13 Maximizing the Lifetime of Wireless Sensor Networks by Optimal Network Design

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CONTENTS

13.1 INTRODUCTION

The Internet of things (IoT) has been defined in Recommendation ITU-T Y.2060 (06/2012) as a global infrastructure for the information society, enabling advanced services by interconnecting physical and virtual things based on existing and evolving interoperable information and communication technologies [1]. The IoT is the network of physical objects (e.g., goods, products, vehicles, buildings) embedded with electronics, sensors, software, and network connectivity, which enable objects to collect and process data. The IoT allows objects to be sensed and controlled remotely through existing network infrastructure, creating opportunities for tight integration of the physical world into computer and communication systems. Each thing is uniquely identifiable through its embedded devices and is able to interoperate within the existing Internet infrastructure. It is estimated that the IoT will consist of 50 billion objects by 2020 [2] and contribute 19 trillion USD in the global economy [3].

One of the major enabling technologies for the IoT is low-energy wireless sensor networks [4–6]. When IoT is augmented with sensors and actuators, the technology becomes cyber-physical systems, such as smart grids, smart homes, smart cities, and intelligent transportation systems. A wireless sensor network (WSN) consists of spatially distributed autonomous sensors which are able to monitor physical and environmental conditions and to cooperatively transmit their sensed data through the network to a base station. Originally motivated by military applications such as battlefield surveillance, WSNs are now deployed and used widely in various applications, such as environmental and earth monitoring (air and water quality and pollution monitoring, forest fire, landslide, and natural disaster detection and prevention); industrial monitoring (machine health monitoring, data logging, and industrial sense and control applications); agriculture (accurate agriculture, irrigation management, greenhouses); passive localization and tracking; smart home monitoring; and IoT [7,8].

Due to very limited power supply and severe energy constraint in sensors [9], the lifetime of a WSN has gained substantial research attention in recent years [10]. Energy consumption in WSNs contains two components, namely, the energy required for data sensing and the energy used for data transmission. Research in lifetime maximization of WSNs has been focused on the first component only [11–14], the second component only [15–20], and both components [21–23]. We believe that the lifetime maximization problem of WSNs should be studied by taking both components of energy consumption into consideration [24].

Several methods have been proposed to increase the lifetime of a WSN, including redundant sensors [25], nonuniform sensor distributions [26], and aggregation and forwarding nodes for data transmission [27,28]. All these methods are based on the observation that sensors at different locations consume their battery power at different speeds. In particular, sensors close to a base station consume energy much faster than sensors far away from the base station [29,30]. Therefore, the most effective way to maximize the lifetime of a WSN is to allocate initial energy to sensors such that they exhaust their energy at the same time [21,31–33].

It has been found that the lifetime of a WSN and an optimal initial energy allocation are determined by a network design. Network lifetime maximization is a two-stage process, namely, optimal network design and optimal energy allocation. In reality, a WSN design includes the locations, sensing ranges, communication ranges, and data generation rates of all sensors, energy consumption for both data sensing and data transmission, as well as a routing algorithm for data transmission to a base station (i.e., a sink). All these factors have an impact on sensor and network lifetime as well as optimal energy allocation. For a given network design, an optimal initial energy allocation which maximizes the network lifetime can be determined [31]. Hence, the lifetime of a WSN is essentially determined by a network design.

It has been known that the lifetime of a WSN can be maximized by an optimal network design. In [34], the network lifetime obtained by optimal energy allocation is represented as a function of the number m of annuli, and it is shown that m has a significant impact on network lifetime. It is proved that for annuli of identical widths, if the energy consumed by data transmission is proportional to $d^{\alpha} + c$, where d is the distance of data transmission, and α and c are some constants, then for a circular area of interest with radius R, the optimal number of annuli that maximizes the network lifetime is

$$m = R\left(\frac{\alpha - 1}{c}\right)^{1/\alpha}, \tag{13.1}$$

for an arbitrary sensor density function.

The investigation in [34] assumes that all annuli have identical widths based on the observation that the energy consumption of a data transmission is minimized when all hops have the same distance [32,35]. However, whether identical annulus widths give the maximum network lifetime is worth further investigation. In this chapter, we show that it is indeed the case that an optimal network design has different widths of annuli. The main contribution of the present chapter is to develop an algorithm to find an optimal network design which maximizes the lifetime of a WSN obtained by optimal initial energy allocation for an arbitrary sensor distribution. We show that the optimal WSN design problem can be formulated as a nonlinear system of equations. Our results reveal the fact that an optimal network design has different widths of annuli. In particular, in an optimal network design, an annulus closer to a sink has larger width. Compared with a network design with identical annulus widths, a network design with variable annulus widths can lead to a noticeable increment of the network lifetime.

The chapter is organized as follows. In Section 13.2, we present preliminary information, including the network design model, the sensor distribution model, and the energy consumption model used in our study, as well as an example to motivate our investigation. In Section 13.3, we develop analytical forms of network lifetime and optimal energy allocation. In Section 13.4, we give an algorithm to find an optimal network design which maximizes the network lifetime obtained by optimal energy allocation for a uniform sensor distribution, where the optimal network design problem is formulated as a nonlinear system of equations. In Section 13.5, we give an algorithm to find an optimal network design for a nonuniform sensor distribution. In Section 13.6, we demonstrate numerical examples. We conclude the chapter in Section 13.7.

13.2 PRELIMINARIES

In this section, we provide preliminary information, including the network design model, the sensor distribution model, and the energy consumption model. We also give a motivational example which inspires our research.

13.2.1 The Network Design Model

Let us consider a circular area of interest A which has radius R meters (see Figure 13.1). Assume that A is divided into m annuli (also called coronae) $A_1, A_2, \ldots, A_m$ by m circles with radii $r_1, r_2, \ldots, r_m$ centered at a sink, where $0 < r_1 < r_2 < \cdots < r_m = R$ [32]. For convenience, we assume that there is A_0 with width $r_0 = 0$ which contains a sink. All sensors report sensory data to the sink. Annulus A_j has width $r_j - r_{j-1}$, where $1 \le j \le m$. For a fixed R, the number m of annuli as well as the sequence of

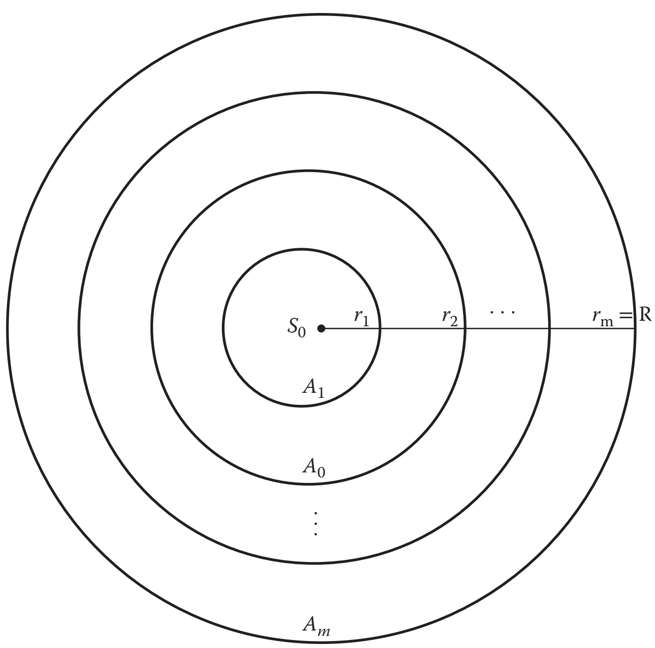

FIGURE 13.1 A circular area of radius R with m annuli.

values $(r_1, r_2, ..., r_{m-1})$ is called a *network design* or a *network configuration*, which has significant impact on energy consumption and network lifetime.

Assume that there are N sensors, $s_1, s_2, ..., s_N$, randomly distributed in A. We use s_0 to represent a sink. All sensors in A_j are designed in such a way that they have the same transmission range, $r_j - r_{j-1}$. All sensors also have a certain sensing range. It is assumed that N is sufficiently large such that a WSN is connected. Furthermore, it is assumed that the sensing range is sufficiently large such that A is well covered. Let N_j be the number of sensors in A_j.

13.2.2 The Sensor Distribution Model

Let $f(r)$ be any *sensor density function* (or *sensor distribution function*) in a circular area of interest, A, with radius R, where $0 \le r \le R$. In other words, the number of sensors in a small area, z, with distance r to the sink is $f(r)z$. The function $f(r)$ should satisfy

$$\int_0^R 2\pi r f(r)\,dr = N. \tag{13.2}$$

The number of sensors in A_j is

$$N_j = \int_{r_{j-1}}^{r_j} 2\pi r f(r)\,dr, \tag{13.3}$$

for all $1 \le j \le m$. For instance, for a uniform distribution, we have

$$f(r) = \frac{N}{\pi R^2}, \tag{13.4}$$

$$\int_0^R 2\pi r f(r)\,dr = \frac{N}{R^2}\int_0^R 2r\,dr = \left(\frac{N}{R^2}\right) r^2 \,|_0^R = N, \tag{13.5}$$

and

$$N_j = \frac{N}{R^2}\int_{r_{j-1}}^{r_j} 2r\,dr = \left(\frac{N}{R^2}\right) r^2 \,|_{r_{j-1}}^{r_j} = \left(\frac{r_j^2 - r_{j-1}^2}{R^2}\right) N, \tag{13.6}$$

for all $1 \le j \le m$.

As an example of nonuniform sensor distribution, let us consider

$$f(r) = \left(\frac{N}{\pi \ln\,(1 + 1/u)}\right)\left(\frac{1}{r^2 + uR^2}\right), \tag{13.7}$$

AU: Equations (13.8–13.97) have been renumbered to maintain sequential order. Please confirm.

where $u > 0$. It is easy to see that

$$\int_0^R 2\pi r f(r)\,dr = \int_0^R 2\pi r \left(\frac{N}{\pi \ln\,(1 + 1/u)}\right)\left(\frac{1}{r^2 + uR^2}\right) dr \tag{13.8}$$

$$
\begin{aligned}
&= \frac{N}{\ln\,(1+1/u)}\int_{0}^{R}\left(\frac{1}{r^2+uR^2}\right)d\left(r^2+uR^2\right) \\
&= \frac{N}{\ln\,(1+1/u)}\int_{uR^2}^{(u+1)R^2}\frac{dx}{x} \quad \left(\text{letting } x=r^2+uR^2\right) \\
&= \frac{N}{\ln\,(1+1/u)}\ln x\,\Big|_{uR^2}^{(u+1)R^2} \\
&= \frac{N}{\ln\,(1+1/u)}\left(\ln\left((u+1)R^2-\ln\,(uR^2)\right)\right) \\
&= N.
\end{aligned}
$$

Notice that the ratio of the largest density (when $r=0$) to the smallest density (when $r=R$) is $(1+1/u)$. Thus, the parameter u indicates uniformity of sensor distribution. For a small u, sensors are more densely distributed in the area closer to the sink. As $u \to 0$, the sensor density near the sink can be arbitrarily large. One the other hand, as u increases, sensors are more evenly distributed in A. For a very large u, we have $\ln\,(1+1/u) \approx 1/u$, and

$$
f(r) \approx \left(\frac{N}{\pi(1/u)}\right)\left(\frac{1}{r^2+uR^2}\right) = \frac{N}{\pi\left(r^2/u+R^2\right)} \approx \frac{N}{\pi R^2}. \tag{13.9}
$$

That is, as $u \to \infty$, $f(r)$ approaches a uniform distribution.

The previous $f(r)$ gives rise to

$$
N_j = \frac{N}{\ln\,(1+1/u)}\ln\left(\frac{r_j^2+uR^2}{r_{j-1}^2+uR^2}\right), \tag{13.10}
$$

for all $1 \le j \le m$. For a very large u, we have

$$
\ln\left(\frac{r_j^2+uR^2}{r_{j-1}^2+uR^2}\right) = \ln\left(1+\frac{r_j^2-r_{j-1}^2}{r_{j-1}^2+uR^2}\right) \approx \frac{r_j^2-r_{j-1}^2}{r_{j-1}^2+uR^2}, \tag{13.11}
$$

and

$$
N_j \approx \frac{N}{(1/u)}\cdot\frac{r_j^2-r_{j-1}^2}{r_{j-1}^2+uR^2} \approx \left(\frac{r_j^2-r_{j-1}^2}{R^2}\right)N, \tag{13.12}
$$

which is the N_j for a uniform distribution.

13.2.3 The Energy Consumption Model

The amount of energy consumed by a sensor to sense and receive data in one unit of time is p mJ/second.

The amount of energy needed to transmit one bit over distance d meters is $\left(a_1 d^{\alpha}+a_2\right)$ pJ, where a_1 is the energy required to run a transmitter amplifier, a_2 is the energy used to activate a transmitter circuitry, and $2 \le \alpha \le 6$ is a constant [36]. The previous expression has a significant implication in minimizing the energy cost of data transmission in WSNs.

Assume that each datum has size b bytes = $8b$ bits. Then, the amount of energy needed to transmit one datum over distance d meters is

$$q = 8b\left(a_1 d^{\alpha} + a_2\right)\text{pJ} = 8a_1 b\left(d^{\alpha} + \frac{a_2}{a_1}\right)\text{pJ} = \frac{8a_1 b}{10^6}\left(d^{\alpha} + c\right)\text{mJ} = a\left(d^{\alpha} + c\right)\text{mJ}, \tag{13.13}$$

where $a = a_1 b / 125{,}000$ mJ/m$^{\alpha}$ and $c = a_2 / a_1$ m$^{\alpha}$. (Notation: pJ = pico Joule, nJ = nano Joule, mJ = micro Joule, m$^{\alpha}$ = meter raised to the power α.) For instance, when $b = 25$ bytes, $a_1 = 10$ pJ/bit/m$^{\alpha}$, $a_2 = 50$ nJ / bit $= 50{,}000$ pJ / bit, we have $a = 0.002$ mJ/m$^{\alpha}$, $c = 5000$ m$^{\alpha}$, and $q = 0.002d^{\alpha} + 10$ mJ. Based on the previous discussion, we know that the amount of energy consumed by a sensor in A_j to transmit a datum is

$$q_j = a\left(\left(r_j - r_{j-1}\right)^{\alpha} + c\right)\text{mJ}, \tag{13.14}$$

for all $1 \le j \le m$.

13.3 A MOTIVATIONAL EXAMPLE

Before we proceed, let us consider the following motivational example. Consider $(k+1)$ sensor $s_0, s_1, s_2, \ldots, s_k$ along a line, where the distance between s_{j-1} and s_j is d_j, for all $1 \le j \le k$, and $d = d_1 + d_2 + \cdots + d_k$ (see Figure 13.2). Each sensor s_j needs to send a bit to s_0 along the path $\left(s_j, s_{j-1}, \ldots, s_0\right)$ with j hops, for all $1 \le j \le k$. Then, for fixed d, the energy consumed by the above k data transmissions is a function of $d_1, d_2, \ldots, d_{k-1}$ (since $d_k = d - d_1 - d_2 - \cdots - d_{k-1}$),

$$E\left(d_1, d_2, \ldots, d_{k-1}\right) = \sum_{j=1}^{k}(k-j+1)\left(a_1 d_j^{\alpha} + a_2\right) = a_1 \sum_{j=1}^{k}(k-j+1)d_j^{\alpha} + a_2 \cdot \frac{k(k+1)}{2}. \tag{13.15}$$

It is clear that to minimize $E\left(d_1, d_2, \ldots, d_{k-1}\right)$, we need only to minimize

$$f\left(d_1, d_2, \ldots, d_{k-1}\right) = \sum_{j=1}^{k}(k-j+1)d_j^{\alpha} = \sum_{j=1}^{k-1}(k-j+1)d_j^{\alpha} + \left(d - \sum_{j=1}^{k-1} d_j\right)^{\alpha}. \tag{13.16}$$

Consider the case when $\alpha = 2$. Since the terms in $f\left(d_1, d_2, \ldots, d_{k-1}\right)$ that include d_j are

$$\left(k-j+2\right)d_j^2 + 2d_j \sum_{j' \ne j} d_{j'} - 2dd_j, \tag{13.17}$$

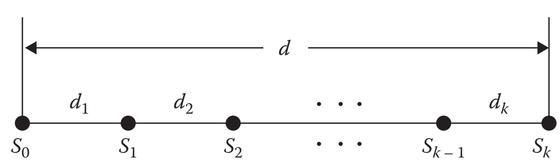

FIGURE 13.2 A data transmission path.

we have

$$\frac{\partial f(d_1,d_2,\ldots,d_{k-1})}{\partial d_j} = 2(k-j+2)d_j + 2\sum_{j'\neq j} d_{j'} - 2d, \tag{13.18}$$

for all $1 \le j \le k-1$. Hence, we get a linear system of equations.

$$(k-j+2)d_j + \sum_{j'\neq j} d_{j'} = d, \tag{13.19}$$

for all $1 \le j \le k-1$. It is straightforward to verify that when $k = 2$, we have $d_1 = \left(\frac{1}{3}\right)d$ and $d_2 = \left(\frac{2}{3}\right)d$; when $k = 3$, we have $d_1 = \left(\frac{2}{11}\right)d$, $d_2 = \left(\frac{3}{11}\right)d$, and $d_3 = \left(\frac{6}{11}\right)d$; when $k = 4$, we have $d_1 = \left(\frac{3}{25}\right)d$, $d_2 = \left(\frac{4}{25}\right)d$, $d_3 = \left(\frac{6}{25}\right)d$, and $d_4 = \left(\frac{12}{25}\right)d$. We observe that the d_j's are different, a quite different conclusion from the fact that for a single data transmission, energy consumption is minimized when all hops have the same distance [34]. Such phenomenon inspires the optimal network configuration problem solved in this chapter.

13.4 NETWORK LIFETIME AND OPTIMAL ENERGY ALLOCATION

In this section, we develop analytical forms of network lifetime and optimal energy allocation, which are necessary to formulate our optimization problems (i.e., network lifetime maximization and optimal network design).

When a datum is transmitted from a sensor, s_j, in A_j to the sink, s_0, the datum is sent along a path $(s_j, s_{j-1}, s_{j-2}, \ldots, s_1, s_0)$ from s_j to s_0, where $s_i \in A_i$ for all $j \ge i \ge 0$. Assume that each sensor senses and transmits μ data to a sink per second. This implies that sensors in A_j contribute $N_j\mu$ data transmissions per second to all A_i's, where $1 \le i \le j$. It is also assumed that all sensors in A_j are treated equally such that they all perform the same amount of data transmission. Since there are $(N_j + N_{j+1} + \cdots + N_m)\mu$ data transmissions to be performed by N_j sensors in A_j per second, a sensor in A_j performs β_j data transmissions in one unit of time, where

$$\beta_j = \frac{1}{N_j}(N_j + N_{j+1} + \cdots + N_m)\mu = \left(\frac{r_m^2 - r_{j-1}^2}{r_j^2 - r_{j-1}^2}\right)\mu, \tag{13.20}$$

for all $1 \le j \le m$.

A sensor in A_j is equipped with E_j of initial energy. Let E denote the total energy budget, that is,

$$E = \sum_{j=1}^{m} N_j E_j. \tag{13.21}$$

Once a sensor is deployed, E_j is not renewable or replenishable. The network lifetime is determined by the *initial energy allocation* $(E_1, E_2, \ldots, E_m)$, which is determined by a network design, that is, $(r_1, r_2, \ldots, r_{m-1})$.

The energy consumed by a sensor in A_j in one unit of time is $p + \beta_j q_j$, which implies that the lifetime of a sensor in A_j is

$$L_j = \frac{E_j}{p + \beta_j q_j}. \tag{13.22}$$

A reasonable definition of *network lifetime* L is $L = \min(L_1, L_2, ..., L_m)$, since when all sensors in A_j run out of battery power, a WSN becomes disconnected and inoperational. It is clear that L is maximized if and only if $L_1 = L_2 = \cdots = L_m$, that is, all sensors die at the same time; otherwise, we can initially move energy from sensors which work longer to sensors which die sooner so that the network lifetime is increased.

To have an identical lifetime L for all the sensors, that is, $E_j / \left(p + \beta_j q_j\right) = L$, we need $E_j = L\left(p + \beta_j q_j\right)$, for all $1 \le j \le m$. Since $N_1 E_1 + N_2 E_2 + \cdots + N_m E_m = E$, that is,

$$\sum_{j=1}^{m} N_j L\left(p + \beta_j q_j\right) = E, \tag{13.23}$$

we get

$$L = \frac{E}{\sum_{j=1}^{m} N_j\left(p + \beta_j q_j\right)}, \tag{13.24}$$

and

$$E_j = \frac{\left(p + \beta_j q_j\right)E}{\sum_{j=1}^{m} N_j\left(p + \beta_j q_j\right)}, \tag{13.25}$$

for all $1 \le j \le m$. An initial energy allocation $\left(E_1, E_2, ..., E_m\right)$ that results in $L_1 = L_2 = \cdots = L_m = L$ is called an *optimal energy allocation*.

We notice that

$$N_j + N_{j+1} + \cdots + N_m = \int_{r_{j-1}}^{r_m} 2\pi r f(r)\, dr. \tag{13.26}$$

Hence, the network lifetime is

$$\begin{aligned} L &= \frac{E}{Np + \sum_{j=1}^{m} (N_j + N_{j+1} + \cdots + N_m)\mu q_j} \\ &= \frac{E}{Np + \mu a \sum_{j=1}^{m} \left(\int_{r_{j-1}}^{r_m} 2\pi r f(r)\, dr\right)\left((r_j - r_{j-1})^{\alpha} + c\right)}. \end{aligned} \tag{13.27}$$

Since

$$p + \beta_j q_j = p + \frac{1}{N_j}\left(N_j + N_{j+1} + \cdots + N_m\right)\mu a\left((r_j - r_{j-1})^{\alpha} + c\right) \tag{13.28}$$

$$= p + \frac{\int_{r_{j-1}}^{r_m} 2\pi r f(r)\,dr}{\int_{r_{j-1}}^{r_j} 2\pi r f(r)\,dr} \cdot \mu a\left((r_j - r_{j-1})^{\alpha} + c\right),$$

the optimal energy allocation is

$$E_j = E \cdot \frac{p + \left(\int_{r_{j-1}}^{r_m} 2\pi r f(r)\,dr\right)\left(\int_{r_{j-1}}^{r_j} 2\pi r f(r)\,dr\right)^{-1} \mu a\left((r_j - r_{j-1})^{\alpha} + c\right)}{Np + \mu a \sum_{j=1}^{m}\left(\int_{r_{j-1}}^{r_m} 2\pi r f(r)\,dr\right)\left((r_j - r_{j-1})^{\alpha} + c\right)}, \tag{13.29}$$

for all $1 \le j \le m$.

13.5 OPTIMAL NETWORK DESIGN FOR UNIFORM DISTRIBUTIONS

In this section, for a uniform sensor distribution, we define our network lifetime maximization problem as a multivariable minimization problem. Then, we give an algorithm to find an optimal network design which maximizes the network lifetime obtained by optimal energy allocation.

13.5.1 The Optimization Problem

Notice that for a uniform distribution of sensors, we have

$$N_j = \left(\frac{\pi r_j^2 - \pi r_{j-1}^2}{\pi R^2}\right) N = \left(\frac{r_j^2 - r_{j-1}^2}{R^2}\right) N, \tag{13.30}$$

and $N = N_1 + N_2 + \cdots + N_m$. Then, we have

$$\begin{aligned}
\sum_{j=1}^{m} N_j\left(p + \beta_j q_j\right) &= \sum_{j=1}^{m} N_j p + \sum_{j=1}^{m} N_j \beta_j q_j \\
&= Np + \sum_{j=1}^{m} (N_j + N_{j+1} + \cdots + N_m)\mu q_j \\
&= Np + \sum_{j=1}^{m} \left(\frac{r_m^2 - r_{j-1}^2}{R^2}\right) N \mu a\left((r_j - r_{j-1})^{\alpha} + c\right) \quad C \\
&= Np + \frac{N\mu a}{R^2} \sum_{j=1}^{m} (r_m^2 - r_{j-1}^2)\left((r_j - r_{j-1})^{\alpha} + c\right) \\
&= N\left(p + \frac{\mu a}{R^2} \sum_{j=1}^{m} (r_m^2 - r_{j-1}^2)\left((r_j - r_{j-1})^{\alpha} + c\right)\right).
\end{aligned} \tag{13.31}$$

Hence, the network lifetime is

$$L = \frac{E}{N\left(p + \frac{\mu a}{R^2}\sum_{j=1}^{m}(r_m^2 - r_{j-1}^2)\left((r_j - r_{j-1})^{\alpha} + c\right)\right)}. \tag{13.32}$$

Since

$$\begin{aligned} p + \beta_j q_j &= p + \frac{1}{N_j}\left(N_j + N_{j+1} + \cdots + N_m\right)\mu a\left((r_j - r_{j-1})^{\alpha} + c\right) \\ &= p + \left(\frac{r_m^2 - r_{j-1}^2}{r_j^2 - r_{j-1}^2}\right)\mu a\left((r_j - r_{j-1})^{\alpha} + c\right), \end{aligned} \tag{13.33}$$

we obtain the optimal energy allocation

$$E_j = \frac{E}{N}\cdot\frac{p + \left(\frac{r_m^2 - r_{j-1}^2}{r_j^2 - r_{j-1}^2}\right)\mu a\left((r_j - r_{j-1})^{\alpha} + c\right)}{p + \frac{\mu a}{R^2}\sum_{j=1}^{m}(r_m^2 - r_{j-1}^2)\left((r_j - r_{j-1})^{\alpha} + c\right)}, \tag{13.34}$$

for all $1 \le j \le m$.

It is clear that the network lifetime L is a function of $r_1, r_2, \ldots, r_{m-1}$. To maximize the network lifetime, we need to minimize the following function:

$$F(r_1, r_2, \ldots, r_{m-1}) = \sum_{j=1}^{m}(r_m^2 - r_{j-1}^2)\left((r_j - r_{j-1})^{\alpha} + c\right), \tag{13.35}$$

where $r_0 = 0$ and $r_m = R$. Notice that $F(r_1, r_2, \ldots, r_{m-1})/R^2$ gives the average number of m^{α} taken by a single data transmission. It is clear that $F(r_1, r_2, \ldots, r_{m-1})$ is a quantity determined by a network design $(r_1, r_2, \ldots, r_{m-1})$ and $F(r_1, r_2, \ldots, r_{m-1})$ determines the energy expenditure of data transmission.

13.5.2 Optimal Network Design

Now, we develop an algorithm to solve our optimization problem. Our main idea is to formulate the optimal network design problem as a nonlinear system of equations. (For technically less-experienced readers, this section can be skipped.)

To minimize the function

$$F(r_1, r_2, \ldots, r_{m-1}) = \sum_{j=1}^{m}(R^2 - r_{j-1}^2)\left((r_j - r_{j-1})^{\alpha} + c\right), \tag{13.36}$$

we should have

$$\frac{\partial F}{\partial r_j} = f_j(r_1, r_2, \ldots, r_{m-1}) = 0, \tag{13.37}$$

for all $1 \le j \le m-1$. Since the terms in $F(r_1, r_2, ..., r_{m-1})$ that include r_j are

$$\left(R^2 - r_{j-1}^2\right)\left((r_j - r_{j-1})^\alpha + c\right) + \left(R^2 - r_j^2\right)\left((r_{j+1} - r_j)^\alpha + c\right), \tag{13.38}$$

we get

$$f_j(r_1, r_2, ..., r_{m-1}) = \alpha\left(R^2 - r_{j-1}^2\right)(r_j - r_{j-1})^{\alpha-1} - 2r_j\left((r_{j+1} - r_j)^\alpha + c\right) - \alpha\left(R^2 - r_j^2\right)(r_{j+1} - r_j)^{\alpha-1}. \tag{13.39}$$

Therefore, we have a nonlinear system of equations, that is,

$$\begin{aligned} f_1(r_1, r_2, ..., r_{m-1}) &= 0, \\ f_2(r_1, r_2, ..., r_{m-1}) &= 0 \\ &\vdots \\ f_{m-1}(r_1, r_2, ..., r_{m-1}) &= 0. \end{aligned} \tag{13.40}$$

AU: Should a comma be added after the middle zero?

By using vector notation to represent the variables $r_1, r_2, ..., r_{m-1}$, we write

$$\mathbf{r} = (r_1, r_2, ..., r_{m-1}), \tag{13.41}$$

AU: Is the boldng of the variables here and below correct?

and $f_j(r_1, r_2, ..., r_{m-1}) = f_j(\mathbf{r})$, where $f_j : \mathbb{R}^{m-1} \to \mathbb{R}$ maps $(m-1)$-dimensional space $\mathbb{R}^{m-1}$ into the real line $\mathbb{R}$. By defining a function $\mathbf{F} : \mathbb{R}^{m-1} \to \mathbb{R}^{m-1}$ which maps $\mathbb{R}^{m-1}$ into $\mathbb{R}^{m-1}$,

$$\mathbf{F}(r_1, r_2, ..., r_{m-1}) = \left(f_1(r_1, r_2, ..., r_{m-1}), f_2(r_1, r_2, ..., r_{m-1}), ..., f_{m-1}(r_1, r_2, ..., r_{m-1})\right), \tag{13.42}$$

namely,

$$\mathbf{F}(\mathbf{r}) = \left(f_1(\mathbf{r}), f_2(\mathbf{r}), ..., f_{m-1}(\mathbf{r})\right), \tag{13.43}$$

then our nonlinear system of equations is

$$\mathbf{F}(\mathbf{r}) = 0, \tag{13.44}$$

where $0 = (0, 0, ..., 0)$.

The previous nonlinear system of equations can be solved by using Newton's method. To this end, we need the Jacobian matrix $J(\mathbf{r})$ defined as

$$J(r) = \begin{bmatrix} \frac{\partial f_1(r)}{\partial r_1} & \frac{\partial f_1(r)}{\partial r_2} & \cdots & \frac{\partial f_1(r)}{\partial r_{m-1}} \\ \frac{\partial f_2(r)}{\partial r_1} & \frac{\partial f_2(r)}{\partial r_2} & \cdots & \frac{\partial f_2(r)}{\partial r_{m-1}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_{m-1}(r)}{\partial r_1} & \frac{\partial f_{m-1}(r)}{\partial r_2} & \cdots & \frac{\partial f_{m-1}(r)}{\partial r_{m-1}} \end{bmatrix}, \tag{13.45}$$

where

$$\frac{\partial f_j(\mathbf{r})}{\partial r_{j-1}} = -2\alpha r_{j-1}(r_j - r_{j-1})^{\alpha-1} - \alpha(\alpha - 1)\left(R^2 - r_{j-1}^2\right)(r_j - r_{j-1})^{\alpha-2}, \tag{13.46}$$

ALGORITHM 13.1 AN ALGORITHM FOR FINDING AN OPTIMAL NETWORK DESIGN.

Algorithm: Optimal Network Design

Input: Parameters α, c, R, m.

Output: An optimal network design $\mathbf{r} = (r_1, r_2, ..., r_{m-1})$ which satisfies $\mathbf{F}(\mathbf{r}) = 0$

$\mathbf{r} \leftarrow (R/m, 2R/m, 3R/m, ..., (m-1)R/m)$; (1)
Repeat; (2)
Calculate $\mathbf{F}(\mathbf{r}) = (f_1(\mathbf{r}), f_2(\mathbf{r}), ..., f_{m-1}(\mathbf{r}))$; (3)
Calculate $J(\mathbf{r})$, where $J(\mathbf{r})_{j,k} = \partial f_j(\mathbf{r}) / \partial r_k$ for $1 \le j, k \le m-1$; (4)
Solve the linear system of equations $J(\mathbf{r})\mathbf{x} = -\mathbf{F}(\mathbf{r})$; (5)
$\mathbf{r} \leftarrow \mathbf{r} + \mathbf{x}$; (6)
until $\|\mathbf{x}\| \le \epsilon$. (7)

for all $2 \le j \le m-1$, and

$$\frac{\partial f_j(\mathbf{r})}{\partial r_j} = \alpha(\alpha-1)\left(R^2 - r_{j-1}^2\right)(r_j - r_{j-1})^{\alpha-2} - 2\left((r_{j+1} - r_j)^{\alpha} + c\right) + 4\alpha r_j (r_{j+1} - r_j)^{\alpha-1} + \alpha(\alpha-1)\left(R^2 - r_j^2\right)(r_{j+1} - r_j)^{\alpha-2}, \tag{13.47}$$

for all $1 \le j \le m-1$, and

$$\frac{\partial f_j(\mathbf{r})}{\partial r_{j+1}} = -2\alpha r_j (r_{j+1} - r_j)^{\alpha-1} - \alpha(\alpha-1)\left(R^2 - r_j^2\right)(r_{j+1} - r_j)^{\alpha-2}, \tag{13.48}$$

for all $1 \le j \le m-2$, and

$$\frac{\partial f_j(\mathbf{r})}{\partial r_k} = 0, \tag{13.49}$$

for all $1 \le j \le m-1$ and $k \ne j-1, j, j+1$.

Our algorithm for finding an optimal network design $\mathbf{r} = (r_1, r_2, ..., r_{m-1})$ which satisfies the nonlinear system of equations $\mathbf{F}(\mathbf{r}) = 0$ is given in Algorithm 13.1. This is essentially Newton's standard iterative method ([37], p. 451). Our initial approximation of $\mathbf{r}$ is $r_j = jR/m$ for all $1 \le j \le m-1$ [line (1)]. The value of $\mathbf{r}$ is then repeated modified as $\mathbf{r} + \mathbf{x}$ [line (6)], where $\mathbf{x}$ is the solution to the linear system of equations $J(\mathbf{r})\mathbf{x} = -\mathbf{F}(\mathbf{r})$ [line (5)]. Such modification is repeated until $\|\mathbf{x}\| \le \epsilon$ [line (7)], where

$$\|\mathbf{x}\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_{m-1}^2}, \tag{13.50}$$

and ϵ is a sufficiently small constant, say, 10^{-10}. The linear system of equations in line (5) can be solved by using the classic Gaussian elimination with backward substitution algorithm ([37], pp. 268–269).

13.6 OPTIMAL NETWORK DESIGN FOR NONUNIFORM DISTRIBUTIONS

In this section, for an arbitrary nonuniform sensor distribution, we define our network lifetime maximization problem as a multivariable minimization problem. We also give an algorithm to find an optimal network design which maximizes the network lifetime obtained by optimal energy allocation by extending our method in Section 13.4.

13.6.1 The Optimization Problem

We follow the method in Section 13.4.1.

For the nonuniform sensor distribution in Section 13.2.2, since

$$\begin{aligned} N_j + N_{j+1} + \cdots + N_m &= \frac{N}{\ln(1+1/u)} \ln\left(\prod_{i=j}^{m} \frac{r_i^2 + uR^2}{r_{i-1}^2 + uR^2} \right) \\ &= \frac{N}{\ln(1+1/u)} \ln\left(\frac{r_m^2 + uR^2}{r_{j-1}^2 + uR^2} \right), \end{aligned} \tag{13.51}$$

we have

$$L = \frac{E}{N\left(p + \dfrac{\mu a}{\ln(1+1/u)} \displaystyle\sum_{j=1}^{m} \ln\left(\frac{r_m^2 + uR^2}{r_{j-1}^2 + uR^2} \right)\left((r_j - r_{j-1})^{\alpha} + c \right) \right)}. \tag{13.52}$$

Because

$$\begin{aligned} p + \beta_j q_j &= p + \frac{1}{N_j}\left(N_j + N_{j+1} + \cdots + N_m \right)\mu a\left((r_j - r_{j-1})^{\alpha} + c \right) \\ &= p + \frac{\ln\left(\dfrac{r_m^2 + uR^2}{r_{j-1}^2 + uR^2} \right)}{\ln\left(\dfrac{r_j^2 + uR^2}{r_{j-1}^2 + uR^2} \right)} \cdot \mu a\left((r_j - r_{j-1})^{\alpha} + c \right), \end{aligned} \tag{13.53}$$

we obtain

$$E_j = \frac{E}{N} \cdot \frac{p + \left(\dfrac{r_m^2 - r_{j-1}^2}{r_j^2 - r_{j-1}^2} \right)\mu a\left((r_j - r_{j-1})^{\alpha} + c \right)}{p + \dfrac{\mu a}{R^2} \displaystyle\sum_{j=1}^{m} (r_m^2 - r_{j-1}^2)\left((r_j^2 - r_{j-1}^2)^{\alpha} + c) \right)}, \tag{13.54}$$

AU: Is a ")" missing at the end of the top line?

for all $1 \le j \le m$.

To maximize the network lifetime, we need to minimize the following function:

$$F(r_1, r_2, \ldots, r_{m-1}) = \sum_{j=1}^{m} \ln\left(\frac{r_m^2 + uR^2}{r_{j-1}^2 + uR^2} \right)\left((r_j - r_{j-1})^{\alpha} + c \right), \tag{13.55}$$

where $r_0 = 0$ and $r_m = R$.

13.6.2 Optimal Network Design

Now, we follow the method in Section 13.4.2. (For technically less-experienced readers, this section can be skipped.)

To minimize the previous function, we should have

$$\frac{\partial F}{\partial r_j} = f_j(r_1, r_2, \ldots, r_{m-1}) = 0, \tag{13.56}$$

for all $1 \le j \le m-1$. Since the terms in $F(r_1, r_2, \ldots, r_{m-1})$ that include r_j are

$$\ln\left(\frac{r_m^2 + uR^2}{r_{j-1}^2 + uR^2}\right)\left((r_j - r_{j-1})^\alpha + c\right) + \ln\left(\frac{r_m^2 + uR^2}{r_j^2 + uR^2}\right)\left((r_{j+1} - r_j)^\alpha + c\right), \tag{13.57}$$

we get

$$\begin{aligned} f_j(r_1, r_2, \ldots, r_{m-1}) = {} & \ln\left(\frac{r_m^2 + uR^2}{r_{j-1}^2 + uR^2}\right)\alpha(r_j - r_{j-1})^{\alpha-1} \\ & -\left(\frac{2r_j}{r_j^2 + uR^2}\right)\left((r_{j+1} - r_j)^\alpha + c\right) - \ln\left(\frac{r_m^2 + uR^2}{r_j^2 + uR^2}\right)\alpha(r_{j+1} - r_j)^{\alpha-1}. \end{aligned} \tag{13.58}$$

Therefore, we have a nonlinear system of equations, that is,

AU: Is a comma needed after the middle line?

$$\begin{aligned} f_1(r_1, r_2, \ldots, r_{m-1}) &= 0, \\ f_2(r_1, r_2, \ldots, r_{m-1}) &= 0 \\ &\vdots \\ f_{m-1}(r_1, r_2, \ldots, r_{m-1}) &= 0. \end{aligned} \tag{13.59}$$

Again, by using vector notation to represent the variables $r_1, r_2, \ldots, r_{m-1}$, we write

$$\mathbf{r} = (r_1, r_2, \ldots, r_{m-1}), \tag{13.60}$$

and $f_j(r_1, r_2, \ldots, r_{m-1}) = f_j(\mathbf{r})$. By defining a function $\mathbf{F} : \mathbb{R}^{m-1} \to \mathbb{R}^{m-1}$

$$\mathbf{F}(r_1, r_2, \ldots, r_{m-1}) = \left(f_1(r_1, r_2, \ldots, r_{m-1}), f_2(r_1, r_2, \ldots, r_{m-1}), \ldots, f_{m-1}(r_1, r_2, \ldots, r_{m-1})\right), \tag{13.61}$$

namely,

$$\mathbf{F}(\mathbf{r}) = \left(f_1(\mathbf{r}), f_2(\mathbf{r}), \ldots, f_{m-1}(\mathbf{r})\right), \tag{13.62}$$

then our nonlinear system of equations is

$$\mathbf{F}(\mathbf{r}) = 0. \tag{13.63}$$

By using the same algorithm in Section 13.4.2, the previous nonlinear system of equations can be solved by using Newton's method, where the Jacobian matrix $J(\mathbf{r})$ is defined as

$$J(\mathbf{r}) = \left[\frac{\partial f_j(\mathbf{r})}{\partial r_k} \right]_{1 \le j,k \le m-1}, \tag{13.64}$$

with

$$\frac{\partial f_j(\mathbf{r})}{\partial r_{j-1}} = -\left(\frac{2r_{j-1}}{r_{j-1}^2 + uR^2} \right) \alpha (r_j - r_{j-1})^{\alpha-1} - \ln\left(\frac{r_m^2 + uR^2}{r_{j-1}^2 + uR^2} \right) \alpha(\alpha-1)(r_j - r_{j-1})^{\alpha-2}, \tag{13.65}$$

for all $2 \le j \le m-1$, and

$$\begin{aligned} \frac{\partial f_j(\mathbf{r})}{\partial r_j} = {} & \ln\left(\frac{r_m^2 + uR^2}{r_{j-1}^2 + uR^2} \right) \alpha(\alpha-1)(r_j - r_{j-1})^{\alpha-2} \\ & - 2\left(\frac{uR^2 - r_j^2}{(r_m^2 + uR^2)^2} \right)\left((r_{j+1} - r_j)^{\alpha} + c\right) + \left(\frac{4r_j}{r_j^2 + uR^2} \right) \alpha (r_{j+1} - r_j)^{\alpha-1} \\ & + \ln\left(\frac{r_m^2 + uR^2}{r_j^2 + uR^2} \right) \alpha(\alpha-1)(r_{j+1} - r_j)^{\alpha-2}, \end{aligned} \tag{13.66}$$

for all $1 \le j \le m-1$, and

$$\frac{\partial f_j(\mathbf{r})}{\partial r_{j+1}} = -\left(\frac{2r_j}{r_j^2 + uR^2} \right) \alpha (r_{j+1} - r_j)^{\alpha-1} - \ln\left(\frac{r_m^2 + uR^2}{r_j^2 + uR^2} \right) \alpha(\alpha-1)(r_{j+1} - r_j)^{\alpha-2}, \tag{13.67}$$

for all $1 \le j \le m-2$, and

$$\frac{\partial f_j(\mathbf{r})}{\partial r_k} = 0, \tag{13.68}$$

for all $1 \le j \le m-1$ and $k \ne j-1, j, j+1$.

13.7 NUMERICAL EXAMPLES

In this section, we demonstrate numerical examples for both uniform and nonuniform sensor distributions.

13.7.1 Uniform Distributions

To show a numerical example of optimal network design for a uniform distribution of sensors, we set $c = 5000$ and $R = 200$. We notice that Newton's algorithm can only find $\mathbf{r} = (r_1, r_2, \ldots, r_{m-1})$ for m not exceeding a certain limit, m^*. The value of m^* is 3 for $\alpha = 2$, 15 for $\alpha = 3$, 32 for $\alpha = 4$, 48 for $\alpha = 5$, and 63 for $\alpha = 6$. In fact, m^* is the optimal number of annuli when all the annuli have the same width [34].

In Table 13.1, we show the optimal network design $(r_1, r_2, \ldots, r_m)$ when $\alpha = 3$ and $m = 15$. We also compare r_j with $j(R/m)$, that is, the value of r_j when all the annuli have the same width. It is easily

TABLE 13.1
Optimal Network Design

j	r_j	$j(R/m)$
0	0.00	0.00
1	13.74	13.33
2	27.44	26.67
3	41.12	40.00
4	54.76	53.33
5	68.38	66.67
6	81.96	80.00
7	95.51	93.33
8	109.01	106.67
9	122.47	120.00
10	135.86	133.33
11	149.18	146.67
12	162.39	160.00
13	175.41	173.33
14	188.12	186.67
15	200.00	200.00

Note: $\alpha = 3$, $m = 15$, uniform distribution.

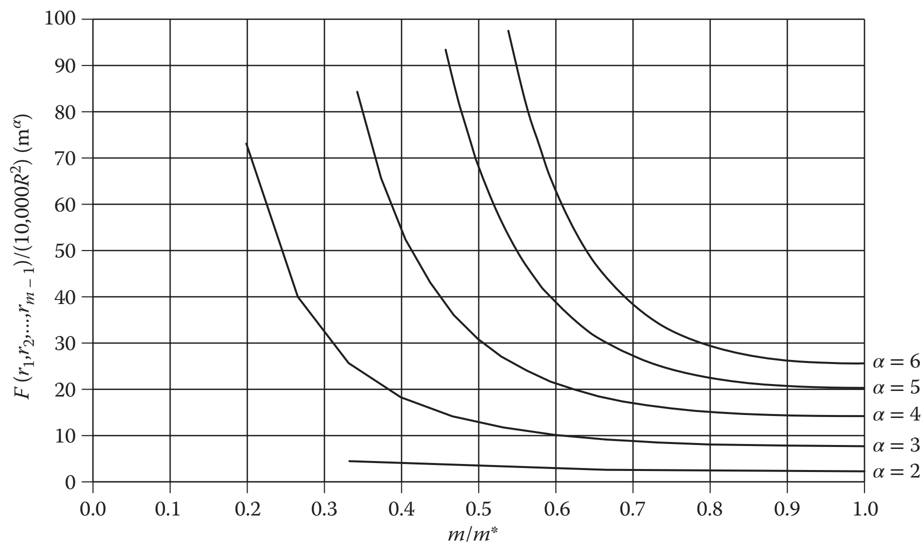

FIGURE 13.3 $F(r_1, r_2, \ldots, r_{m-1})$ vs. number of annuli m (uniform distribution).

observed that $F(r_1, r_2, \ldots, r_{m-1})$ is minimized when annuli have different widths. Furthermore, from A_1 to A_m, annuli have decreasing widths, that is, $r_1 - r_0 > r_2 - r_1 > \cdots > r_m - r_{m-1}$.

In Figure 13.3, we display the value of $F(r_1, r_2, \ldots, r_{m-1})$ (actually,

$$F(r_1, r_2, \ldots, r_{m-1}) / (10{,}000R^2) \tag{13.69}$$

is shown) for $\alpha = 2,3,4,5,6$, where $1 \le m \le m^*$. We observe that for all $\alpha > 2$, as m increases, $F(r_1, r_2, \ldots, r_{m-1})$ decreases rapidly. $F(r_1, r_2, \ldots, r_{m-1})$ reaches its minimum value at m^*. Thus, the energy used for data transmission is very sensitive to the choice of m.

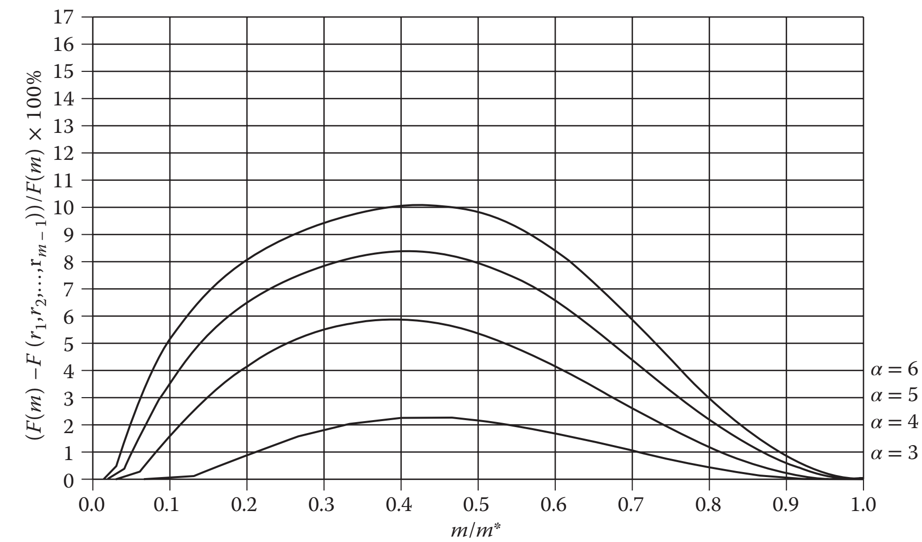

FIGURE 13.4 Relative reduction of $F(r_1, r_2, \ldots, r_{m-1})$ vs. number of annuli m (uniform distribution).

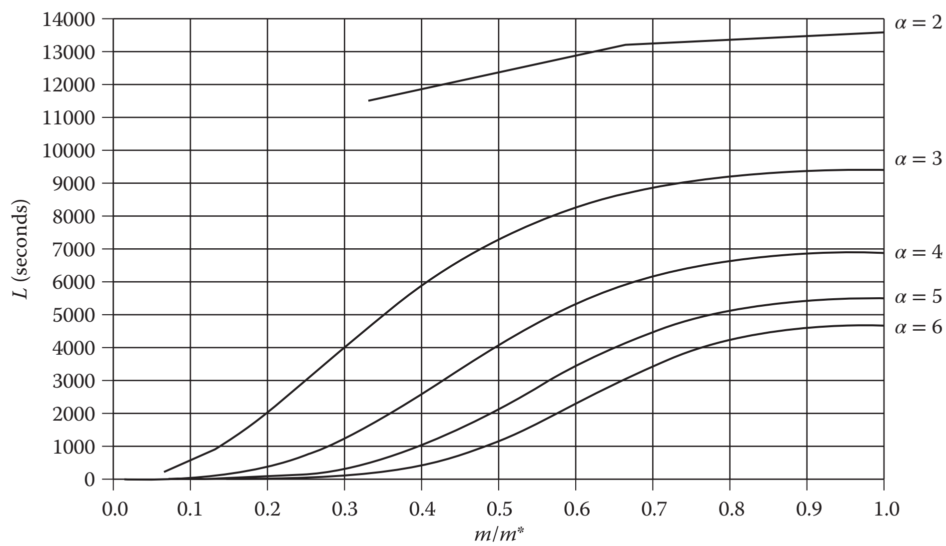

FIGURE 13.5 Network lifetime L vs. number of annuli m (uniform distribution).

Let $F(m)$ denote $F(r_1, r_2, \ldots, r_{m-1})$ when all the annuli have the same width, that is, $r_j = jr$ with $r = R/m$, for all $1 \le j \le m$. In Figure 13.4, we show the relative reduction of $F(r_1, r_2, \ldots, r_{m-1})$ when compared with $F(m)$, that is,

$$\left(\frac{F(m) - F(r_1, r_2, \ldots, r_{m-1})}{F(m)}\right) \times 100\%. \tag{13.70}$$

For $\alpha = 2$, the relative reduction is small (less than 0.7%) and not shown in the figure. For $\alpha > 2$, the relative reduction is noticeable, except for m close to 1 and m^*.

In Figure 13.5, we demonstrate network lifetime L as a function of m, where $1 \le m \le m^*$, and show the effect of α on L. We set $p = 6$, $a = 0.002$, $c = 5000$, $\mu = 0.03$, $R = 200$, $E/N = 100$ J, and $\alpha = 2, 3, 4, 5, 6$. We observe that for all α, as m increases, L increases rapidly. L reaches its maximum value at m^*. Thus, the network lifetime is very sensitive to the choice of m. Furthermore, the value of α also has a noticeable impact on the network lifetime.

Let L^* denote network lifetime when all the annuli have the same width. In Figure 13.6, we show the relative increment of L when compared with L^*, that is,

$$\left(\frac{L-L^*}{L^*}\right)\times 100\%. \tag{13.71}$$

For $\alpha = 2$, the relative increment is small (less than 0.13%) and not shown in the figure. For $\alpha > 2$, the relative increment is noticeable, except for m close to 1 and m^*.

AU: Please verify the accuracy of this sequence. In similar arrangements, these are written as "$\alpha = 6$ for $m = 63$" Or "$\alpha = 6$ with $m = 63$ "

In Figure 13.7, we show the normalized optimal energy allocation $E_j/(E/N)/E_1$, where $1 \le j \le m$. We set $p = 6$, $a = 0.002$, $c = 5000$, $\mu = 0.03$, $R = 200$, and $\alpha = 2$ and $m = 3$, $\alpha = 3$ and $m = 15$, $\alpha = 4$ and $m = 32$, $\alpha = 5$ and $m = 48$, $\alpha = 6$ and $m = 63$. Each m is the optimal choice for the

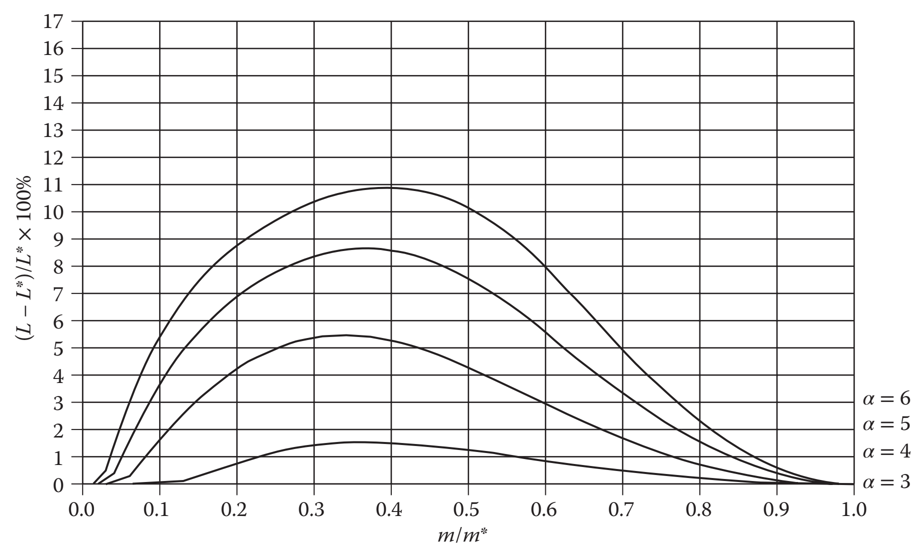

FIGURE 13.6 Relative increment of L vs. number of annuli m (uniform distribution).

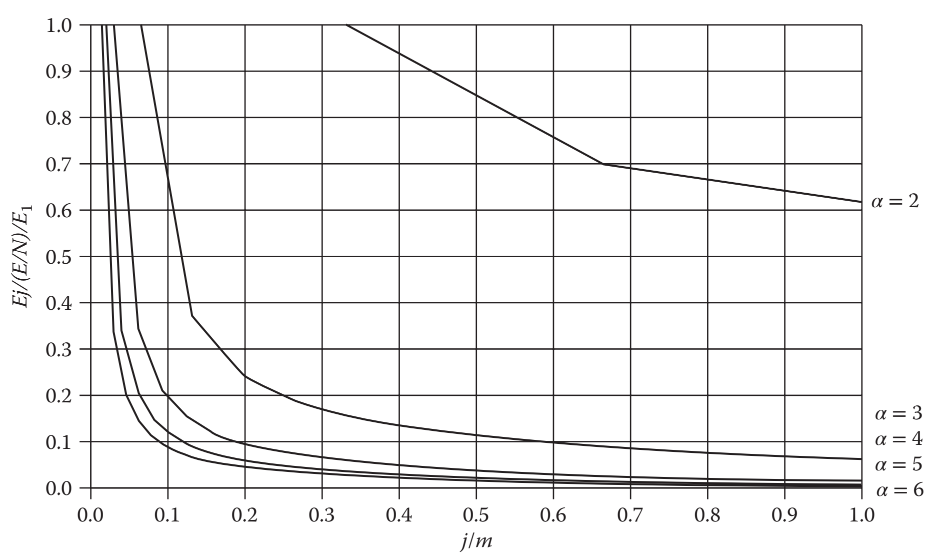

FIGURE 13.7 Optimal energy allocation E_j vs. j (uniform distribution).

corresponding α. It is observed that an optimal energy allocation is not balanced. In particular, we have $E_1 > E_2 > \cdots > E_m$. Sensors closer to a sink receive significantly more energy than sensors far away from the sink. Such an imbalance increases as α increases.

13.7.2 Nonuniform Distributions

To show a numerical example of optimal network design for a nonuniform distribution of sensors, we consider the following nonuniform sensor distribution function:

$$f(r) = \left(\frac{N}{\pi \ln(1+1/u)}\right)\left(\frac{1}{r^2+uR^2}\right). \tag{13.72}$$

In Figure 13.8, we display the above $f(r)$, where $0 \le r \le R$, assuming that $N = 10{,}000$, $R = 200$, and $u = 0.125, 0.250, 0.500, 1.000, 2.000, 4.000, 8.000$. It can be seen that as u increases, $f(r)$ approaches the uniform distribution $f(r) = N/(\pi R^2) = 0.0795774$.

In the following, we continue to use the previous nonuniform sensor density function, where $u = 0.5$, that is, the ratio of the largest density to the smallest density is 3. Again, we set $c = 5000$ and $R = 200$. We notice that the value of m^*, that is, the optimal number of annuli when all the annuli have the same width, is identical to that of the uniform distribution [34].

In Table 13.2, we show the optimal network design $(r_1, r_2, ..., r_m)$ for a nonuniform distribution of sensors when $\alpha = 3$ and $m = 15$. We also compare r_j with $j(R/m)$, that is, the value of r_j when all the annuli have the same width. It is observed that $F(r_1, r_2, ..., r_{m-1})$ is minimized when the annuli have different widths. Furthermore, from A_1 to A_m, annuli have decreasing widths, that is, $r_1 - r_0 > r_2 - r_1 > \cdots > r_m - r_{m-1}$.

In Figure 13.9, we display the value of $F(r_1, r_2, ..., r_{m-1})$ (actually,

$$F(r_1, r_2, ..., r_{m-1}) / (10{,}000 \ln(1+1/u)) \tag{13.73}$$

is shown for comparison with Figure 13.3) for $\alpha = 2, 3, 4, 5, 6$, where $1 \le m \le m^*$. As expected, the behavior of $F(r_1, r_2, ..., r_{m-1})$ is similar to and less than that of the uniform distribution in Figure 13.3.

In Figure 13.10, we show the relative reduction of $F(r_1, r_2, ..., r_{m-1})$ when compared with $F(m)$, that is, $F(r_1, r_2, ..., r_{m-1})$ when all the annuli have the same width. For $\alpha = 2$, the relative reduction is small (less than 0.6%) and not shown in the figure. As expected, the behavior of the relative reduc

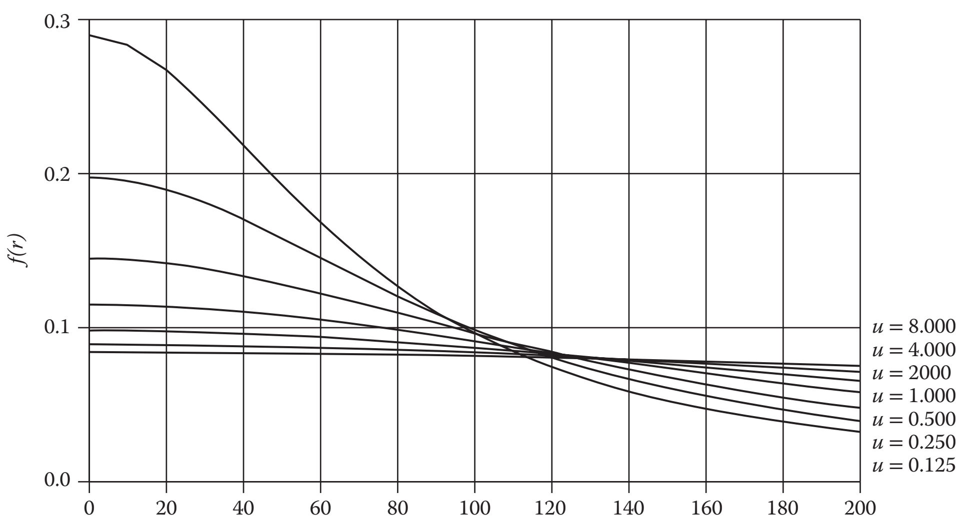

FIGURE 13.8 Nonuniform sensor distribution functions.

TABLE 13.2
Optimal Network Design

j	r_j	$j(R/m)$
0	0.00	0.00
1	13.72	13.33
2	27.39	26.67
3	41.00	40.00
4	54.57	53.33
5	68.09	66.67
6	81.58	80.00
7	95.02	93.33
8	108.43	106.67
9	121.79	120.00
10	135.11	133.33
11	148.38	146.67
12	161.58	160.00
13	174.66	173.33
14	187.56	186.67
15	200.00	200.00

Note: $\alpha = 3$, $m = 15$, nonuniform distribution.

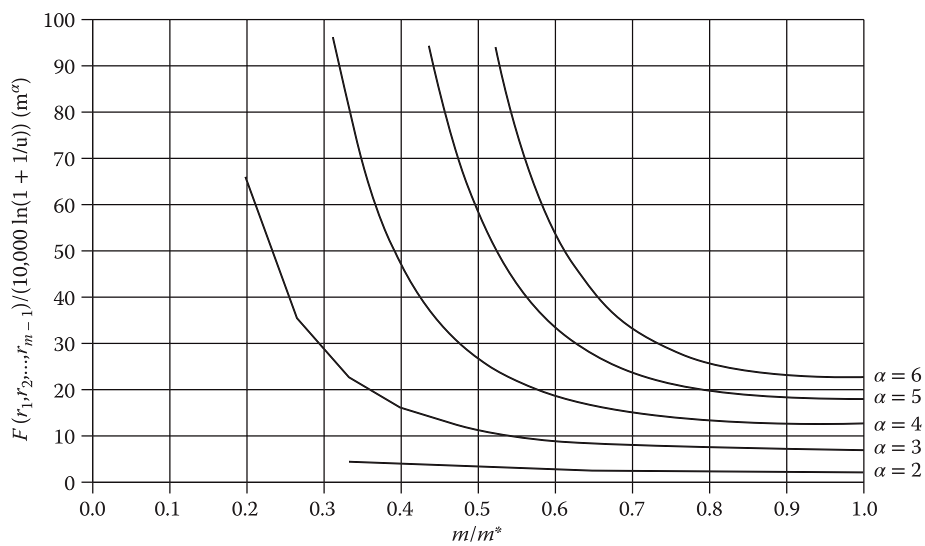

FIGURE 13.9 $F(r_1, r_2, \ldots, r_{m-1})$ vs. number of annuli m (nonuniform distribution).

tion is similar to that of a uniform distribution in Figure 13.4. Furthermore, the relative reduction is greater than that of the uniform distribution in Figure 13.4, which is less obvious.

In Figure 13.11, we demonstrate network lifetime L as a function of m, where $1 \le m \le m^*$, and show the effect of α on L. We set $p = 6$, $a = 0.002$, $c = 5000$, $\mu = 0.03$, $R = 200$, $E/N = 100$ J, and $\alpha = 2,3,4,5,6$. As expected, the behavior of L is similar to and greater than that of the uniform distribution in Figure 13.5.

In Figure 13.12, we show the relative increment of L when compared with L^*, that is, the network lifetime when all the annuli have the same width. For $\alpha = 2$, the relative increment is small (less than 0.1%) and not shown in the figure. As expected, the behavior of the relative increment is similar to that of the uniform distribution in Figure 13.6. Furthermore, the relative increment is greater than

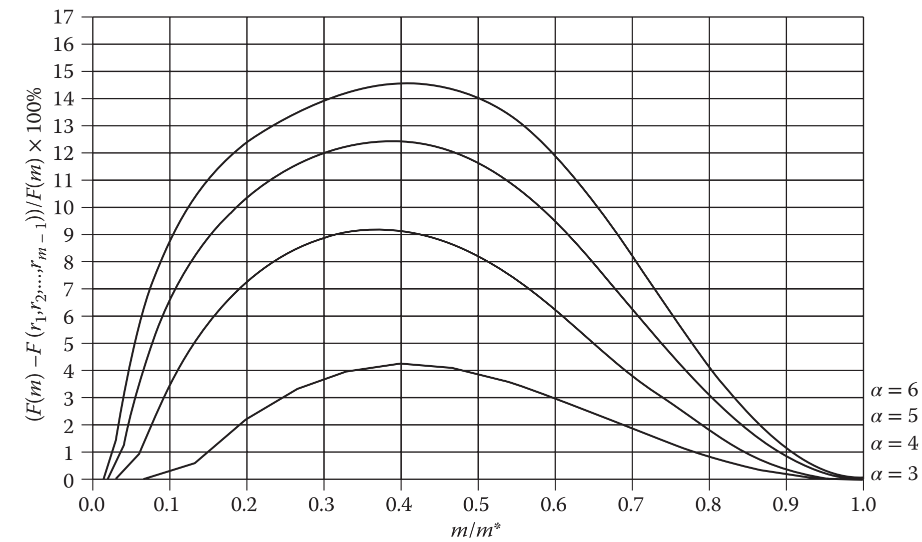

FIGURE 13.10 Relative reduction of $F(r_1, r_2, \ldots, r_{m-1})$ vs. number of annuli m (nonuniform distribution).

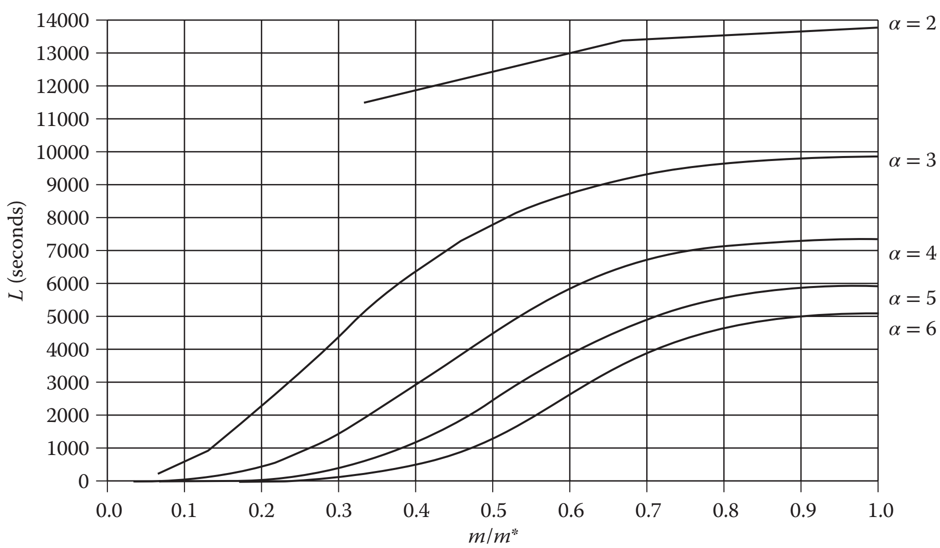

FIGURE 13.11 Network lifetime L vs. number of annuli m (nonuniform distribution).

that of the uniform distribution in Figure 13.6, which is less obvious but a direct consequence of Figure 13.10.

In Figure 13.13, we show the normalized optimal energy allocation, that is, $E_j/(E/N)/E_1$, where $1 \le j \le m$. We set $p = 6$, $a = 0.002$, $c = 5000$, $\mu = 0.03$, $R = 200$, and $\alpha = 2$ and $m = 3$, $\alpha = 3$ and $m = 15$, $\alpha = 4$ and $m = 32$, $\alpha = 5$ and $m = 48$, and $\alpha = 6$ and $m = 63$. Each m is the optimal choice for the corresponding α. As expected, the optimal energy allocation is similar to but more balanced than that in Figure 13.7.

Finally, in Figure 13.14, we demonstrate network lifetime L as a function of u, where $0 < u \le 5$, and show the impact of u on L. We set $p = 6$, $a = 0.002$, $c = 5000$, $\mu = 0.03$, $R = 200$, $E/N = 100$, and $\alpha = 2$ with $m = 3$, $\alpha = 3$ with $m = 15$, $\alpha = 4$ with $m = 32$, $\alpha = 5$ with $m = 48$, and $\alpha = 6$ with $m = 63$. We observe that when $u < 1$, the network lifetime can be increased noticeably by using a nonuniform sensor distribution. As u increases, the network lifetime approaches that of a uniform distribution.

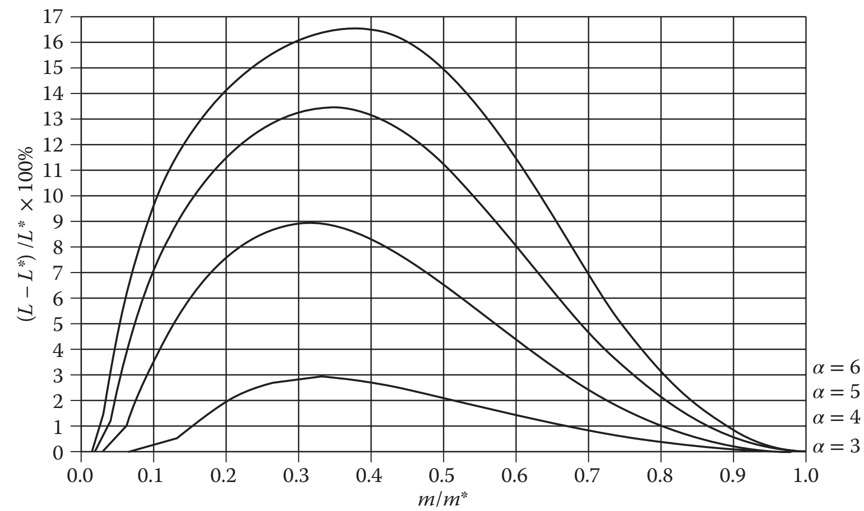

FIGURE 13.12 Relative increment of L vs. number of annuli m (nonuniform distribution).

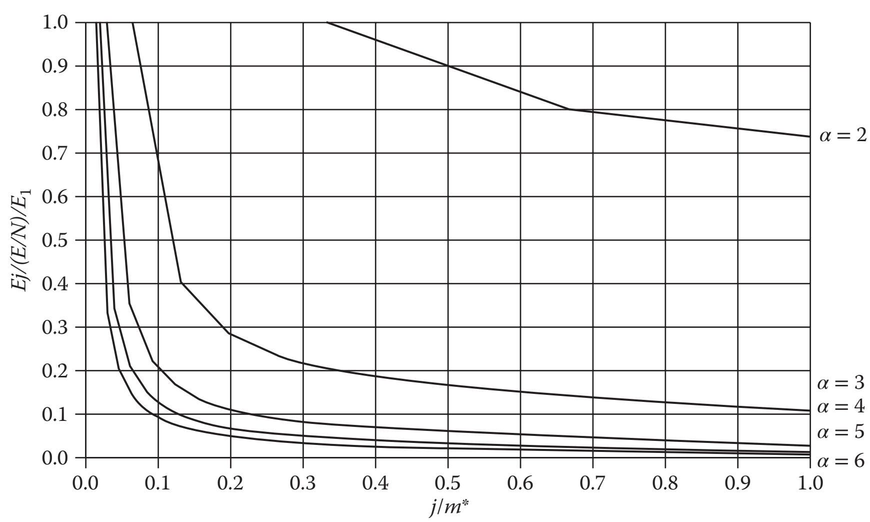

FIGURE 13.13 Optimal energy allocation E_j vs. j (nonuniform distribution).

13.8 FUTURE WORK

It has been observed that an optimal energy allocation that results in the maximum network lifetime is not balanced. Sensors close to a sink bear much heavier data transmission loads and consume much more energy than sensors far away from the sink. If such uneven energy depletion and allocation cause problems in real implementation and applications of WSNs, additional effort should be made to produce balanced energy allocation. Fortunately, just as with network lifetime, an optimal energy allocation is determined by a network design. This means that a WSN can be designed in such a way that an optimal energy allocation that yields the maximum network lifetime also satisfies certain balance constraints. For instance, the ratio of the maximum energy allocated to a sensor to the minimum energy allocated to a sensor does not exceed a certain limit. Future research efforts can be directed toward solving the problem of optimal network design with energy balance constraints, namely, to design a WSN such that an optimal energy allocation satisfies a given balance constraint.

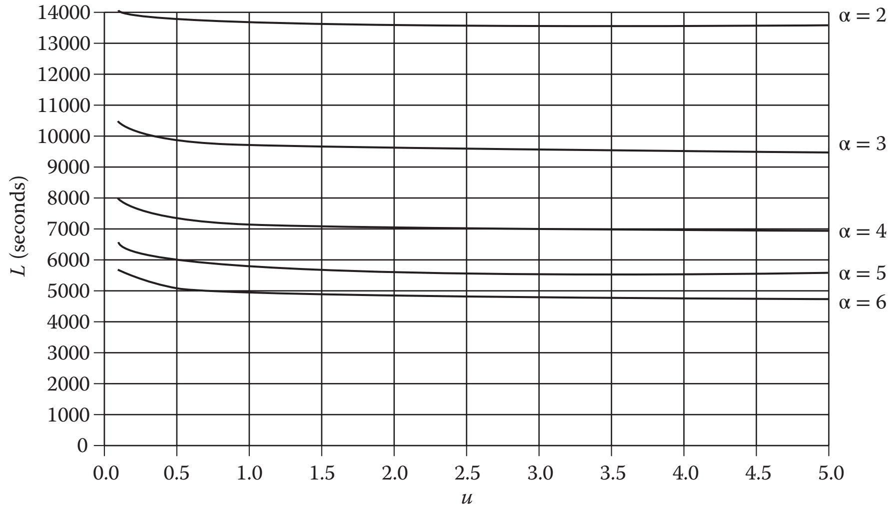

FIGURE 13.14 Network lifetime L vs. u (nonuniform distribution, varying α).

13.9 CONCLUSIONS

AU: References have been renumbered in both list and text citation in order to maintain sequential order. Please confirm whether it is appropriate.

We have developed an algorithm to find an optimal network design which maximizes the lifetime of a wireless sensor network obtained by optimal initial energy allocation for an arbitrary sensor distribution. Our strategy is to solve a nonlinear system of equations. We have shown that an optimal network design has different widths of annuli. Furthermore, for the same number of annuli, an optimal network design with variable annulus widths can yield a noticeably longer network lifetime than a network design with identical annulus widths.

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