cost in TBLMS–CPLU (q = 4,  $\tau = N(d)E(T_s)$ )

$$C_{ ext{CWLU}}( au, d) = rac{\Delta_u}{\lambda_c au} + \Delta_p (q d^2 - q d + 1),$$

under the CWLU model. The QoS for an arriving phone call under the CPLU model or the  $\gamma$ th phone call under the CWLU model is given by Theorem 11 or Theorem 12.

### 8.1 Simulation results

Extensive simulations have been conducted to validate our analytical results on the expected number  $E(X_u)$  of location updates between two consecutive phone calls. In Table 9, we display and compare our analytical data and simulations results of  $E(X_u)$ . Notice that  $E(X_u)$  is independent of q, i.e., the hexagonal cell structure with q = 3or the mesh cell structure with q = 4. Our parameters are set as  $\lambda_c = 1$ ,  $\lambda_s = 20$ , and  $\tau = dE(T_s) = d/\lambda_s$ . The analytical data of  $E(X_u)$  for the CPLU model are calculated by using Theorem 14, and the analytical data of  $E(X_u)$  for the CWLU model are calculated by using Theorem 15. The simulation results of  $E(X_u)$  are obtained by simulating a mobile terminal with random inter-call





**Fig. 11** Location management cost in TBLMS–CPLU (q = 4,  $\tau = dE(T_s)$ )

Fig. 10 Location management

cost in TBLMS–CWLU (q = 3,

 $\tau = d\boldsymbol{E}(T_s))$ 

time and random cell residence time specified by the above pdfs  $f_{T_c}(t)$  and  $f_{T_s}(t)$ . For every pair of consecutive phone calls, we record the number of location updates between the two consecutive phone calls. We then report the average number of location updates between all consecutive phone calls. The number of phone calls is as large as 100,000, such that the maximum 99 % confidence interval of our simulation results is about  $\pm$  0.87565 %. The relative difference between a simulation result and its corresponding analytical datum is also given, where we observe that the relative differences of

all our simulation results are no more than  $\pm 0.73$  %. These simulation results validate the correctness of our analytical data.

To better illustrate our results, we show in the "Appendix" the expected number of location updates between two consecutive phone calls.

## 8.2 Numerical data

In this section, we demonstrate numerical data for location management costs in a TBLMS. We assume the  $\tau = d\boldsymbol{E}(T_s))$ 

following parameter settings:  $\lambda_c = 1, \lambda_s = 50.00, 25.00,$  $12.50, 6.25, \rho = 0.02, 0.04, 0.08, 0.16, \Delta_p = 1, \text{ and } \Delta_u =$ 30. In Figs. 5, 6, 7 and 8, we show our data for TBLMS-CPLU and q = 3, TBLMS-CWLU and q = 3, TBLMS-CPLU and q = 4, TBLMS–CWLU and q = 4, respectively, where  $\tau = E(T_m) = N(d)E(T_s)$ . In Figs. 9, 10, 11 and 12, we show our data for TBLMS–CPLU and q = 3, TBLMS– CWLU and q = 3, TBLMS-CPLU and q = 4, TBLMS-CWLU and q = 4, respectively, where  $\tau = dE(T_s)$ . The location update cycle  $\tau$  is set as  $\tau = E(T_m) = N(d)E(T_s)$ , where  $E(T_m)$  is the expected time for a mobile terminal to reach the boundary (i.e., ring d) of a PA, i.e., the expected PA residence time and the expected location update time in a DBLMS, or  $\tau = dE(T_s)$ , i.e., the expected location update time in an MBLMS.

When  $\tau = N(d)E(T_s)$ , the total cost of location management in a TBLMS  $(\tau, d)$  is

$$C_{ ext{CPLU}}(d) = rac{\Delta_u}{1-e^{-
ho N(d)}} + \Delta_p (qd^2-qd+1),$$

under the CPLU model, and





$$C_{\text{CWLU}}(d) = \frac{\Delta_u}{\rho N(d)} + \Delta_p (qd^2 - qd + 1),$$

under the CWLU model. When  $\tau = dE(T_s)$ , the total cost of location management in a TBLMS  $(\tau, d)$  is

$$C_{\text{CPLU}}(d) = \frac{\Delta_u}{1 - e^{-\rho d}} + \Delta_p (qd^2 - qd + 1),$$

under the CPLU model, and

$$C_{\text{CWLU}}(d) = rac{\Delta_u}{
ho d} + \Delta_p (qd^2 - qd + 1),$$

Fig. 14 Reachability in TBLMS (q = 4,  $\tau = N(d)E(T_s)$ ) under the CWLU model.

We have the following observations.

- When *d* is small, the total cost of location management is dominated by location update cost, which is determined by  $\Delta_u$  and  $\rho$ . An increased  $\Delta_u$  will increase the total cost of location management noticeably. Similarly, a decreased  $\rho$ , which is caused by an increased  $\lambda_s$ , will increase the total cost of location management noticeably.
- When *d* is large, the total cost of location management is dominated by terminal paging cost, which is





determined by  $\Delta_p$ , q, and d. An increased  $\Delta_p$  will increase the total cost of location management noticeably. Also, the different values of q = 3 and q = 4 give rise to noticeable difference in the total cost of location management.

- A TBLMS operated under the CWLU model has lower location update cost than a TBLMS operated under the CPLU model. This is due to the fact that 1 − e<sup>-x</sup> < x for all x > 0.
- There is an optimal value of  $d^*$  which minimizes  $C_{\text{CPLU}}(d)$  or  $C_{\text{CWLU}}(d)$ . However, such cost optimiza-

tion is not very useful, since there is no consideration of QoS. We will address this issue in the next section.

In addition to location management cost, QoS is another consideration in a TBLMS. We show reachability of the first phone call within a location update cycle in a TBLMS with the following parameter settings:  $\gamma = 1$ ,  $\lambda_c = 1$ ,  $\lambda_s = 50$ , 25, 12.5, 6.25,  $\rho = 0.02$ , 0.04, 0.08, 0.16. In Figs. 13, 14, 15 and 16, we show our data for q = 3 and  $\tau = N(d)E(T_s)$ , q = 4 and  $\tau = N(d)E(T_s)$ , q = 3 and  $\tau = dE(T_s)$ , q = 4 and  $\tau = dE(T_s)$ , respectively. All the data are cal-



culated by using Theorem 12. It is observed that when  $\tau$  is large, different  $\rho$  results in noticeable difference in the reachability, i.e., an increased  $\rho$  can significantly increase the QoS. However, when  $\tau$  is small, different  $\rho$  results in about the same reachability.

## 8.3 Cost minimization with guaranteed QoS

The performance (i.e., the cost of mobility management and the QoS) of a time-based location management scheme TBLMS ( $\tau$ , d) is determined by  $\tau$  and d. The time threshold  $\tau$  determines the cost of location update, i.e., the less  $\tau$  is, the

higher the cost is. The radius *d* determines the cost of terminal paging, i.e., the greater *d* is, the higher the cost is. Both  $\tau$  and *d* affect the QoS, i.e., the less  $\tau$  is, the higher the QoS is, and the greater *d* is, the higher the QoS is.

Both  $\tau$  and d can be determined by the required level Q of QoS. For a given  $\tau$  (i.e., fixed cost of location update), we can set d large enough (i.e., increasing the cost of paging) such that  $I(\tau, d) \ge Q$  or  $I(\tau, d, \gamma) \ge Q$ . Equivalently, for a given d (i.e., fixed cost of paging), we can set  $\tau$  small enough (i.e., increasing the cost of location update) such that  $I(\tau, d) \ge Q$  or  $I(\tau, d, \gamma) \ge Q$ . For a given mobile terminal, we can find a TBLMS $(\tau, d)$ , which achieves the



**Fig. 20** Location management cost in TBLMS–CWLU (q = 4,  $\tau = \tau(Q)$ )



required QoS, such that the total location update and terminal paging cost is minimized.

In the following, we display our results of cost minimization with guaranteed QoS. Let  $\tau(Q)$  denote the maximum value of  $\tau$  such that  $I(\tau, d, \gamma) \ge Q$ . For a given  $d, \tau(Q)$  can be found numerically by using the bisection method, since we know that  $I(\tau, d, \gamma)$  is a decreasing function of  $\tau$ . We assume the following parameter settings:  $\gamma = 1$ ,  $\lambda_c = 1$ ,  $\lambda_s = 30$ ,  $\Delta_p = 1$ ,  $\Delta_u = 30$ . In Figs. 17, 18, 19 and 20, we show the minimized cost of location management with Q = 0.90, 0.95, 0.99 for TBLMS–CPLU and q = 3, TBLMS–CWLU and q = 3, TBLMS–CPLU and q = 4, TBLMS–CWLU and q = 4, respectively.

Once  $\tau(Q)$  is found for each *d*, we can then determine the best combination of  $\tau$  and *d*, such that the total cost of mobility management is minimized and the QoS is *Q*. Again, such a search of *d* can be performed algorithmically, since there is a unique *d* which minimizes the cost. Our optimization results are summarized as follows. For a TBLMS-CPLU with q = 3, we have

- $Q = 0.90, d = 5, \tau = 0.4598727831,$  $C_{\text{CPLU}}(\tau, d) = 142.3811;$
- $Q = 0.95, d = 6, \tau = 0.4765203080,$  $C_{CPLU}(\tau, d) = 170.1432;$
- $Q = 0.99, d = 6, \tau = 0.2523264976,$  $C_{CPLU}(\tau, d) = 225.5237.$

For a TBLMS–CWLU with q = 3, we have

- $Q = 0.90, d = 5, \tau = 0.4598727831,$  $C_{CWLU}(\tau, d) = 126.2354;$
- $Q = 0.95, d = 6, \tau = 0.4765203080,$  $C_{CWLU}(\tau, d) = 153.9564;$

•  $Q = 0.99, d = 6, \tau = 0.2523264976,$  $C_{\text{CWLU}}(\tau, d) = 209.8936.$ 

For a TBLMS–CPLU with q = 4, we have

- $Q = 0.90, d = 5, \tau = 0.4519856408,$  $C_{CPLU}(\tau, d) = 163.4999;$
- $Q = 0.95, d = 5, \tau = 0.3051068387,$  $C_{CPLU}(\tau, d) = 195.0878;$
- $Q = 0.99, d = 6, \tau = 0.2445410249,$  $C_{CPLU}(\tau, d) = 259.2895.$

For a TBLMS–CWLU with q = 4, we have

- $Q = 0.90, d = 5, \tau = 0.4519856408,$  $C_{CWLU}(\tau, d) = 147.3738;$
- $Q = 0.95, d = 5, \tau = 0.3051068387,$  $C_{CWLU}(\tau, d) = 179.3262;$
- $Q = 0.99, d = 6, \tau = 0.2445410249,$  $C_{CWLU}(\tau, d) = 243.6788.$

## 9 Concluding remarks

We have emphasized that in a TBLMS, the simple paging method does not guarantee to find a mobile terminal, no matter how small the location update cycle is and how big the radius of a PA is. This implies that in addition to cost analysis and optimization, the QoS, which is the probability that a mobile terminal can be found in the current PA, is a fundamental issue to deal with in a TBLMS. Based on our previous results on random walks among rings of cell structures, we have analyzed the location distribution of a mobile terminal in a PA and the Author's personal copy

reachability of a mobile terminal in a PA when a phone call arrives, where the inter-call time and the cell residence time can have an arbitrary probability distribution. By using results from renewal processes, we have analyzed the cost of dynamic mobility management in a TBLMS, where the inter-call time and the cell residence time can have an arbitrary probability distribution. We have also developed a method to find a TBLMS which has the best combination of the location update cycle and the radius of a PA with the minimum cost of mobility management, while still satisfying the required QoS.

We would like to mention that our results on location distribution and reachability of a mobile terminal in a PA have significant impact on other research in dynamic mobility management. In addition to their application in the analysis of QoS in a TBLMS, these results can be employed to study the expected cost of various progressive paging methods. However, such investigation is beyond the scope of the paper, and we will report our results in a separate paper.

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#### **Appendix: Proofs of theorems**

*Proof of Theorem 2* The probability generating function (pgf) of X(t) is defined as

$$G_{X(t)}(t,z) = \sum_{j=0}^{\infty} \boldsymbol{P}[X(t)=j]z^{j}.$$

That is,

$$egin{aligned} G_{X(t)}(t,z) &= \sum_{j=0}^\infty (F_{S_j}(t) - F_{S_{j+1}}(t)) z^j \ &= 1 + \sum_{j=1}^\infty F_{S_j}(t) z^{j-1}(z-1), \end{aligned}$$

where we note that  $F_{S_0}(t) = 1$ . Applying Laplace transform to  $G_{X(t)}(t, z)$ , we get

$$G_{X(t)}^{*}(s,z) = \frac{1}{s} + \frac{1}{s} \sum_{j=1}^{\infty} z^{j-1}(z-1) f_{S_{j}}^{*}(s),$$

where we notice that  $F_X^*(s) = f_X^*(s)/s$  for any random variable *X*. For any two independent random variables *X* and *Y*, we have

$$f_{X+Y}^*(s) = f_X^*(s)f_Y^*(s)$$

Therefore, for an ordinary renewal process, we have

$$f_{S_j}^*(s) = \prod_{i=1}^{j} f_{T_i}^*(s) = (f_T^*(s))^j.$$

It is easy to verify that

$$G_{X(t)}^*(s,z) = \frac{1}{s} + \frac{1}{s} \sum_{j=1}^{\infty} z^{j-1} (z-1) (f_T^*(s))^j = \frac{1 - f_T^*(s)}{s(1 - f_T^*(s)z)}.$$

The pgf of *X*, the number of renewals in a random time interval  $T_c$ , is

$$G_X(z) = \sum_{j=0}^{\infty} \mathbf{P}[X=j] z^j = \int_0^{\infty} G_{X(t)}(t,z) f_{T_c}(t) dt.$$

The Laplace transform of  $G_{X(t)}(t, z)$  is

$$G^*_{X(t)}(s,z) = \int_0^\infty G_{X(t)}(t,z)e^{-st}dt.$$

Notice that

$$\left(-\frac{\partial}{\partial s}\right)^{\gamma_c-1}G^*_{X(t)}(s,z) = \int_0^\infty G_{X(t)}(t,z)t^{\gamma_c-1}e^{-st}dt$$

and

$$\frac{\lambda_c^{\gamma_c}}{(\gamma_c-1)!} \left(-\frac{\partial}{\partial s}\right)^{\gamma_c-1} G_{X(t)}^*(s,z) \\ = \int_0^\infty G_{X(t)}(t,z) \frac{\lambda_c(\lambda_c t)^{\gamma_c-1} e^{-st}}{(\gamma_c-1)!} dt.$$

The last equation implies that if  $T_c$  has an Erlang distribution with parameters ( $\gamma_c$ ,  $\lambda_c$ ),

$$f_{T_c}(t) = rac{\lambda_c (\lambda_c t)^{\gamma_c - 1} e^{-\lambda_c t}}{(\gamma_c - 1)!},$$

then we should have

$$G_X(z) = \frac{\lambda_c^{\gamma_c}}{(\gamma_c - 1)!} \left( -\frac{\partial}{\partial s} \right)^{\gamma_c - 1} [G^*_{X(t)}(s, z)]_{s = \lambda_c}.$$

For an ordinary renewal process, we get

$$G_X(z) = \frac{\lambda_c^{\gamma_c}}{(\gamma_c - 1)!} \left(-\frac{\partial}{\partial s}\right)^{\gamma_c - 1} \left[\frac{1 - f_T^*(s)}{s(1 - f_T^*(s)z)}\right]_{s = \lambda_c}$$

The Erlang distribution of  $T_c$  can be easily generalized to a hyper-Erlang distribution with pdf

$$f_{T_c}(t) = \sum_{i=1}^{k_c} w_{c,i} \left( \frac{\lambda_{c,i}(\lambda_{c,i}t)^{\gamma_{c,i}-1} e^{-\lambda_{c,i}t}}{(\gamma_{c,i}-1)!} \right),$$

where  $w_{c,1} + w_{c,2} + \cdots + w_{c,k_c} = 1$ . For an ordinary renewal process, we get

$$G_X(z) = \sum_{i=1}^{k_c} w_{c,i} \left( \frac{\lambda_{c,i}^{\gamma_{c,i}}}{(\gamma_{c,i}-1)!} \right) \left( -\frac{\partial}{\partial s} \right)^{\gamma_{c,i}-1} \\ \times \left[ \frac{1 - f_T^*(s)}{s(1 - f_T^*(s)z)} \right]_{s=\lambda_{c,i}}.$$

Notice that

$$\boldsymbol{P}[X=j] = \frac{1}{j!} \cdot \frac{d^j}{dz^j} G_X(z) \bigg|_{z=0},$$

for all  $j \ge 0$ . This proves the theorem.

Proof of Theorem 6 Assume that

$$f_T(t) = \sum_{i=1}^k w_i \left( \frac{\lambda_i e^{-\lambda_i t} (\lambda_i t)^{\gamma_i - 1}}{(\gamma_i - 1)!} \right),$$

where  $w_1 + w_2 + \cdots + w_k = 1$ . Then, we have

$$\lambda_T = \left(\sum_{i=1}^k w_i\left(\frac{\gamma_i}{\lambda_i}\right)\right)^{-1},$$

and

$$F_T(t) = \sum_{i=1}^k w_i \left( 1 - e^{-\lambda_i t} \sum_{j=0}^{\gamma_i - 1} \frac{(\lambda_i t)^j}{j!} \right)$$
  
=  $1 - \sum_{i=1}^k \sum_{j=0}^{\gamma_i - 1} w_i e^{-\lambda_i t} \frac{(\lambda_i t)^j}{j!}.$ 

It is well known that the residual time T' has pdf

$$f_{T'}(t) = \lambda_T (1 - F_T(t)).$$

(See [27], Eq. (5.10), p. 172). Consequently, we get

$$f_{T'}(t) = \lambda_T (1 - F_T(t)) \\ = \left(\sum_{i=1}^k w_i \left(\frac{\gamma_i}{\lambda_i}\right)\right)^{-1} \sum_{i=1}^k \sum_{j=0}^{\gamma_i - 1} \frac{w_i}{\lambda_i} \left(\frac{\lambda_i e^{-\lambda_i t} (\lambda_i t)^j}{j!}\right),$$

which implies that T' has a hyper-Erlang distribution.  $\Box$ 

*Proof of Theorem 7* Under the condition that  $X'_s = j$ , the probability that *u* is in ring *r* after crossing *j* cell boundaries is  $p_{0r}^{(j)}$ , where  $r \ge 0$ , and  $p_{0r}^{(j)}$  is defined in Theorem 5. Thus, the probability that *u* is in ring *r* when a phone call arrives is

$$\xi_r = \sum_{j=0}^{\infty} \mathbf{P}[X'_s = j] p_{0r}^{(j)},$$

for all  $r \ge 0$ . The result for  $P[X'_s = j]$  can be obtained from Theorem 1, where the renewal precess is  $T'_s, T_s, T_s, \ldots$ , and  $S_j = T'_s + (j-1)T_s, S_{j+1} = T'_s + jT_s$ , for all  $j \ge 0$ .

*Proof of Theorem 8* If the last location update is irregular, we have Case 1, and

$$\begin{split} P[X'_{s} &= j] \\ &= \int_{0}^{\infty} \left( F_{\text{Erlang}}(\lambda_{s}, j, t) - F_{\text{Erlang}}(\lambda_{s}, j+1, t) \right) f_{T'_{c}}(t) dt \\ &= \int_{0}^{\tau} e^{-\lambda_{s}t} \frac{(\lambda_{s}t)^{j}}{j!} \cdot \frac{1}{F_{T_{c}}(\tau)} \sum_{i=1}^{k_{c}} w_{c,i} f_{\text{Erlang}}(\lambda_{c,i}, \gamma_{c,i}, t) dt \\ &= \frac{1}{F_{T_{c}}(\tau)} \sum_{i=1}^{k_{c}} w_{c,i} \int_{0}^{\tau} e^{-\lambda_{s}t} \frac{(\lambda_{s}t)^{j}}{j!} \cdot \frac{\lambda_{c,i}e^{-\lambda_{c,i}t}(\lambda_{c,i}t)^{\gamma_{c,i}-1}}{(\gamma_{c,i}-1)!} dt \\ &= \frac{1}{F_{T_{c}}(\tau)} \sum_{i=1}^{k_{c}} w_{c,i} \frac{(j+\gamma_{c,i}-1)!\lambda_{s}^{j}\lambda_{c,i}^{\gamma_{c,i}}}{j!(\gamma_{c,i}-1)!(\lambda_{s}+\lambda_{c,i})t)^{j+\gamma_{c,i}-1}} \\ &\int_{0}^{\tau} \frac{(\lambda_{s}+\lambda_{c,i})e^{-(\lambda_{s}+\lambda_{c,i})t}((\lambda_{s}+\lambda_{c,i})t)^{j+\gamma_{c,i}-1}}{(j+\gamma_{c,i}-1)!} dt \\ &= \frac{1}{F_{T_{c}}(\tau)} \sum_{i=1}^{k_{c}} w_{c,i} \binom{j+\gamma_{c,i}-1}{j} \frac{\lambda_{s}^{j}\lambda_{c,i}^{\gamma_{c,i}}}{(\lambda_{s}+\lambda_{c,i})t)^{j+\gamma_{c,i}-1}} \\ &\times F_{\text{Erlang}}(\lambda_{s}+\lambda_{c,i},j+\gamma_{c,i},\tau), \end{split}$$

for all  $j \ge 0$ . Similarly, if the last location update is regular, we have Case 2, and the claim that

$$\begin{split} \boldsymbol{P}[X'_{s} = j] = & \frac{1}{F_{T'_{c}}(\tau)} \sum_{i=1}^{k_{c}} \sum_{j'=1}^{\gamma_{c,i}} w_{c,i,j'} \binom{j+j'-1}{j} \\ & \times \frac{\lambda_{s}^{j} \lambda_{c,i}^{j'}}{(\lambda_{s} + \lambda_{c,i})^{j+j'}} F_{\text{Erlang}}(\lambda_{s} + \lambda_{c,i}, j+j', \tau) \end{split}$$

for all  $j \ge 0$ , can be proved in a similar way.

Proof of Theorem 13 Since  $\lfloor t/\tau \rfloor + 1 = h$  for all (h - 1) $\tau \le t < h\tau$ , where  $h \ge 1$ , we have

 $\boldsymbol{E}(X_u)$ 

$$= \int_{0}^{\infty} \left( \left\lfloor \frac{t}{\tau} \right\rfloor + 1 \right) f_{T_c}(t) dt$$
  
$$= \sum_{h=1}^{\infty} \int_{(h-1)\tau}^{h\tau} hf_{T_c}(t) dt$$
  
$$= \sum_{h=1}^{\infty} h(F_{T_c}(h\tau) - F_{T_c}((h-1)\tau))$$
  
$$= \lim_{h \to \infty} (hF_{T_c}(h\tau) - (F_{T_c}((h-1)\tau) + F_{T_c}((h-2)\tau))$$
  
$$+ \dots + F_{T_c}(\tau) + F_{T_c}(0))).$$

By replacing  $F_{T_c}(t) = 1 - R_{T_c}(t)$ , the above quantity is





$$\begin{split} &\lim_{h\to\infty} \left(R_{T_c}(0)+R_{T_c}(\tau)+R_{T_c}(2\tau)+\cdots+R_{T_c}((h-1)\tau)\right.\\ &-hR_{T_c}(h\tau))\\ &=\sum_{h=0}^{\infty}R_{T_c}(h\tau). \end{split}$$

The above analysis applies to arbitrary  $T_c$ .

Fig. 21 shows the expected number of location updates between two consecutive phone calls.

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