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Location distribution of a mobile terminal and its application to paging cost reduction and minimization

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HIGHLIGHTS

- Analyze the location distribution of a mobile terminal in a paging area.
- Obtain expected costs of several selective paging methods.
- Find an important fact of progressive paging.

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ABSTRACT

Reducing the cost of dynamic mobility management in wireless communication networks has been an interesting and important issue. It is well known that by using a selective paging method, both costs for location update and terminal paging can be reduced significantly. However, an efficient selective paging method needs the information of the location distribution of a mobile terminal. Based on our previous results on random walks among rings of cell structures, we analyze the location distribution of a mobile terminal in a paging area when a phone call arrives, where the inter-call time and the cell residence time can have arbitrary probability distributions. Our analysis is conducted for both distance-based and movement-based location management schemes, and for two different call handling models, i.e., the call plus location update model and the call without location update model. Together with our earlier results on location distribution in time-based location management schemes, for several selective paging methods, including progressive paging methods, ring paging methods, and cell paging methods, we are able to obtain their expected costs of paging for distance-based, movement-based, and time-based location management schemes. We find that a progressive paging method with very small time delay can reduce the terminal paging cost dramatically, while further increasing the time delay does not result in noticeable reduction of terminal paging cost. Our work reported in this paper significantly extends our understanding of cost reduction and minimization of dynamic location management in wireless communication networks.

1. Introduction

1.1. Motivation

Reducing the cost of dynamic mobility management in wireless communication networks has been an interesting and important issue in the research community. The cost of location management includes two components, i.e., the cost of location update and the cost of terminal paging. It is well known that by using the simple paging method, there is a tradeoff between the costs of location update and terminal paging. In particular, increasing (reducing, respectively) the location update time reduces (increases, respectively) the location update cost, while increases (reduces, respectively) the terminal paging area and cost. It seems that reducing both location update cost and terminal paging cost is conflicting requirements. Fortunately, it is also well known that by using a selective paging method, which takes longer time than the simple paging method, both costs for location update and terminal paging can be reduced significantly. However, an efficient selective paging method needs the information of the location distribution of a mobile terminal. Due to lack of accurate analytical results of location distribution, there is lack of solid analytical results of the performance of selective paging methods.

The motivation of the present paper is to develop analytical results of location distribution of a mobile terminal inside a paging area (i.e., the probability that a mobile terminal is in a ring of

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a paging area) when a phone call arrives for distance-based and movement-based location management schemes. Such results are extremely useful in designing an efficient selective paging method to find a mobile terminal and analyzing its expected cost. Our results are obtained by using a very accurate ring level Markov chain as a mobility model to describe the movement of a mobile terminal [33]. Such results not only help to design and analyze selective paging methods, they are also very useful in finding more aggressive paging methods with faster paging speed and high quality of service.

1.2. Our contributions

The investigation in this paper makes the following significant contributions.

- First, based on our previous results on random walks among rings of cell structures [33], we analyze the location distribution of a mobile terminal in a paging area when a phone call arrives, where the inter-call time and the cell residence time can have arbitrary probability distributions. Our analysis is conducted for both distance-based and movement-based location management schemes, and for two different call handling models, i.e., the call plus location update model and the call without location update model.
- Second, together with our earlier results on location distribution in time-based location management schemes [35], for several selective paging methods, including progressive paging methods, ring paging methods, and cell paging methods, we are able to obtain their expected costs of paging for distance-based, movement-based, and time-based location management schemes.
- Third, we find that a progressive paging method with very small time delay can reduce the terminal paging cost dramatically, while further increasing the time delay does not result in noticeable reduction of terminal paging cost. Such an important finding has not been well documented in the existing literature.

Our work reported in this paper significantly extends our understanding of cost reduction and minimization of dynamic location management in wireless communication networks. It is worth to mention that while our earlier work [32,34,33,35] focused on the analysis of location update cost, this paper focuses on terminal paging cost reduction and minimization based on our new results on location distribution of a mobile terminal in a paging area when a phone call arrives.

The rest of the paper is organized as follows. In Section 2, we review related research. In Section 3, we provide preliminary information of our study. In Section 4, we describe mathematical background results used in this paper. In Section 5, we analyze the location distribution of a mobile terminal in a paging area. In Section 6, we present numerical data of location distribution. In Section 7, we discuss various selective paging methods. In Section 8, we demonstrate examples of paging cost reduction and minimization. In Section 9, we display numerical data to show paging cost reduction and minimization. Finally, in Section 10, we conclude the paper.

2. Related research

In this section, we review related research in analyzing dynamic mobility management and in reducing paging cost.

There are mainly three basic location update methods studied in the literature, i.e., the distance-based method, the movement-based method, and the time-based method [11]. Accordingly, there are three types of dynamic location management schemes, i.e., *distance-based location management schemes* (DBLMS), *movement-based location management schemes* (MBLMS), and *time-based location management schemes* (TBLMS). A DBLMS (an MBLMS and a TBLMS, respectively) employs the distance-based (the movement-based and the time-based, respectively) location update method. Furthermore, a DBLMS or an MBLMS or a TBLMS can use various terminal paging methods.

The design and analysis of any dynamic location management scheme depend on a mobility model of mobile terminals. Various mobility models have been proposed in the literature, including the shortest distance mobility model [3,2], the fluid flow model [7,12], the big move and the random walk models [14], the user mobility pattern scheme [16], the cell coordinates system [41], the isotropic diffusive motion model [44], one-dimensional Markov chains [4,11,15,40,49], and two-dimensional Markov chains [5,7,20,27,65].

Recently, we developed a ring level random walk model to accurately represent the movement of a mobile terminal in two-dimensional cellular structures. This Markov chain model has been used to analyze the paging area residence time and the cost of dynamic mobility management in a DBLMS [33]. It has also been used to study location distribution and reachability of a mobile terminal in a paging area, and to analyze the quality of service in a TBLMS [35]. It will continue to be employed in this paper to investigate location distribution in a DBLMS and an MBLMS, and to study paging cost reduction methods.

Dynamic mobility management is an important and fundamental research issue in wireless communication, and significant effort has been devoted by many researchers. The performance of movement-based location management schemes has been investigated in [5,11,22,36,37,32,34,38,43,61,66]. The performance of distance-based location management schemes has been studied in [2,9,11,15,27,33,40,41,63,67,68]. The performance of time-based location management schemes has been considered in [4,11,12,35,44,62]. Other studies were reported in [7,14,16,19–21,23,45,49,58], and some comparative studies were in [13,30,31,48,50]. Dynamic location management in a wireless communication network with a finite number of cells has been treated as an optimization problem which is solved by using bio-inspired methods such as simulated annealing, neural networks, and genetic algorithms [8,51,54,53,55,56]. The reader is also referred to the surveys in [6,29,52,25, (Ch. 15)], and [26, (Ch. 11)].

Terminal paging methods with low cost and time delay have been studied by several researchers [3,5,10,27,28,39,46,47,57,59,60,64]. Virtually all these studies focus on various selective paging methods. Two most important considerations for these methods are time delay and paging cost. A mobile terminal is located within certain geographical area divided into cells. At any moment, the mobile terminal resides in one of the cells. Each cell is associated with some probability that the mobile terminal is in the cell. A location distribution is a probability distribution of a mobile terminal in a geographical area. A selective paging method partitions the area into several disjoint regions, and proceeds in paging rounds when a phone call arrives. During each round, all cells in one region are polled by sending polling signals. The process is repeated until the mobile terminal is found, so that an incoming phone call can be routed to the mobile terminal. The number of paging rounds is the time delay, and the number of cells polled is the paging cost.

Minimizing both time delay and paging cost is conflicting requirements. A common strategy is to minimize paging cost with a time delay constraint. Given a location distribution over a search area and a time delay, the selective paging problem is to find a sequence of disjoint regions, such that the expected paging cost is minimized, subject to the constraint that the maximum or expected time delay does not exceed the given time limit [3,10,27,28,39,46,59,60]. The most notable result is a

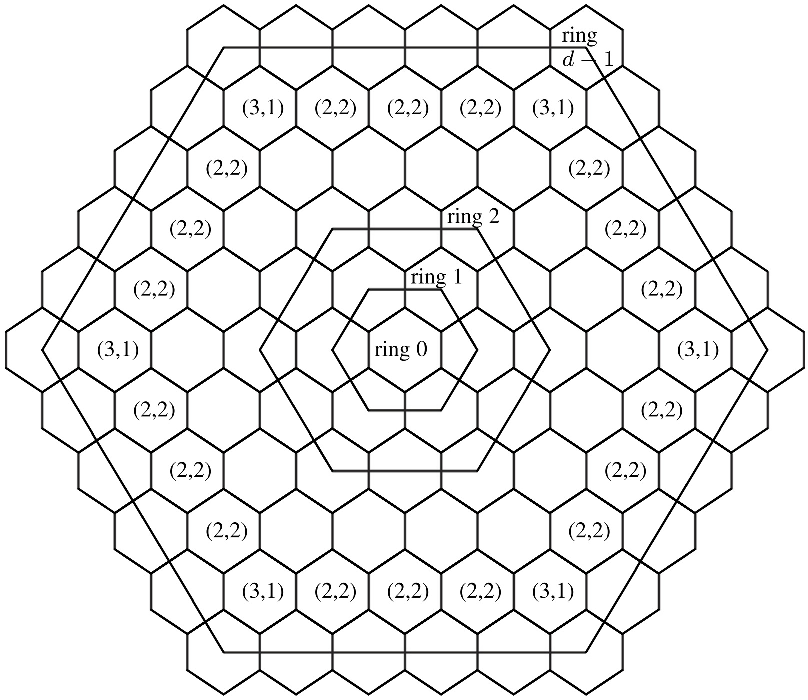

Fig. 1. The hexagonal cell configuration. A cell s in ring r marked with (x, y) means that s has x neighbors in ring $r + 1$ and y neighbors in ring $r - 1$. Such values are useful in calculating the probabilities of moving into adjacent rings, i.e., the probability a_r of moving into ring $r + 1$ and the probability b_r of moving into ring $r - 1$.

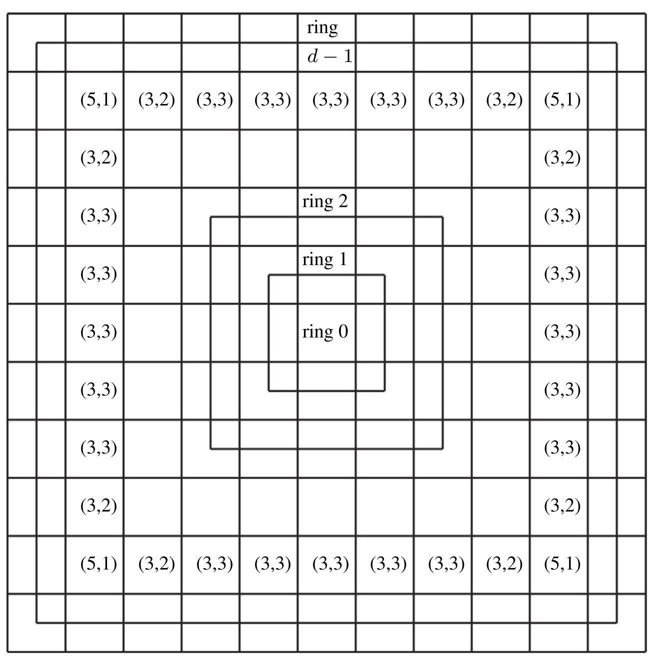

Fig. 2. The mesh cell configuration. A cell s in ring r marked with (x, y) has the same meaning as that in Fig. 1.

dynamic programming algorithm [28,39] to solve the problem in time proportional to the time delay and the number of cells [10]. However, all these studies suffer from a key issue which has not been well addressed, namely, location distribution of a mobile terminal in a paging area, although some effort was made for distance-based [27] and movement-based [5] location management schemes by using less accurate Markov chains.

3. Preliminary information

In this section, we provide preliminary information of our study.

3.1. Wireless communication networks

A wireless communication network has the common hexagonal cell configuration or mesh cell configuration. In the *hexagonal cell structure* (see Fig. 1), cells are hexagons of identical size and each cell has six neighbors. In the *mesh cell structure* (see Fig. 2), cells are squares of identical size and each cell has eight neighbors. Throughout the paper, we let q be a constant such that $q = 3$ for the hexagonal cell configuration and $q = 4$ for the mesh cell configuration. By using the constant q, the hexagonal cell configuration and the mesh cell configuration can be treated in a unified way. For instance, we can say that each cell has $2q$

neighbors without mentioning the particular cell structure. The network is homogeneous in the sense that the behavior of a mobile terminal is statistically the same in all the cells.

Let s be the cell registered by a mobile terminal in the last location update. The cells in a wireless networks can be divided into rings, where s is the center of the network and called ring 0. The $2q$ neighbors of s constitute ring 1. In general, the neighbors of all the cells in ring r, except those neighbors in rings $r-1$ and r, constitute ring $r+1$. For all $r \geq 0$, the cells in ring r have distance r to s. For all $r \geq 1$, the number of cells in ring r is $2qr$. Notice that the rings are defined with respect to s. When a mobile terminal updates its location to another cell s', s' becomes the center of the network, and ring r consists of the $2qr$ cells whose distance to s' is r.

When a mobile terminal u moves out of a cell s, it is normally assumed that u moves into one of s's $2q$ neighbors with equal probability [22,36], although this assumption is irrelevant in analyzing location update cost of movement-based schemes [32]. However, how u moves into the neighboring cells is very important in analyzing location update cost of distance-based schemes [33]; and in analyzing location distribution, reachability, and quality of service of time-base schemes [35]. In this paper, we further show that the movement characteristics of a mobile terminal decides its location distribution in a paging area of a movement-based scheme or a distance-based scheme, and such knowledge is critical in reducing paging cost of all schemes. The reason is that the way u moves into the neighboring cells determines how fast or slow u reaches the boundary of a paging area.

3.2. Location update methods

A mobile terminal u constantly moves from cell to cell. Such movement also results in movement from ring to ring. Let the sequence of cells visited by u before the next phone call be denoted as $s_0, s_1, s_2, \ldots, s_d, \ldots$, where $s_0 = s$ is u's last registered cell (not the cell in which u received the previous phone call) and considered as u's current location.

There are three location update methods proposed in the current literature, namely, the distance-based method, the movement-based method, and the time-based method. In the *distance-based location update method*, location update is performed as soon as u moves into a cell s_j in ring d, where d is a distance threshold, i.e., the distance of u from the last registered cell s is d, such that s_j is registered as u's current location. It is clear that $j \geq d$, i.e., it takes at least d steps for u to reach ring d. In the *movement-based location update method*, location update is performed as soon as u has crossed cell boundaries for d times since the last location update, where d is a movement threshold. It is clear that the sequence of registered cells for u is $s_d, s_{2d}, s_{3d}, \ldots$. In the *time-based location update method*, location update is performed every τ unit of time, where τ is a time threshold, regardless of the current location of u.

3.3. Terminal paging methods

In all dynamic location management schemes, a current *paging area* (PA) consists of rings $0, 1, 2, \ldots, d-1$, where d is some value appropriately chosen. We say that such a PA has radius d. Since the number of cells in ring r is $2qr$, for all $r \geq 1$, the total number of cells in a PA is $qd^2 - qd + 1$. It should be noticed that a PA is defined with respect to the current location of a mobile terminal, and is changed whenever a mobile terminal updates its location. The radius d of a PA can be adjusted in accordance with various cost and performance considerations. On the other hand, the location and size of a cell are fixed in a wireless network.

Two kinds of terminal paging methods have been proposed in the literature. In the *simple paging method*, the radius of a PA is fixed at d, where d is the distance threshold used by a distance-based location update method, or the movement threshold used by a movement-based location update method, or appropriately chosen in accordance with the time threshold used by a time-based location update method. In a *selective paging method*, cells in a PA or the entire wireless communication network are divided into disjoint regions, such that these regions are paged one after another successively, until a mobile terminal is found.

The simple paging method is the fastest method, since it sends polling signals only once. However, it is also the most expensive, since it sends polling signals to all cells in a paging area. On the other hand, a selective paging method trades cost with time, i.e., it covers a paging area gradually, with increased time delay but reduced paging cost.

3.4. Call handling models

We will consider two different call handling models.

In the *call plus location update* (CPLU) model, the location of a mobile terminal is updated each time a phone call arrives. That is, in addition to distance-based or movement-based or time-based location updates, the arrival of a phone call also initiates location update and defines a new PA. This causes the original location update cycle of a mobile terminal being interrupted. In the *call without location update* (CWLU) model, the arrival of a phone call has nothing to do with location update, that is, a mobile terminal still keeps its original location update cycles.

As seen from our previous studies [32,34,33,35], the analysis of the two different call handling models typically involves different types of renewal processes.

3.5. Notations

Throughout the paper, we use $\boldsymbol{P}[E]$ to denote the probability of an event E. For a random variable T, we use $\boldsymbol{E}(T)$ to represent the expectation of T and $\lambda_T = \boldsymbol{E}(T)^{-1}$. The probability density function (pdf) of T is $f_T(t)$, and the cumulative distribution function (cdf) of T is $F_T(t)$. The Laplace transform of $f_T(t)$ and $F_T(t)$ for a nonnegative random variable T is defined as

$$f_T^*(s) = \boldsymbol{E}(e^{-sT}) = \int_0^\infty e^{-st} f_T(t)dt,$$

and

$$F_T^*(s) = \int_0^\infty e^{-st} F_T(t)dt.$$

There are several important random variables in the study of dynamic location management. The *inter-call time* T_c is defined as the length of the time interval between two consecutive phone calls. The *cell residence time* T_s is defined as the time a mobile terminal stays in a cell before it moves into a neighboring cell. The *paging area residence time* T_m is defined as the time a mobile terminal stays in the current PA before it moves out of the PA. The *location update time* T_u is defined as the time between two consecutive location updates, which is actually the time for a mobile terminal to across d cell boundaries in an MBLMS, or the paging area residence time in a DBLMS, or the time threshold τ in a TBLMS. The quantity $\rho = \lambda_{T_c}/\lambda_{T_s}$ is the *call-to-mobility ratio*.

Dynamic location management is per-terminal based. A mobile terminal is specified by $f_{T_c}(t)$ and $f_{T_s}(t)$, where $f_{T_c(t)}$ is the call pattern and $f_{T_s(t)}$ is the mobility pattern.

The cost of dynamic location management contains two components, i.e., the cost of location update and the cost of terminal paging. The cost of location update is proportional to the number of location updates. If there are X_u location update between two consecutive phone calls, the cost of location update is $\Delta_u X_u$, where Δ_u is a constant. Since X_u is a random variable, the location update cost is actually calculated as $\Delta_u \boldsymbol{E}(X_u)$. The cost of terminal paging is proportional to the number of cells paged. For instance, if a PA has radius d, the cost of the simple paging method is $\Delta_p(qd^2 - qd + 1)$, where Δ_p is a constant. The cost of a selective paging method needs to be more carefully defined (see Section 7).

4. Mathematical background

In this section, we describe mathematical background results used in this paper.

4.1. Hyper-Erlang distributions

Let the pdf of an Erlang distribution be represented as

$$f_{\text{Erlang}}(\lambda, \gamma, t) = \frac{\lambda e^{-\lambda t}(\lambda t)^{\gamma-1}}{(\gamma-1)!},$$

and the cdf of an Erlang distribution be represented as

$$F_{\text{Erlang}}(\lambda, \gamma, t) = 1 - e^{-\lambda t} \sum_{j=0}^{\gamma-1} \frac{(\lambda t)^j}{j!}.$$

The class of hyper-Erlang distributions are used extensively in this study. A random variable X with a hyper-Erlang distribution has a pdf

$$f_X(t) = \sum_{i=1}^{k} w_i \left(\frac{\lambda_i e^{-\lambda_i t}(\lambda_i t)^{\gamma_i-1}}{(\gamma_i-1)!} \right) = \sum_{i=1}^{k} w_i f_{\text{Erlang}}(\lambda_i, \gamma_i, t),$$

where $w_1 + w_2 + \cdots + w_k = 1$. Special forms of hyper-Erlang distributions include hyperexponential distributions ($\gamma_i = 1$ for all $1 \le i \le k$); exponential distributions ($k = 1$ and $\gamma_1 = 1$); chi-square distributions ($k = 1$ and $\lambda_1 = 1/2$); Erlang distributions ($k = 1$).

Let T' be the residual time of T. The following theorem will be used in Section 5.1. The proof of the theorem is given in [35] (see Theorem 6 in [35]).

Theorem 1. *If T has a hyper-Erlang distribution, T' also has a hyper-Erlang distribution.*

4.2. Renewal processes

A *renewal process* is defined by a sequence of independent random variables $T_1, T_2, T_3, \ldots$, where $T_2, T_3, \ldots$ are a sequence of independent and identically distributed (i.i.d.) random variables with a common pdf, but T_1 may have a different pdf. (The reader is referred to [1] for a general introduction to the renewal theory.) A renewal process has many associated random variables and properties. The most interesting property related to our study is the number of renewals in a random period of time. We use $X(t)$ to denote the number of renewals in a time interval of length t. Let $S_j = T_1 + T_2 + \cdots + T_j$. If $S_j \le t < S_{j+1}$, we say that the number of renewals $X(t)$ in a time interval of length t is j.

Let X be the number of renewals in a random time interval of length T_c.

4.2.1. Ordinary renewal processes

An *ordinary renewal process* is defined by a sequence of i.i.d. random variables $T_1, T_2, T_3, \ldots$, with a common pdf $f_T(t)$ [18].

The following theorem gives the probability distribution of X in an ordinary renewal process, where T_c has a hyper-Erlang distribution with pdf

$$\begin{aligned} f_{T_c}(t) &= \sum_{i=1}^{k_c} w_{c,i} \left(\frac{\lambda_{c,i} e^{-\lambda_{c,i} t}(\lambda_{c,i} t)^{\gamma_{c,i}-1}}{(\gamma_{c,i}-1)!} \right) \\ &= \sum_{i=1}^{k_c} w_{c,i} f_{\text{Erlang}}(\lambda_{c,i}, \gamma_{c,i}, t), \end{aligned}$$

where $w_{c,1} + w_{c,2} + \cdots + w_{c,k_c} = 1$. The theorem generalizes Equation (14) in [17]. The theorem will be used in Sections 5.1 and 5.2. The proof of the theorem is given in [35] (see Theorem 2 in [35]).

Theorem 2. *For an ordinary renewal process, if T_c has a hyper-Erlang distribution, we have*

$$\boldsymbol{P}[X = j] = \sum_{i=1}^{k_c} w_{c,i} \left(\frac{\lambda_{c,i}^{\gamma_{c,i}}}{(\gamma_{c,i}-1)!} \right) \left(-\frac{\partial}{\partial s} \right)^{\gamma_{c,i}-1} \left[\left(\frac{1 - f_T^*(s)}{s} \right) (f_T^*(s))^j \right]_{s=\lambda_{c,i}},$$

for all $j \ge 0$.

4.2.2. Equilibrium renewal processes

An ordinary renewal process $T_1, T_2, T_3, \ldots$, where the pdf of T_1 is the residual time of an ordinary T_i, is called an *equilibrium renewal process*, which can be regarded as an ordinary renewal process that has been running for a long time before it is first observed [18]. Notice that if the T_i's have an exponential distribution, an equilibrium renewal process becomes an ordinary renewal process.

The following theorem gives the probability distribution of X in an equilibrium renewal process when T_c has a hyper-Erlang distribution. The theorem generalizes Equation (14) in [42]. The theorem will be used in Section 5.1. The proof of the theorem is given in [32] (see Theorem 5 in [32]).

Theorem 3. *For an equilibrium renewal process, if T_c has a hyper-Erlang distribution, we have*

$$\boldsymbol{P}[X = 0] = 1 - \lambda_T \sum_{i=1}^{k_c} w_{c,i} \left(\frac{\lambda_{c,i}^{\gamma_{c,i}}}{(\gamma_{c,i}-1)!} \right) \left(-\frac{\partial}{\partial s} \right)^{\gamma_{c,i}-1} \left[\frac{1 - f_T^*(s)}{s^2} \right]_{s=\lambda_{c,i}},$$

and

$$\boldsymbol{P}[X = j] = \lambda_T \sum_{i=1}^{k_c} w_{c,i} \left(\frac{\lambda_{c,i}^{\gamma_{c,i}}}{(\gamma_{c,i}-1)!} \right) \left(-\frac{\partial}{\partial s} \right)^{\gamma_{c,i}-1} \left[\left(\frac{1 - f_T^*(s)}{s} \right)^2 (f_T^*(s))^{j-1} \right]_{s=\lambda_{c,i}},$$

for all $j \ge 1$.

4.3. Random walks among rings

Consider a mobile terminal u in a cell s. After staying in s for T_s amount of time, u moves out of s and enters into one of the $2q$ neighbors of s. It is clear that the movement of a mobile terminal can be described by a random walk among the cells. Such a random walk can be characterized by a two-dimensional Markov chain, where for each cell, we have a state in the Markov chain associated with the cell. The transition probability from a cell to a neighboring cell is $1/(2q)$. While the random walk among the cells and the cell level Markov chain accurately describe the movement of a mobile terminal in any cell structure, the number of states is exactly the same as the number of cells, which causes excessive computation cost in obtaining numerical data [33].

To reduce the number of states, we consider a random walk among the rings and construct a ring level Markov chain which contains states $K_0, K_1, K_2, \ldots, K_d, \ldots$, where state K_r means that a mobile terminal u is in ring r, $r \geq 0$. Initially, u is in state K_0. Instead of the probabilities of moving into neighboring cells, we are interested in the probabilities of moving into adjacent rings, i.e., the probability a_r of moving into ring $r+1$ and the probability b_r of moving into ring $r-1$. Let p_{ij} denote the transition probability from K_i to K_j, where $i, j \geq 0$. Then, we have $p_{01} = 1$, $p_{r,r+1} = a_r$, $p_{r,r-1} = b_r$, $p_{r,r} = 1 - a_r - b_r$, for all $r \geq 1$. All other p_{ij}'s not specified above are zeros. The exact values of the a_r's and the b_r's are extremely difficult to obtain. Fortunately, very accurate approximate values can be derived [33].

We use $p_{ij}^{(n)}$ to denote the n-step transition probability from K_i to K_j, where $i, j \geq 0$. The following result is well known [24, p. 383], which will be used in Theorems 5–10.

Theorem 4. *If the nth power of P is $P^n = [g_{ij}]$, we have $p_{ij}^{(n)} = g_{ij}$, for all $i, j \geq 0$.*

Let $N(d)$ denote the expected number of steps for a mobile terminal to move out of a paging area of radius d. $N(d)$ is actually the expected number of steps for a random walk starting from K_0 to reach K_d. It was shown in [33] that $N(d) \approx \alpha_q \cdot d(qd-1)/(q-1)$, where α_3 is roughly 0.55 and α_4 is roughly 0.60.

5. Location distribution in a paging area

Let ξ_r, $r \geq 0$, denote the probability that a mobile terminal u is in ring r when a phone call arrives. The sequence $(\xi_0, \xi_1, \xi_2, \ldots)$ is called a *location distribution* of u. In this section, we analyze the location distribution of a mobile terminal in a paging area when a phone call arrives.

We have seen in [33,35] that although the location update cost can be significantly reduced in a DBLMS and a TBLMS, the total cost of mobility management can still be high for large d due to the quadratic growth rate of terminal paging cost of the simple paging method. The aim of this paper is to significantly reduce the cost of terminal paging. The only way to reduce paging cost is to reduce the radius of a PA. This is based on the observation that when a phone call arrives, or even after crossing cell boundaries many times, it is very likely that a mobile terminal is still far away from the boundary of a PA. To reduce the radius of a PA, we need to know the location distribution of a mobile terminal u in a PA of radius d when a phone call arrives, i.e., the probability ξ_r that u is in ring r, where $0 \leq r \leq d-1$. From such a probability distribution, we are able to know whether it is possible to reduce the size of a PA, and if possible, to what extent.

Before we proceed, we have the following observations.

An *ordinary location update* (OLU) is performed at the boundary of a cell residence time. In a DBLMS-CWLU and an MBLMS-CWLU, all location updates are ordinary. A *modified location update* (MLU) is performed in the midst of a cell residence time. In a DBLMS-CPLU and an MBLMS-CPLU, the arrival of a phone call initiates a modified location update. However, all subsequent location updates before the next phone call are ordinary.

Notice that in the time dimension, in a DBLMS-CPLU or an MBLMS-CPLU, each arriving phone call in the midst of a cell residence time initiates a modified location update and creates a new PA whose left boundary is in the midst of a cell residence time; however, all subsequent location updates before the next phone call coincide with the boundaries of cell residence times. In Fig. 3, the phone call C_1 initiates an MLU_1, such that the OLU shown in a dashed line after OLU_1 is not performed. The subsequent location updates OLU_2 and OLU_3 before C_2 are ordinary. The phone calls C_2 and C_3 initiate MLU_2 and MLU_3 respectively. The boundaries of paging area residence times in a DBLMS-CWLU and the boundaries of location update times in an MBLMS-CWLU coincide with the boundaries of cell residence times. All location updates in a DBLMS-CWLU or an MBLMS-CWLU are ordinary. The OLU U and U' in Fig. 4 are performed at the boundaries of cell residence times.

5.1. The CPLU model

In the CPLU model, there can be at most one phone call between two successive location updates, because each arriving phone call initiates a location update immediately. Consider any phone call C. Let X_s' denote the number of cell boundary crossings during the time interval of length T_c', i.e., the time between the moment of the last location update before C arrives (when the current PA is established) and the moment when C arrives. We consider two cases.

- *Case* 1. If the last location update is ordinary (e.g., C_1 and C_2 in Fig. 3), the sequence of cell residence times since the last location update is an ordinary renewal process. Furthermore, since OLU_1 and OLU_3 are random events before C_1 and C_2, the time between OLU_1 and C_1 and the time between OLU_3 and C_2 are residual inter-call times. If T_c has a hyper-Erlang distribution (as mentioned earlier, hyper-Erlang distributions include many other distributions as special cases, such as exponential distributions, which have been adopted by many researchers, e.g., [36–38]) with pdf

$$f_{T_c}(t) = \sum_{i=1}^{k_c} w_{c,i} \left(\frac{\lambda_{c,i}(\lambda_{c,i}t)^{\gamma_{c,i}-1} e^{-\lambda_{c,i}t}}{(\gamma_{c,i}-1)!} \right),$$

where $w_{c,1} + w_{c,2} + \cdots + w_{c,k_c} = 1$, by Theorem 1, T_c' also has a hyper-Erlang distribution with

$$f_{T_c'}(t) = \sum_{i=1}^{k_c} \sum_{l=0}^{\gamma_{c,i}-1} w_{c,i,l} \left(\frac{\lambda_{c,i} e^{-\lambda_{c,i}t} (\lambda_{c,i}t)^l}{l!} \right),$$

where

$$w_{c,i,l} = \frac{w_{c,i}}{\lambda_{c,i}} \left(\sum_{i=1}^{k_c} w_{c,i} \left(\frac{\gamma_{c,i}}{\lambda_{c,i}} \right) \right)^{-1},$$

for all $1 \leq i \leq k_c$ and $0 \leq l \leq \gamma_{c,i} - 1$. (See the proof of the theorem in [35].) By Theorem 2, the probability that there are j cell boundary crossings is

$$\boldsymbol{P}[X_s' = j] = \sum_{i=1}^{k_c} \sum_{l=0}^{\gamma_{c,i}-1} w_{c,i,l} \left(\frac{\lambda_{c,i}^l}{l!} \right) \left(-\frac{\partial}{\partial s} \right)^l \left[\left(\frac{1 - f_{T_s}^*(s)}{s} \right) (f_{T_s}^*(s))^j \right]_{s=\lambda_{c,i}},$$

for all $j \geq 0$.

- *Case* 2. If the last location update is modified (e.g., C_3 in Fig. 3), the sequence of cell residence times since the last location update is an equilibrium renewal process. Notice that the arrival of C_2 in the midst of a cell residence time causes the sequence of cell residence times after C_2 to be an equilibrium renewal

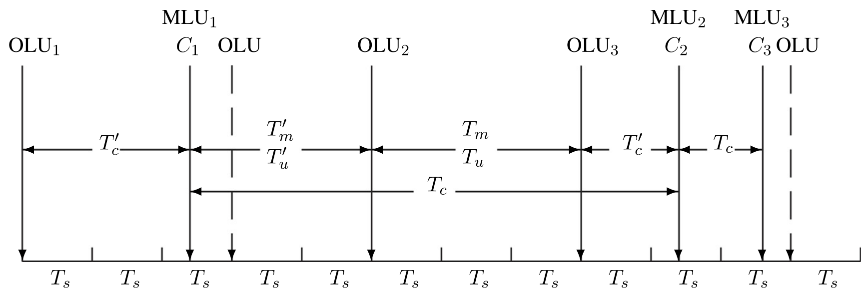

Fig. 3. Illustration of various time variables in a DBLMS-CPLU or an MBLMS-CPLU. The arriving phone calls C_1, C_2, and C_3 in the midst of cell residence times initiate modified location updates MLU$_1$, MLU$_2$, and MLU$_3$ respectively.

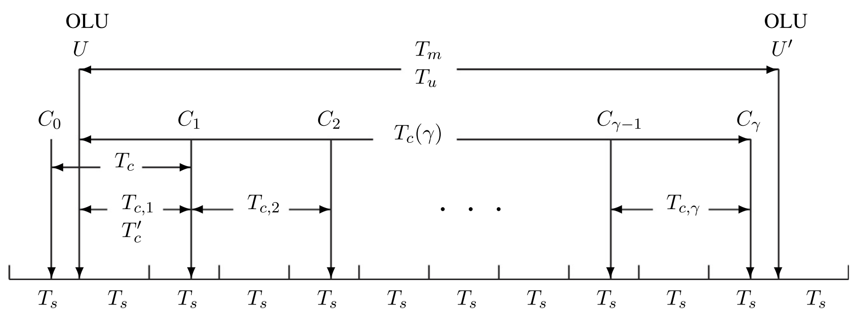

Fig. 4. Illustration of various time variables in a DBLMS-CWLU or an MBLMS-CWLU. All location updates are ordinary.

process. Since T'_c is simply T_c, by Theorem 3, the probability that there are j cell boundary crossings is

$$\boldsymbol{P}[X'_s = 0] = 1 - \lambda_{T_s} \sum_{i=1}^{k_c} w_{c,i} \left(\frac{\lambda_{c,i}^{\gamma_{c,i}}}{(\gamma_{c,i}-1)!} \right) \left(-\frac{\partial}{\partial s} \right)^{\gamma_{c,i}-1} \left[\frac{1 - f^*_{T_s}(s)}{s^2} \right]_{s=\lambda_{c,i}},$$

and

$$\boldsymbol{P}[X'_s = j] = \lambda_{T_s} \sum_{i=1}^{k_c} w_{c,i} \left(\frac{\lambda_{c,i}^{\gamma_{c,i}}}{(\gamma_{c,i}-1)!} \right) \left(-\frac{\partial}{\partial s} \right)^{\gamma_{c,i}-1} \left[\left(\frac{1 - f^*_{T_s}(s)}{s} \right)^2 (f^*_{T_s}(s))^{j-1} \right]_{s=\lambda_{c,i}},$$

for all $j \geq 1$.

Notice that if T_c and T_s have exponential distributions, the above two cases are identical.

Under the condition that $X'_s = j$, the probability that u is in ring r after crossing j cell boundaries is $p_{0r}^{(j)}$, where $r \geq 0$, and $p_{0r}^{(j)}$ is defined in Theorem 4. Thus, the probability that u is in ring r when a phone call arrives is

$$\xi_r = \sum_{j=0}^{\infty} \boldsymbol{P}[X'_s = j] p_{0r}^{(j)},$$

for all $r \geq 0$. The ξ_r's give a *location distribution* of u.

However, $\boldsymbol{P}[X'_s = j]$ is not exactly what given above. The above value of $\boldsymbol{P}[X'_s = j]$ is given as if there will be no more location update. In a real wireless communication network, we need to consider additional constraints caused by specific location update methods to calculate the exact value of $\boldsymbol{P}[X'_s = j]$, so that the actual location distribution of a mobile terminal can be obtained.

In the following, we show effective ways (Theorems 5, 6, 7, 9) to calculate the exact location distribution of a mobile terminal based on $\boldsymbol{P}[X'_s = j]$ and $\boldsymbol{P}[X_s(\gamma) = j]$ (to be defined later), $j = 0, 1, 2, \ldots$, once they are available, through analysis or simulation or experimentation, for arbitrary probability distribution of the inter-call time and the cell residence time.

5.1.1. Distance-based schemes

The following theorem gives the location distribution of a mobile terminal in a DBLMS-CPLU.

Theorem 5. *In a DBLMS-CPLU, the probability that a mobile terminal is in ring r of a PA of radius d when a phone call arrives is*

$$\xi_r = \frac{\sum_{j=0}^{\infty} \boldsymbol{P}[X'_s = j] p_{0r}^{(j)}}{\sum_{j=0}^{\infty} \left(\sum_{r=0}^{d-1} p_{0r}^{(j)} \right) \boldsymbol{P}[X'_s = j]},$$

for all $0 \leq r \leq d-1$, and any probability distributions of T_c and T_s. If T_c has a hyper-Erlang distribution, $\boldsymbol{P}[X'_s = j]$ is given by Case 1 or Case 2, depending on the type of the last location update.

Proof. In a distance-based scheme, when a phone call arrives, a mobile terminal u is still in the current PA, although the number X'_s of cell boundary crossings since the last location update can be any $j \geq 0$. So we need to figure out the probability of $X'_s = j$ under the condition that u is still in the current PA.

Let E denote the event that u is still in the current PA. Then, we have

$$\boldsymbol{P}[E] = \sum_{j=0}^{\infty} \boldsymbol{P}[E|X'_s = j] \boldsymbol{P}[X'_s = j].$$

Under the condition that $X'_s = j$, the probability that u is in ring r after crossing j cell boundaries is $p_{0r}^{(j)}$, where $0 \leq r \leq d-1$. Since u is in the current PA if and only if u is in rings $0, 1, 2, \ldots, d-1$, we have

$$\boldsymbol{P}[E|X'_s = j] = \sum_{r=0}^{d-1} p_{0r}^{(j)},$$

which implies that

$$\boldsymbol{P}[E] = \sum_{j=0}^{\infty} \left(\sum_{r=0}^{d-1} p_{0r}^{(j)} \right) \boldsymbol{P}[X_s' = j].$$

By Bayes' formula, under the condition that u is still in the current PA, the probability that u has made j cell boundary crossings is

$$\boldsymbol{P}[X_s' = j | E] = \frac{\boldsymbol{P}[E | X_s' = j] \boldsymbol{P}[X_s' = j]}{\boldsymbol{P}[E]},$$

which is actually

$$\boldsymbol{P}[X_s' = j | E] = \frac{\boldsymbol{P}[E | X_s' = j] \boldsymbol{P}[X_s' = j]}{\sum_{j=0}^{\infty} \boldsymbol{P}[E | X_s' = j] \boldsymbol{P}[X_s' = j]},$$

for all $j \geq 0$. Under the condition that u is in the current PA and that u has made j cell boundary crossings, the probability that u is in ring r is

$$\frac{p_{0r}^{(j)}}{\sum_{r=0}^{d-1} p_{0r}^{(j)}}.$$

Finally, the probability that a mobile terminal is in ring r of a PA of radius d when a phone call arrives is

$$\xi_r = \sum_{j=0}^{\infty} \boldsymbol{P}[X_s' = j | E] \frac{p_{0r}^{(j)}}{\sum_{r=0}^{d-1} p_{0r}^{(j)}}.$$

We have proved the theorem. ■

5.1.2. Movement-based schemes

The following theorem gives the location distribution of a mobile terminal in an MBLMS-CPLU.

Theorem 6. *In an MBLMS-CPLU, the probability that a mobile terminal is in ring r of a PA of radius d when a phone call arrives is*

$$\xi_r = \left(\sum_{j=0}^{d-1} \boldsymbol{P}[X_s' = j] \right)^{-1} \sum_{j=0}^{d-1} \boldsymbol{P}[X_s' = j] \frac{p_{0r}^{(j)}}{\sum_{r=0}^{d-1} p_{0r}^{(j)}},$$

for all $0 \leq r \leq d-1$*, and any probability distributions of* T_c *and* T_s*. If* T_c *has a hyper-Erlang distribution,* $\boldsymbol{P}[X_s' = j]$ *is given by Case* 1 *or Case* 2*, depending on the type of the last location update.*

Proof. In a movement-based scheme, when a phone call arrives, the number of cell boundary crossings since the last location update is no more than $d-1$, that is, we should have $0 \leq X_s' \leq d-1$. Under the condition that $0 \leq X_s' \leq d-1$, the probability that a mobile terminal u has made j cell boundary crossings is

$$\boldsymbol{P}[X_s' = j | 0 \leq X_s' \leq d-1] = \boldsymbol{P}[X_s' = j] \left(\sum_{j=0}^{d-1} \boldsymbol{P}[X_s' = j] \right)^{-1},$$

for all $0 \leq j \leq d-1$. Under the condition that $X_s' = j$, the probability that u is in ring r after crossing j cell boundaries is

$$\frac{p_{0r}^{(j)}}{\sum_{r=0}^{d-1} p_{0r}^{(j)}},$$

where $0 \leq r \leq d-1$. The probability that a mobile terminal is in ring r of a PA of radius d when a phone call arrives is

$$\xi_r = \sum_{j=0}^{d-1} \boldsymbol{P}[X_s' = j | 0 \leq X_s' \leq d-1] \frac{p_{0r}^{(j)}}{\sum_{r=0}^{d-1} p_{0r}^{(j)}}.$$

Thus, we have proved the theorem. ■

5.2. The CWLU model

In the CWLU model, there can be many phone calls between two successive location updates. Let $C_1, C_2, \ldots, C_\gamma, \ldots$ be a sequence of phone calls between two successive ordinary location updates U and U' (see Fig. 4).

Let $T_{c,\gamma}$ be the inter-call time between $C_{\gamma-1}$ and C_γ, where $\gamma \geq 2$, and

$$T_c(\gamma) = T_{c,1} + T_{c,2} + \cdots + T_{c,\gamma}$$

denote the time between the moment of the last location update before C_γ arrives (when the current PA is established) and the moment when C_γ arrives, where $\gamma \geq 1$, and $T_{c,1} = T_c'$. If T_c has an exponential distribution with pdf

$$f_{T_c}(t) = \lambda_c e^{-\lambda_c t},$$

all the $T_{c,\gamma}$'s have the same pdf, where $\gamma \geq 1$, and $T_c(\gamma)$ has an Erlang distribution,

$$f_{T_c(\gamma)}(t) = \frac{\lambda_c (\lambda_c t)^{\gamma-1} e^{-\lambda_c t}}{(\gamma-1)!}.$$

Let $X_s(\gamma)$ denote the number of cell boundary crossings during the time interval of length $T_c(\gamma)$. Since every location update is ordinary, the sequence of cell residence times since the last location update is an ordinary renewal process. By Theorem 2, the probability that there are j cell boundary crossings is

$$\boldsymbol{P}[X_s(\gamma) = j] = \frac{\lambda_c^\gamma}{(\gamma-1)!} \left(-\frac{\partial}{\partial s} \right)^{\gamma-1} \left[\left(\frac{1 - f_{T_s}^*(s)}{s} \right) (f_{T_s}^*(s))^j \right]_{s=\lambda_c},$$

for all $j \geq 0$.

5.2.1. Distance-based schemes

The following theorem gives the location distribution of a mobile terminal in a DBLMS-CWLU.

Theorem 7. *In a DBLMS-CWLU, the probability that a mobile terminal is in ring r of a PA of radius d when the* γ*th phone call arrives is*

$$\xi_r = \frac{\sum_{j=0}^{\infty} \boldsymbol{P}[X_s(\gamma) = j] p_{0r}^{(j)}}{\sum_{j=0}^{\infty} \left(\left(\sum_{r=0}^{d-1} p_{0r}^{(j)} \right) \boldsymbol{P}[X_s(\gamma) = j] \right)},$$

for all $0 \leq r \leq d-1$*, and any probability distributions of* T_c *and* T_s*.*

Proof. Following the same reasoning in the proof of Theorem 5, we get the theorem. ■

The following result gives the location distribution of a mobile terminal in a DBLMS-CWLU in the special case when T_s has an exponential distribution.

Theorem 8. *If T_s has an exponential distribution with*

$$f_{T_s}(t) = \lambda_s e^{-\lambda_s t},$$

we have

$$\boldsymbol{P}[X_s(\gamma)=j] = \frac{\rho^\gamma}{(\rho+1)^{\gamma+j}} \cdot \frac{(j+1)(j+2)\cdots(j+\gamma-1)}{(\gamma-1)!} = \frac{\rho^\gamma}{(\rho+1)^{\gamma+j}} \binom{j+\gamma-1}{\gamma-1},$$

and

$$\xi_r = \frac{\sum_{j=0}^{\infty}\left(\frac{\rho^\gamma}{(\rho+1)^{\gamma+j}}\binom{j+\gamma-1}{\gamma-1} p_{0r}^{(j)}\right)}{\sum_{j=0}^{\infty}\left(\left(\sum_{r=0}^{d-1} p_{0r}^{(j)}\right)\frac{\rho^\gamma}{(\rho+1)^{\gamma+j}}\binom{j+\gamma-1}{\gamma-1}\right)},$$

for all $0 \le r \le d-1$, *where* $\rho = \lambda_c/\lambda_s$ *is the* call—to—mobility ratio.

5.2.2. Movement-based schemes

The following theorem gives the location distribution of a mobile terminal in an MBLMS-CWLU.

Theorem 9. *In an MBLMS-CWLU, the probability that a mobile terminal is in ring r of a PA of radius d when the γth phone call arrives is*

$$\xi_r = \left(\sum_{j=0}^{d-1}\boldsymbol{P}[X_s(\gamma)=j]\right)^{-1}\sum_{j=0}^{d-1}\boldsymbol{P}[X_s(\gamma)=j]\frac{p_{0r}^{(j)}}{\sum_{r=0}^{d-1} p_{0r}^{(j)}},$$

for all $0 \le r \le d-1$, *and any probability distributions of* T_c *and* T_s.

Proof. Following the same reasoning in the proof of Theorem 6, we get the theorem. ■

The following theorem gives the location distribution of a mobile terminal in an MBLMS-CWLU in the special case when T_s has an exponential distribution.

Theorem 10. *If T_s has an exponential distribution with*

$$f_{T_s}(t) = \lambda_s e^{-\lambda_s t},$$

for both ordinary and modified last location updates, we have

$$\xi_r = \left(\sum_{j=0}^{d-1}\frac{\rho^\gamma}{(\rho+1)^{\gamma+j}}\binom{j+\gamma-1}{\gamma-1}\right)^{-1} \times \sum_{j=0}^{d-1}\frac{\rho^\gamma}{(\rho+1)^{\gamma+j}}\binom{j+\gamma-1}{\gamma-1}\frac{p_{0r}^{(j)}}{\sum_{r=0}^{d-1} p_{0r}^{(j)}},$$

for all $0 \le r \le d-1$.

6. Numerical data of location distribution

In this section, we present numerical data of location distribution.

Let us consider a DBLMS with $d = 20$. Both T_c and T_s have exponential distributions with $\rho = 0.1$. In Tables 1 and 2, we show ξ_r, the probability that a mobile terminal is in ring r of a PA of radius d when the γth phone call arrives in a DBLMS-CWLU, obtained by using Theorem 8. It is noticed that when $\gamma = 1$, ξ_r is also the probability that a mobile terminal is in ring r of a PA of radius d when a phone call arrives in a DBLMS-CPLU obtained by using Theorem 5.

Let us consider an MBLMS with $d = 20$. Both T_c and T_s have exponential distributions with $\rho = 0.1$. In Tables 3 and 4, we show ξ_r, the probability that a mobile terminal is in ring r of a PA of radius d when the γth phone call arrives in an MBLMS-CWLU, obtained by using Theorem 10. It is noticed that when $\gamma = 1$, ξ_r is also the probability that a mobile terminal is in ring r of a PA of radius d when a phone call arrives in an MBLMS-CPLU obtained by using Theorem 6.

It is observed that in an MBLMS, the probability that a mobile terminal is far away from the center of a PA is less than that in a DBLMS. Also, in the case where $q = 4$, a mobile terminal moves more quickly away from the center of a PA than the case where $q = 3$.

We would like to mention that results on location distribution similar to Sections 5–6 have been obtained for a TBLMS in [35].

7. Selective paging methods

In this section, we discuss various selective paging methods.

7.1. Progressive paging

The progressive paging method realizes paging cost reduction with increased paging time delay. Let $d_0, d_1, d_2, \ldots, d_k$ be a sequence of integers such that $0 = d_0 < d_1 < d_2 < \cdots < d_{k-1} < d_k = d$. We define k paging areas PA(d_1), PA(d_2), ..., PA(d_k), where PA(d_i) consists of all cells in rings $0, 1, 2, \ldots, d_i - 1$ in the current PA of radius d. The progressive paging method consists of k steps. In the ith step, where $1 \le i \le k$, cells in PA(d_i)–PA(d_{i-1}) (i.e., cells in PA(d_i) but not in PA(d_{i-1})) are paged. The k steps are executed successively until a mobile terminal is found. It is clear that the total time to find a mobile terminal is proportional to the number of steps in progressive paging. The parameter k guarantees that the running time of the progressive paging method does not exceed certain limit, that is, k is the worst-case time delay. When $k = 1$, the progressive paging method becomes the simple paging method. So we will assume that $k \ge 2$ for a nontrivial case. Also notice that $k \le d$.

A DBLMS (an MBLMS and a TBLMS, respectively) using the progressive paging method characterized by parameters $d_1, d_2, \ldots, d_k$ is represented by DBLMS($d_1, d_2, \ldots, d_k$) (MBLMS ($d_1, d_2, \ldots, d_k$) and TBLMS($\tau, d_1, \ldots, d_k$), respectively).

The performance of the progressive paging method is analyzed as follows. For $0 \le i \le k$, define $\pi(d_i)$ to be the probability that a mobile terminal u is in PA(d_i). It is easy to see that

$$\pi(d_i) = \sum_{r=0}^{d_i-1}\xi_r,$$

with $\pi(d_0) = 0$. Clearly, the probability that the ith step is executed, i.e., a mobile terminal is not in PA(d_{i-1}), is $1 - \pi(d_{i-1})$. Thus, the expected number of steps executed is

$$\sum_{i=1}^{k}(1-\pi(d_{i-1})) = k - \sum_{i=1}^{k-1}\pi(d_i).$$

Since the number of cells in PA(d_i) is $A(d_i) = qd_i^2 - qd_i + 1$ for $d_i > 0$ and $A(0) = 0$, the number of cells paged in the ith step is $A(d_i) - A(d_{i-1})$, and the cost of the ith step is $\Delta_p(A(d_i) - A(d_{i-1}))$. With probability $\pi(d_i) - \pi(d_{i-1})$, the number of cells paged is $A(d_i)$.

The following three theorems give the expected cost to find a mobile terminal in a DBLMS($d_1, d_2, \ldots, d_k$) and an MBLMS($d_1, d_2, \ldots, d_k$) and a TBLMS($\tau, d_1, \ldots, d_k$), respectively.

Table 1
Location distribution in a DBLMS ($q = 3, d = 20$).

r	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 4$	$\gamma = 5$
0	0.1269843033	0.0316470625	0.0152434216	0.0101561282	0.0077513740
1	0.2380144084	0.1326271720	0.0815814810	0.0578479346	0.0450242990
2	0.2114034661	0.1760602513	0.1295768189	0.0990601384	0.0797845794
3	0.1505316809	0.1660576089	0.1429908874	0.1188797038	0.1002520073
4	0.1005831200	0.1379229218	0.1363391884	0.1233506403	0.1095791933
5	0.0650100257	0.1064966263	0.1189890683	0.1168531976	0.1096426597
6	0.0411691293	0.0784019409	0.0977635329	0.1038206545	0.1029565304
7	0.0257143478	0.0558044441	0.0768528043	0.0878921200	0.0920715632
8	0.0159025468	0.0387328354	0.0584017585	0.0716351266	0.0791760951
9	0.0097612284	0.0263638354	0.0432029048	0.0566155047	0.0659208672
10	0.0059566118	0.0176667406	0.0312666565	0.0436160639	0.0534062479
11	0.0036178108	0.0116880702	0.0222183942	0.0328809444	0.0422603012
12	0.0021887141	0.0076498194	0.0155441093	0.0243258783	0.0327518373
13	0.0013195931	0.0049600144	0.0107258716	0.0176948389	0.0249033112
14	0.0007929033	0.0031879057	0.0073057638	0.0126645819	0.0185857797
15	0.0004743849	0.0020294803	0.0049075504	0.0089071167	0.0135898866
16	0.0002815914	0.0012745648	0.0032357434	0.0061218025	0.0096735474
17	0.0001639328	0.0007794673	0.0020641029	0.0040472494	0.0065900516
18	0.0000901016	0.0004456631	0.0012202615	0.0024606603	0.0041010864
19	0.0000400996	0.0002035756	0.0005696802	0.0011697150	0.0019787820
	1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000

Table 2
Location distribution in a DBLMS ($q = 4, d = 20$).

r	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 4$	$\gamma = 5$
0	0.1188356360	0.0266517068	0.0123883429	0.0083440697	0.0064809483
1	0.2456012941	0.1393782100	0.0876425971	0.0634677396	0.0503023841
2	0.2143902340	0.1803241786	0.1345874213	0.1044781774	0.0853918318
3	0.1488894737	0.1644978418	0.1426048573	0.1197311092	0.1020855083
4	0.0988386666	0.1348422077	0.1333809040	0.1212573645	0.1085005635
5	0.0640609340	0.1038657392	0.1155424211	0.1135088691	0.1068870214
6	0.0409090956	0.0767864023	0.0949315913	0.1004541239	0.0996353701
7	0.0258624396	0.0551300409	0.0750101445	0.0851852536	0.0889716490
8	0.0162312659	0.0387216758	0.0575083485	0.0698341625	0.0767360431
9	0.0101306624	0.0267351948	0.0430412651	0.0556899143	0.0642961958
10	0.0062956758	0.0182070165	0.0315841169	0.0433967425	0.0525621310
11	0.0038987248	0.0122593890	0.0227960774	0.0331562435	0.0420577577
12	0.0024072246	0.0081756265	0.0162200906	0.0248971238	0.0330128829
13	0.0014823132	0.0054059498	0.0113942239	0.0184014830	0.0254527420
14	0.0009101326	0.0035452452	0.0079055315	0.0133900680	0.0192732556
15	0.0005564780	0.0023030684	0.0054094646	0.0095743809	0.0142979593
16	0.0003373886	0.0014749234	0.0036303473	0.0066845052	0.0103170773
17	0.0002002728	0.0009180679	0.0023525524	0.0044803866	0.0071113618
18	0.0001118314	0.0005323270	0.0014078710	0.0027524770	0.0044639574
19	0.0000502563	0.0002451884	0.0006618314	0.0013158058	0.0021633593
	1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000

Table 3
Location distribution in an MBLMS ($q = 3, d = 20$).

r	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 4$	$\gamma = 5$
0	0.1472941171	0.0473923662	0.0285772011	0.0228588049	0.0203743576
1	0.2688641460	0.1872545458	0.1442424540	0.1238608408	0.1130704281
2	0.2295284790	0.2306698352	0.2087952607	0.1923535902	0.1815255766
3	0.1536873607	0.1959718495	0.2012931227	0.1983544433	0.1943580064
4	0.0937717473	0.1415263958	0.1606116923	0.1679804521	0.1707728813
5	0.0533209565	0.0911462096	0.1119624203	0.1232398701	0.1296147090
6	0.0283898895	0.0533279679	0.0697904060	0.0802184237	0.0869663820
7	0.0141301269	0.0285420117	0.0393074210	0.0468589978	0.0521739546
8	0.0065439890	0.0139898508	0.0200798335	0.0246837788	0.0281301014
9	0.0028021549	0.0062650069	0.0093009723	0.0117332556	0.0136441946
10	0.0011008277	0.0025508726	0.0038935805	0.0050204239	0.0059412343
11	0.0003930818	0.0009375466	0.0014643231	0.0019235007	0.0023111713
12	0.0001261470	0.0003080427	0.0004904410	0.0006545149	0.0007969008
13	0.0000358729	0.0000893143	0.0001445120	0.0001954942	0.0002407869
14	0.0000088750	0.0000224557	0.0000368342	0.0000504160	0.0000627282
15	0.0000018628	0.0000047775	0.0000079288	0.0000109634	0.0000137629
16	0.0000003197	0.0000008295	0.0000013906	0.0000019400	0.0000024548
17	0.0000000423	0.0000001109	0.0000001875	0.0000002636	0.0000003360
18	0.0000000039	0.0000000102	0.0000000174	0.0000000247	0.0000000317
19	0.0000000002	0.0000000005	0.0000000009	0.0000000012	0.0000000016
	1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000

Table 4
Location distribution in an MBLMS ($q = 4, d = 20$).

r	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 4$	$\gamma = 5$
0	0.1380467363	0.0400249628	0.0228962429	0.0181135812	0.0161509157
1	0.2767288175	0.1946665651	0.1511938754	0.1306311940	0.1197705152
2	0.2320746990	0.2339835230	0.2124970125	0.1963250078	0.1856934583
3	0.1517466956	0.1929124752	0.1979230438	0.1950009894	0.1911189666
4	0.0923384930	0.1383867836	0.1563815528	0.1631469434	0.1656142801
5	0.0530445981	0.0898392033	0.1096768392	0.1202274950	0.1260943282
6	0.0288253527	0.0535956582	0.0696364816	0.0796322711	0.0860122235
7	0.0147805526	0.0295481448	0.0403842766	0.0478712976	0.0530744781
8	0.0071178781	0.0150655082	0.0214602723	0.0262273588	0.0297537214
9	0.0031997915	0.0070877303	0.0104460659	0.0131021674	0.0151659788
10	0.0013330887	0.0030630052	0.0046436280	0.0059546569	0.0070150665
11	0.0005103144	0.0012079827	0.0018750350	0.0024503699	0.0029315846
12	0.0001776490	0.0004309331	0.0006822908	0.0009062778	0.0010990222
13	0.0000555182	0.0001374348	0.0002212863	0.0002980970	0.0003658254
14	0.0000153172	0.0000385682	0.0000629975	0.0000859098	0.0001065446
15	0.0000036464	0.0000093148	0.0000154045	0.0000212338	0.0000265819
16	0.0000007243	0.0000018733	0.0000031317	0.0000043579	0.0000055015
17	0.0000001138	0.0000002975	0.0000005021	0.0000007047	0.0000008965
18	0.0000000128	0.0000000337	0.0000000574	0.0000000812	0.0000001041
19	0.0000000008	0.0000000021	0.0000000037	0.0000000052	0.0000000067
	1.0000000000	1.0000000000	1.0000000000	1.0000000000	1.0000000000

Theorem 11. *The expected paging cost in a* DBLMS$(d_1, d_2, \ldots, d_k)$-*CPLU (respectively,* DBLMS$(d_1, d_2, \ldots, d_k)$-*CWLU) is*

$$\begin{aligned} D_p(d_1, d_2, \ldots, d_k) &= \Delta_p \sum_{i=1}^{k} (1 - \pi(d_{i-1}))(A(d_i) - A(d_{i-1})) \\ &= \Delta_p \sum_{i=1}^{k} (\pi(d_i) - \pi(d_{i-1}))A(d_i), \end{aligned}$$

where the $\pi(d_i)$'s are based on the ξ_r's given by Theorem 5 (respectively, Theorem 7).

Theorem 12. *The expected paging cost in an* MBLMS$(d_1, d_2, \ldots, d_k)$-*CPLU (respectively,* MBLMS$(d_1, d_2, \ldots, d_k)$-*CWLU) is*

$$\begin{aligned} M_p(d_1, d_2, \ldots, d_k) &= \Delta_p \sum_{i=1}^{k} (1 - \pi(d_{i-1}))(A(d_i) - A(d_{i-1})) \\ &= \Delta_p \sum_{i=1}^{k} (\pi(d_i) - \pi(d_{i-1}))A(d_i), \end{aligned}$$

where the $\pi(d_i)$'s are based on the ξ_r's given by Theorem 6 (respectively, Theorem 9).

Theorem 13. *The expected paging cost in a* TBLMS$(\tau, d_1, \ldots, d_k)$-*CPLU (respectively,* TBLMS$(\tau, d_1, \ldots, d_k)$-*CWLU) is*

$$\begin{aligned} C_p(\tau, d_1, \ldots, d_k) &= \Delta_p \sum_{i=1}^{k} (1 - \pi(d_{i-1}))(A(d_i) - A(d_{i-1})) \\ &= \Delta_p \sum_{i=1}^{k} (\pi(d_i) - \pi(d_{i-1}))A(d_i), \end{aligned}$$

where the $\pi(d_i)$'s are based on the ξ_r's given by Theorem 7 (respectively, Theorem 9) in [35].

We notice that although in the worst case when a mobile terminal u is not in PA(d_{k-1}), the paging cost to find u can be as large as $\Delta_p A(d)$, the expected cost $D_p(d_1, d_2, \ldots, d_k)$ and $M_p(d_1, d_2, \ldots, d_k)$ and $C_p(\tau, d_1, \ldots, d_k)$ can be much lower.

We are very interested in the following optimization problem, called the *progressive paging problem* (PPP), which is defined as follows: given d and $\xi_0, \xi_1, \xi_2, \ldots, \xi_{d-1}$, and k, where $2 \le k \le d$, find $d_1, d_2, \ldots, d_k$ such that $D_p(d_1, d_2, \ldots, d_k)$ or $M_p(d_1, d_2, \ldots, d_k)$ or $C_p(\tau, d_1, \ldots, d_k)$ is minimized.

The above problem can be solved by using a dynamic programming method as follows. Let $P(d', k)$ denote the minimum progressive paging cost for the PPP with parameters d' and $\xi_0, \xi_1, \xi_2, \ldots, \xi_{d'-1}$, and k, where $1 \le k \le d' \le d$. Then, we have

$$\begin{aligned} P(d', 1) &= \Delta_p \pi(d')A(d'), \quad 1 \le d' \le d; \\ P(d', k) &= \min_{k-1 \le d'' \le d'-1} (P(d'', k-1) + \Delta_p(\pi(d') \\ &\quad - \pi(d''))A(d')), \quad 2 \le k \le d' \le d. \end{aligned}$$

It is clear that the above dynamic programming method can be implemented in $O(d^2k)$ time.

One extreme progressive paging method is to set $k = d$ and $d_r = r$, for all $1 \le r \le d$. This scheme is called DBLMS$^*(d)$ or MBLMS$^*(d)$ or TBLMS$^*(\tau, d)$, and the resulted paging cost is

$$D_p^*(d) = D_p(1, 2, \ldots, d),$$

or

$$M_p^*(d) = M_p(1, 2, \ldots, d),$$

or

$$C_p^*(\tau, d) = C_p(\tau, 1, 2, \ldots, d).$$

When a mobile terminal u in ring $r - 1$, the minimum paging cost to find u is $A(r)$. Hence, the minimum expected paging cost to find u is

$$\Delta_p \sum_{r=1}^{d} \xi_{r-1} A(r),$$

which is exactly $D_p^*(d)$ or $M_p^*(d)$ or $C_p^*(\tau, d)$. The following three theorems state that if there is no paging time limitation, $D_p^*(d)$ and $M_p^*(d)$ and $C_p^*(\tau, d)$ guarantee optimality.

Theorem 14. *In a* DBLMS$(d_1, d_2, \ldots, d_k)$ *(*MBLMS$(d_1, d_2, \ldots, d_k)$, TBLMS$(\tau, d_1, \ldots, d_k)$*, respectively),* $D_p^*(d)$ *(*$M_p^*(d)$, $C_p^*(\tau, d)$*, respectively) gives the minimum expected progressive paging cost for all $1 \le k \le d$.*

Notice that $D_p^*(d)$ and $M_p^*(d)$ and $C_p^*(\tau, d)$ can be represented in various equivalent ways. For instance,

$$D_p^*(d) = \Delta_p \sum_{r=1}^{d} (1 - \pi(r-1))(A(r) - A(r-1))$$

$$= \Delta_p \sum_{r=1}^{d} \xi_{r-1} A(r)$$

$$= \Delta_p \sum_{r=1}^{d} \xi_{r-1}(S_0 + S_1 + \cdots + S_{r-1})$$

$$= \Delta_p \sum_{r=1}^{d} (1 - \xi_0 - \xi_1 - \cdots - \xi_{r-2}) S_{r-1},$$

where S_r is the number of cells in ring r, i.e., $S_0 = 1$ and $S_r = 2qr$ for all $r \geq 1$.

7.2. A search problem

Consider an object x which is somewhere in a search area A. The search area A is divided into cells, i.e., A is a set of cells. Assume that A has n cells $c_1, c_2, \ldots, c_n$. The location of x in A is given by a probability distribution function σ, such that the probability that x is in cell c is $\sigma(c)$, and

$$\sum_{c \in A} \sigma(c) = 1.$$

Cells can be grouped into regions. In other words, a region R is defined as a set of cells. The probability that x is in R is

$$p(R) = \sum_{c \in R} \sigma(c).$$

The cost to search a cell is given by a cost function η, where $\eta(c) > 0$. A search step is to search all the cells in a region R. The cost of a step searching R is

$$W(R) = \sum_{c \in R} \eta(c).$$

A search method is a sequence of regions $(R_1, R_2, \ldots, R_k)$ such that $R_1, R_2, \ldots, R_k$ are disjoint and $R_1 \cup R_2 \cup \cdots \cup R_k$ covers all the cells. We say that x is found in a step searching R_i if $x \in R_i$. The method searches $R_1, R_2, \ldots, R_k$ successively until x is found. The expected cost of searching by using a search method $(R_1, R_2, \ldots, R_k)$ is

$$C(R_1, R_2, \ldots, R_k) = \sum_{i=1}^{k} p(R_i)(W(R_1) + W(R_2) + \cdots + W(R_i)) = \sum_{i=1}^{k} (1 - p(R_1) - p(R_2) - \cdots - p(R_{i-1})) W(R_i).$$

The *search problem* (SP) is defined as follows: given a search area with n cells, a probability distribution function σ, a cost function η, and k, find a search method $(R_1, R_2, \ldots, R_k)$ such that the expected cost $C(R_1, R_2, \ldots, R_k)$ of searching is minimized.

The most difficult part of the SP is to find a partition of A into k disjoint regions $R_1, R_2, \ldots, R_k$. Once the k regions are given, $C(R_1, R_2, \ldots, R_k)$ can be minimized by searching the k regions in an order of $R_{j_1}, R_{j_2}, \ldots, R_{j_k}$ given by the following theorem.

Theorem 15. *For a given partition of a search area into k disjoint regions $R_1, R_2, \ldots, R_k$, the expected cost of searching is minimized by searching the k regions in an order of $R_{j_1}, R_{j_2}, \ldots, R_{j_k}$, where*

$$\frac{p(R_{j_1})}{W(R_{j_1})} \geq \frac{p(R_{j_2})}{W(R_{j_2})} \geq \cdots \geq \frac{p(R_{j_k})}{W(R_{j_k})}.$$

Proof. Consider the expected cost of searching by using a search method $(R_{j_1}, R_{j_2}, \ldots, R_{j_k})$,

$$C(R_{j_1}, R_{j_2}, \ldots, R_{j_k}) = \sum_{r=1}^{k} \left(p(R_{j_r}) \sum_{i=1}^{r} W(R_{j_i}) \right).$$

We compare the cost of two different methods

$$R_{j_1}, R_{j_2}, \ldots, R_{j_l}, R_{j_{l+1}}, \ldots, R_{j_k},$$

and

$$R_{j_1}, R_{j_2}, \ldots, R_{j_{l+1}}, R_{j_l}, \ldots, R_{j_k},$$

where $1 \leq l \leq k-1$, and the only difference is the exchange of R_{j_l} and $R_{j_{l+1}}$. Since

$$C(R_{j_1}, R_{j_2}, \ldots, R_{j_l}, R_{j_{l+1}}, \ldots, R_{j_k}) = \sum_{r=1}^{l-1} \left(p(R_{j_r}) \sum_{i=1}^{r} W(R_{j_i}) \right) + p(R_{j_l}) \sum_{i=1}^{l} W(R_{j_i}) + p(R_{j_{l+1}}) \sum_{i=1}^{l+1} W(R_{j_i}) + \sum_{r=l+2}^{k} \left(p(R_{j_r}) \sum_{i=1}^{r} W(R_{j_i}) \right),$$

and

$$C(R_{j_1}, R_{j_2}, \ldots, R_{j_{l+1}}, R_{j_l}, \ldots, R_{j_k}) = \sum_{r=1}^{l-1} \left(p(R_{j_r}) \sum_{i=1}^{r} W(R_{j_i}) \right) + p(R_{j_{l+1}}) \left(\sum_{i=1}^{l-1} W(R_{j_i}) + W(R_{j_{l+1}}) \right) + p(R_{j_l}) \sum_{i=1}^{l+1} W(R_{j_i}) + \sum_{r=l+2}^{k} \left(p(R_{j_r}) \sum_{i=1}^{r} W(R_{j_i}) \right),$$

we have

$$C(R_{j_1}, R_{j_2}, \ldots, R_{j_l}, R_{j_{l+1}}, \ldots, R_{j_k}) - C(R_{j_1}, R_{j_2}, \ldots, R_{j_{l+1}}, R_{j_l}, \ldots, R_{j_k}) = p(R_{j_{l+1}}) W(R_{j_l}) - p(R_{j_l}) W(R_{j_{l+1}}).$$

The above equation implies that

$$C(R_{j_1}, R_{j_2}, \ldots, R_{j_l}, R_{j_{l+1}}, \ldots, R_{j_k}) \leq C(R_{j_1}, R_{j_2}, \ldots, R_{j_{l+1}}, R_{j_l}, \ldots, R_{j_k})$$

if and only if

$$p(R_{j_{l+1}}) W(R_{j_l}) \leq p(R_{j_l}) W(R_{j_{l+1}}).$$

Hence, $C(R_{j_1}, R_{j_2}, \ldots, R_{j_k})$ is minimized if and only if

$$\frac{p(R_{j_l})}{W(R_{j_l})} \geq \frac{p(R_{j_{l+1}})}{W(R_{j_{l+1}})},$$

for all $1 \leq l \leq k-1$. This proves the theorem. ■

The following theorem states that more regions result in less cost.

Theorem 16. *For a partition of a search area into k disjoint regions $R_1, R_2, \ldots, R_k$, if we further divide R_l into two disjoint subregions R_l' and R_l'', then we have*

$$C(R_1, R_2, \ldots, R_l', R_l'', \ldots, R_k) < C(R_1, R_2, \ldots, R_l, \ldots, R_k).$$

Proof. It is easy to see that

$$C(R_1, R_2, \ldots, R_l', R_l'', \ldots, R_k) - C(R_1, R_2, \ldots, R_l, \ldots, R_k) = -p(R_l') W(R_l'') < 0.$$

This proves the theorem. ■

The above theorem implies that if there is no limitation on search time, i.e., $k = n$, the lowest possible cost of searching is achieved by a search method $(R_1, R_2, \ldots, R_n)$, where $R_i = \{c_{j_i}\}$ for all $1 \le i \le n$, and

$$\frac{\sigma(c_{j_1})}{\eta(c_{j_1})} \ge \frac{\sigma(c_{j_2})}{\eta(c_{j_2})} \ge \cdots \ge \frac{\sigma(c_{j_n})}{\eta(c_{j_n})},$$

with expected cost

$$C(R_1, R_2, \ldots, R_n) = \sum_{i=1}^{n} \sigma(c_{j_i})(\eta(c_{j_1}) + \eta(c_{j_2}) + \cdots + \eta(c_{j_i})).$$

7.3. Beyond progressive paging

The progressive paging method can be extended as follows.

7.3.1. Ring paging

By *ring paging* we mean each ring must be paged as an unbreakable unit. However, rings can be combined into groups and paged together in any way. We can divided the rings $0, 1, 2, \ldots, d-1$ into k disjoint groups $R_1, R_2, \ldots, R_k$, where $R_i \subseteq \{0, 1, 2, \ldots, d-1\}$. A ring paging method consists of k steps. In the ith step, where $1 \le i \le k$, rings in R_i are paged. The k steps are executed successively until a mobile terminal is found. Let

$$\pi(R) = \sum_{r \in R} \xi_r$$

be the probability that a mobile terminal u is in a ring of R. The probability that the ith step is executed is the probability that u is not in any ring of groups $R_1, R_2, \ldots, R_{i-1}$, i.e.,

$$1 - \pi(R_1) - \pi(R_2) - \cdots - \pi(R_{i-1}).$$

The number of cells in R_i is

$$N(R_i) = \sum_{r \in R_i} S_r,$$

where S_r is the number of cells in ring r, i.e., $S_0 = 1$ and $S_r = 2qr$ for all $r \ge 1$. The cost to page rings in group R_i is

$$W(R_i) = \Delta_p N(R_i) = \Delta_p \sum_{r \in R_i} S_r.$$

The expected cost of ring paging is

$$\begin{aligned} &C(R_1, R_2, \ldots, R_k) \\ &\quad = \sum_{i=1}^{k} (1 - \pi(R_1) - \pi(R_2) - \cdots - \pi(R_{i-1}))W(R_i) \\ &\quad = \Delta_p \sum_{i=1}^{k} (1 - \pi(R_1) - \pi(R_2) - \cdots - \pi(R_{i-1}))N(R_i). \end{aligned}$$

The *ring paging problem* (RPP) is defined as follows: given d and $\xi_0, \xi_1, \xi_2, \ldots, \xi_{d-1}$, and k, where $2 \le k \le d$, find $R_1, R_2, \ldots, R_k$ such that the expected cost $C(R_1, R_2, \ldots, R_k)$ of ring paging is minimized.

The following theorem is a direct consequence of Theorem 15.

Theorem 17. *For a partition of rings $0, 1, 2, \ldots, d-1$ into k disjoint groups $R_1, R_2, \ldots, R_k$, by searching the k groups in an order of $R_{j_1}, R_{j_2}, \ldots, R_{j_k}$, where*

$$\frac{\pi(R_{j_1})}{N(R_{j_1})} \ge \frac{\pi(R_{j_2})}{N(R_{j_2})} \ge \cdots \ge \frac{\pi(R_{j_k})}{N(R_{j_k})},$$

the expected cost of ring paging is minimized.

By Theorem 16, an optimal ring paging method can be obtained when $k = d$. Consider the RPP with $k = d$. In this case, we basically need to find a permutation $(j_0, j_1, j_2, \ldots, j_{d-1})$ of $(0, 1, 2, \ldots, d-1)$. Then, we search a mobile terminal u in the rings $j_0, j_1, j_2, \ldots, j_{d-1}$ successively until u is found. According to Theorem 15, the optimal permutation is obtained by sorting the rings $0, 1, 2, \ldots, d-1$ into $j_0, j_1, j_2, \ldots, j_{d-1}$ in terms of nonincreasing order of ξ_r/S_r, that is,

$$\frac{\xi_{j_0}}{S_{j_0}} \ge \frac{\xi_{j_1}}{S_{j_1}} \ge \frac{\xi_{j_2}}{S_{j_2}} \ge \cdots \ge \frac{\xi_{j_{d-1}}}{S_{j_{d-1}}}.$$

We call the ring paging method using the above optimal order $j_0, j_1, j_2, \ldots, j_{d-1}$ the *optimal ring paging method.*

Theorem 18. *The expected paging cost of the optimal ring paging method is*

$$\begin{aligned} &\Delta_p \sum_{r=0}^{d-1} \xi_{j_r}(S_{j_0} + S_{j_1} + \cdots + S_{j_r}) \\ &\quad = \Delta_p \sum_{r=0}^{d-1} (1 - \xi_{j_0} - \xi_{j_1} - \cdots - \xi_{j_{r-1}})S_{j_r}, \end{aligned}$$

where $\xi_{j_0}/S_{j_0} \ge \xi_{j_1}/S_{j_1} \ge \xi_{j_2}/S_{j_2} \ge \cdots \ge \xi_{j_{d-1}}/S_{j_{d-1}}$.

7.3.2. Cell paging

By *cell paging* we mean the cells in a ring can be searched one by one. By Theorem 16, a cell paging method has lower cost than a ring paging method. By the symmetry of a wireless communication network, a mobile terminal u is in the S_r cells of ring r with equal probability ξ_r/S_r. By Theorem 15, cells should be paged in a nonincreasing order of the probabilities that u is in these cells. Therefore, we sort the rings $0, 1, 2, \ldots, d-1$ into an order $j_0, j_1, j_2, \ldots, j_{d-1}$ such that

$$\frac{\xi_{j_0}}{S_{j_0}} \ge \frac{\xi_{j_1}}{S_{j_1}} \ge \frac{\xi_{j_2}}{S_{j_2}} \ge \cdots \ge \frac{\xi_{j_{d-1}}}{S_{j_{d-1}}}.$$

Then, we search the rings in the order of $j_0, j_1, j_2, \ldots, j_{d-1}$ and for each ring j_r, the cells in ring j_r one by one. It is clear that if a mobile terminal u is in the ith cell of ring j_r, the cost to find u is $S_{j_0} + S_{j_1} + \cdots + S_{j_{r-1}} + i$, where $1 \le i \le S_{j_r}$. Thus, the expected cost to find u is

$$\begin{aligned} &\Delta_p \sum_{r=0}^{d-1} \left(\frac{\xi_{j_r}}{S_{j_r}} \sum_{i=1}^{S_{j_r}} (S_{j_0} + S_{j_1} + \cdots + S_{j_{r-1}} + i) \right) \\ &\quad = \Delta_p \sum_{r=0}^{d-1} \frac{\xi_{j_r}}{S_{j_r}} \left(S_{j_r}(S_{j_0} + S_{j_1} + \cdots + S_{j_{r-1}}) + \frac{S_{j_r}(S_{j_r}+1)}{2} \right) \\ &\quad = \Delta_p \sum_{r=0}^{d-1} \xi_{j_r} \left(S_{j_0} + S_{j_1} + \cdots + S_{j_{r-1}} + \frac{S_{j_r}+1}{2} \right). \end{aligned}$$

We call this the *optimal cell paging method*, which has the lowest possible cost of paging.

Theorem 19. *The expected paging cost of the optimal cell paging method is*

$$\Delta_p \sum_{r=0}^{d-1} \xi_{j_r} \left(S_{j_0} + S_{j_1} + \cdots + S_{j_{r-1}} + \frac{S_{j_r}+1}{2} \right),$$

where $\xi_{j_0}/S_{j_0} \ge \xi_{j_1}/S_{j_1} \ge \xi_{j_2}/S_{j_2} \ge \cdots \ge \xi_{j_{d-1}}/S_{j_{d-1}}$.

The above theorem gives the ultimate minimal terminal paging cost. However, the worst-case running time of the method is $O(d^2)$, which makes the method practically less interesting. To make cell paging more attractive, we may add a delay constraint.

We view a paging area A of radius d as a set of $A(d)$ cells. The location of a mobile terminal u in A is given by a probability distribution function σ, such that the probability that u is in cell s is $\sigma(s)$, and

$$\sum_{s\in A}\sigma(s)=1.$$

More specifically, for a cell s in ring r, the probability that u is in s is $\sigma(s)=\xi_r/S_r$, where $0\le r\le d-1$.

Cells can be grouped into regions. In other words, a region R is defined as a set of cells. The probability that u is in R is

$$p(R)=\sum_{s\in R}\sigma(s).$$

The cost to page a cell is Δ_p. A paging step is to page all the cells in a region R. The cost of a step paging R is $\Delta_p|R|$. A paging method is a sequence of regions $(R_1, R_2, \ldots, R_k)$ such that $R_1, R_2, \ldots, R_k$ are disjoint and $R_1\cup R_2\cup\cdots\cup R_k$ covers the paging area A. The method pages $R_1, R_2, \ldots, R_k$ successively until u is found. The expected cost of paging by using the paging method $(R_1, R_2, \ldots, R_k)$ is

$$\begin{aligned}C(R_1, R_2, \ldots, R_k) &= \Delta_p\sum_{i=1}^{k}p(R_i)(|R_1|+|R_2|+\cdots+|R_i|)\\ &= \Delta_p\sum_{i=1}^{k}(1-p(R_1)-p(R_2)-\cdots\\ &\quad -p(R_{i-1}))|R_i|.\end{aligned}$$

The *cell paging problem* (CPP) is defined as follows: given d and $\xi_0, \xi_1, \xi_2, \ldots, \xi_{d-1}$, and k, where $2\le k\le A(d)$, find a paging method $(R_1, R_2, \ldots, R_k)$ such that the expected cost $C(R_1, R_2, \ldots, R_k)$ of paging is minimized.

By using a dynamic programming algorithm [28,39], the CPP can be solved in $O(kd^2)$ time [10].

The CPP is similar to the problem studied in [3,46,57], where the delay constraint is the average-case paging time $p(R_1)+2p(R_2)+\cdots+kp(R_k)$, while our delay constraint is the worst-case paging time k.

8. Examples

Example 1. Let us consider a DBLMS with $q=3$ and $d=20$. Both T_c and T_s have exponential distributions with $\lambda_c=1$ and $\lambda_s=10$. By Theorems 5 and 7, the location distribution $(\xi_0, \xi_1, \xi_2, \ldots, \xi_{d-1})$ when a phone call in a DBLMS-CPLU or the first phone call in a DBLMS-CWLU arrives is given in Table 5. We have the following sample solutions for various k, i.e., the maximum number of steps in progressive paging.

- When $k=2$, the best solution is $d_1=7, d_2=20$, with expected paging cost $D_p(d_1, d_2)=194.2321\Delta_p$.
- When $k=3$, the best solution is $d_1=4, d_2=9$, and $d_3=20$, with expected paging cost $D_p(d_1, d_2, d_3)=108.9627\Delta_p$.
- When $k=4$, the best solution is $d_1=3, d_2=6, d_3=10$, and $d_4=20$, with expected paging cost $D_p(d_1, d_2, d_3, d_4)=81.8296\Delta_p$.
- When $k=20$, we have $d_r=r$ for all $1\le r\le 20$, and expected paging cost $D_p(d_1, d_2, \ldots, d_{20})=45.6188\Delta_p$.

It turns out that the rings are already sorted into an order for both optimal ring paging and optimal cell paging. The optimal ring paging cost is $45.6188\Delta_p$ and the optimal cell paging cost is $38.0977\Delta_p$. All the above solutions are significantly lower than $D_p(d)=A(d)\Delta_p=1141\Delta_p$.

Example 2. Let us consider an MBLMS with $q=3$ and $d=20$. Both T_c and T_s have exponential distributions with $\lambda_c=1$ and $\lambda_s=10$. By Theorems 6 and 9, the location distribution $(\xi_0, \xi_1, \xi_2, \ldots, \xi_{d-1})$ when a phone call in an MBLMS-CPLU or the first phone call in an MBLMS-CWLU arrives is given in Table 5. We have the following sample solutions for various k, i.e., the maximum number of steps in progressive paging.

- When $k=2$, the best solution is $d_1=6, d_2=20$, with expected paging cost $M_p(d_1, d_2)=147.2099\Delta_p$.
- When $k=3$, the best solution is $d_1=4, d_2=8$, and $d_3=20$, with expected paging cost $M_p(d_1, d_2, d_3)=74.1874\Delta_p$.
- When $k=4$, the best solution is $d_1=3, d_2=5, d_3=9$, and $d_4=20$, with expected paging cost $M_p(d_1, d_2, d_3, d_4)=54.6799\Delta_p$.
- When $k=20$, we have $d_r=r$ for all $1\le r\le 20$, and expected paging cost $M_p(d_1, d_2, \ldots, d_{20})=31.4285\Delta_p$.

It turns out that the rings are already sorted into an order for both optimal ring paging and optimal cell paging. The optimal ring paging cost is $31.4285\Delta_p$ and the optimal cell paging cost is $25.2699\Delta_p$. All the above solutions are significantly lower than $M_p(d)=A(d)\Delta_p=1141\Delta_p$.

Example 3. Let us consider a TBLMS with $q=3$ and $d=20$. Both T_c and T_s have exponential distributions with $\lambda_c=1$ and $\lambda_s=10$. We set $\tau=N(d)\boldsymbol{E}(T_s)$. By Theorems 8 and 10 in [35], the location distribution $(\xi_0, \xi_1, \xi_2, \ldots, \xi_{d-1})$ when a phone call in a TBLMS-CPLU or the first phone call in a TBLMS-CWLU arrives is given in Table 5. Notice that $I(\tau, d)=\xi_0+\xi_1+\cdots+\xi_{19}=0.999863363560188$ is the probability that a mobile terminal can be found in a PA of radius d. We have the following sample solutions for various k, i.e., the maximum number of steps in progressive paging.

- When $k=2$, the best solution is $d_1=7, d_2=20$, with expected paging cost $C_p(\tau, d_1, d_2)=194.3615\Delta_p$.
- When $k=3$, the best solution is $d_1=4, d_2=9, d_3=20$, with expected paging cost $C_p(\tau, d_1, d_2, d_3)=109.1037\Delta_p$.
- When $k=4$, the best solution is $d_1=3, d_2=6, d_3=10$, and $d_4=20$, with expected paging cost $C_p(\tau, d_1, d_2, d_3, d_4)=81.9743\Delta_p$.
- When $k=20$, we have $d_r=r$ for all $1\le r\le 20$, and expected paging cost $C_p(\tau, d_1, d_2, \ldots, d_{20})=45.7685\Delta_p$.

It turns out that the rings are already sorted into an order for both optimal ring paging and optimal cell paging. The optimal ring paging cost is $45.7685\Delta_p$ and the optimal cell paging cost is $38.0925\Delta_p$. All the above solutions are significantly lower than $C_p(\tau, d)=A(d)\Delta_p=1141\Delta_p$.

Example 4. Let us consider a TBLMS with $q=3$ and $d=20$. Both T_c and T_s have exponential distributions with $\lambda_c=1$ and $\lambda_s=10$. We set $\tau=d\boldsymbol{E}(T_s)$. By Theorems 8 and 10 in [35], the location distribution $(\xi_0, \xi_1, \xi_2, \ldots, \xi_{d-1})$ when a phone call in a TBLMS-CPLU or the first phone call in a TBLMS-CWLU arrives is given in Table 5. Notice that $I(\tau, d)=\xi_0+\xi_1+\cdots+\xi_{19}=0.999999921954133$ is the probability that a mobile terminal can be found in a PA of radius d. We have the following sample solutions for various k, i.e., the maximum number of steps in progressive paging.

- When $k=2$, the best solution is $d_1=6, d_2=20$, with expected paging cost $C_p(\tau, d_1, d_2)=153.9049\Delta_p$.
- When $k=3$, the best solution is $d_1=4, d_2=8, d_3=20$, with expected paging cost $C_p(\tau, d_1, d_2, d_3)=78.4378\Delta_p$.
- When $k=4$, the best solution is $d_1=3, d_2=5, d_3=9$, and $d_4=20$, with expected paging cost $C_p(\tau, d_1, d_2, d_3, d_4)=58.0782\Delta_p$.

Table 5
Location distribution data of Examples 1–4.

ξ_r	Example 1	Example 2	Example 3	Example 4
ξ_0	0.126984303252809	0.147294117090233	0.126966952569710	0.145140869319645
ξ_1	0.238014408401451	0.268864145991489	0.237981886960079	0.265410663939907
ξ_2	0.211403466114172	0.229528479022705	0.211374580697212	0.227295482079873
ξ_3	0.150531680924762	0.153687360701018	0.150511112811810	0.153104435760744
ξ_4	0.100583119959185	0.093771747279603	0.100569376639772	0.094434314254021
ξ_5	0.065010025701666	0.053320956471823	0.065001142963202	0.054704851669172
ξ_6	0.041169129319087	0.028389889490487	0.041163504115825	0.030017279431395
ξ_7	0.025714347807327	0.014130126874155	0.025710834290388	0.015649888989792
ξ_8	0.015902546775094	0.006543988982861	0.015900373907714	0.007761695232872
ξ_9	0.009761228412939	0.002802154901768	0.009759894673436	0.003663943638694
ξ_{10}	0.005956611777799	0.001100827709469	0.005955797887567	0.001646964510849
ξ_{11}	0.003617810833662	0.000393081750629	0.003617316508865	0.000705340951265
ξ_{12}	0.002188714096661	0.000126147004442	0.002188415038554	0.000288000818377
ξ_{13}	0.001319593076614	0.000035872912955	0.001319412772109	0.000112209962224
ξ_{14}	0.000792903300736	0.000008874986829	0.000792794961247	0.000041756807978
ξ_{15}	0.000474384877042	0.000001862751246	0.000474320058778	0.000014857093977
ξ_{16}	0.000281591409507	0.000000319716574	0.000281552933855	0.000005059618451
ξ_{17}	0.000163932779462	0.000000042307485	0.000163910380268	0.000001650497065
ξ_{18}	0.000090101610581	0.000000003868325	0.000090089299416	0.000000513906288
ξ_{19}	0.000040099569443	0.000000000185900	0.000040094090380	0.000000143471544

- When $k = 20$, we have $d_r = r$ for all $1 \le r \le 20$, and expected paging cost $C_p(\tau, d_1, d_2, \ldots, d_{20}) = 32.9245\Delta_p$.

It turns out that the rings are already sorted into an order for both optimal ring paging and optimal cell paging. The optimal ring paging cost is $32.9245\Delta_p$ and the optimal cell paging cost is $26.6157\Delta_p$. All the above solutions are significantly lower than $C_p(\tau, d) = A(d)\Delta_p = 1141\Delta_p$.

9. Numerical data of paging cost reduction

In this section, we display numerical data to show paging cost reduction and minimization. In particular, we demonstrate numerical data to compare mobility management costs of dynamic location management schemes using the progressive paging methods, the ring paging methods, and the cell paging methods.

In Fig. 5, we consider DBLMS-CPLU with $q = 3$. Assume that both T_c and T_s have exponential distributions. We display the total costs of location management of the following schemes:

- DBLMS(d_1, d_2)-CPLU, with paging cost $D_p(d_1, d_2)$;
- DBLMS(d_1, d_2, d_3)-CPLU, with paging cost $D_p(d_1, d_2, d_3)$;
- DBLMS(d_1, d_2, d_3, d_4)-CPLU, with paging cost $D_p(d_1, d_2, d_3, d_4)$;
- DBLMS$^*(d)$-CPLU $=$ DBLMS$(1, 2, \ldots, d)$-CPLU, with paging cost $D_p^*(d)$;
- DBLMS-CPLU with the minimum ring paging cost given by Theorem 18;
- DBLMS-CPLU with the minimum cell paging cost given by Theorem 19.

The parameters are set as $\lambda_c = 1$, $\lambda_s = 10$, $\rho = 0.1$, $\Delta_p = 1$, and $\Delta_u = 100$. Notice that the paging cost $D_p^*(d)$ is identical to the minimum ring paging cost given by Theorem 18. Therefore, in the figure, we show the total cost of location management with progressive paging when $k = 2$, progressive paging when $k = 3$, progressive paging when $k = 4$, optimal ring paging, and optimal cell paging.

In Figs. 6–12, we consider the total costs of location management of the following schemes in exactly the same way as Fig. 5:

- Fig. 6: DBLMS-CWLU with $q = 3$;
- Fig. 7: DBLMS-CPLU with $q = 4$;
- Fig. 8: DBLMS-CWLU with $q = 4$;
- Fig. 9: MBLMS-CPLU $q = 3$;
- Fig. 10: MBLMS-CWLU $q = 3$;
- Fig. 11: MBLMS-CPLU $q = 4$;
- Fig. 12: MBLMS-CWLU $q = 4$.

In Fig. 13, we consider TBLMS-CPLU with $q = 3$. The location update cycle is set as $\tau = N(d)\boldsymbol{E}(T_s)$. All parameters are set in the same way as Fig. 5. The location distribution is given as $\xi_0, \xi_1, \xi_2, \ldots, \xi_{d-1}$, where $d = 20$, such that the quality of service (QoS), i.e., the reachability $I(\tau, d)$ of a mobile terminal in a paging area, is above 0.999. We display the total costs of location management of the following schemes:

- TBLMS(τ, d_1, d_2)-CPLU, with paging cost $C_p(\tau, d_1, d_2)$;
- TBLMS(τ, d_1, d_2, d_3)-CPLU, with paging cost $C_p(\tau, d_1, d_2, d_3)$;
- TBLMS$(\tau, d_1, d_2, d_3, d_4)$-CPLU, with paging cost $C_p(\tau, d_1, d_2, d_3, d_4)$;
- TBLMS$^*(\tau, d)$-CPLU $=$ TBLMS$(\tau, 1, 2, \ldots, d)$-CPLU, with paging cost $C_p^*(\tau, d)$;
- TBLMS-CPLU with the minimum ring paging cost given by Theorem 18;
- TBLMS-CPLU with the minimum cell paging cost given by Theorem 19.

Notice that the QoS is above 0.999 for both $q = 3$ and 4.

In Figs. 14–16, we consider the total costs of location management of the following schemes in exactly the same way as Fig. 13:

- Fig. 14: TBLMS-CWLU with $q = 3$;
- Fig. 15: TBLMS-CPLU with $q = 4$;
- Fig. 16: TBLMS-CWLU with $q = 4$.

In Figs. 17–20, we repeat the work of Figs. 13–16, except that the time threshold is set as $\tau = d\boldsymbol{E}(T_s)$. Notice that the QoS is above 0.9999999 when $q = 3$ and 0.999999 when $q = 4$.

From the above numerical data, we have the following important observations. First, a progressive paging method with very small time delay (such as $k = 2, 3$) can reduce the terminal paging cost dramatically. The reader is referred to the figures in [32,33,35] with similar settings, where, as d increases, the total cost of location management increases rapidly by using the simple paging method. Second, further increasing the time delay (even to the maximum possible such as a cell paging method) does not result in noticeable reduction of terminal paging cost. Hence, the best approach is to sacrifice the paging time slightly while achieving significant reduction in paging cost.

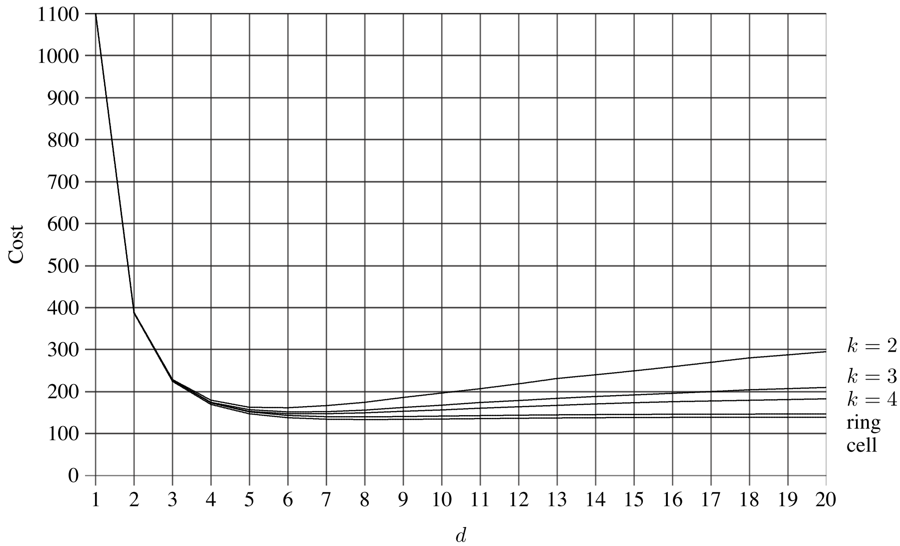

Fig. 5. Location management cost in DBLMS-CPLU with selective paging ($q = 3$).

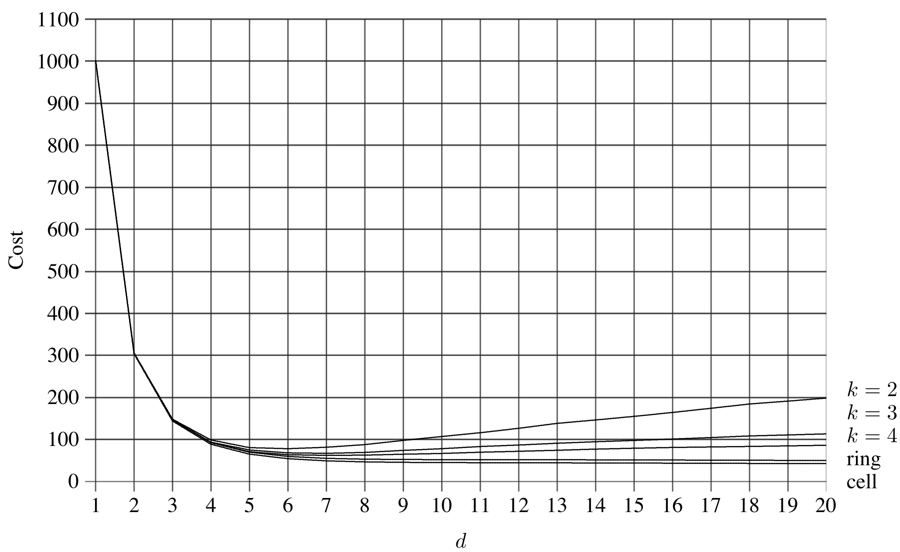

Fig. 6. Location management cost in DBLMS-CWLU with selective paging ($q = 3$).

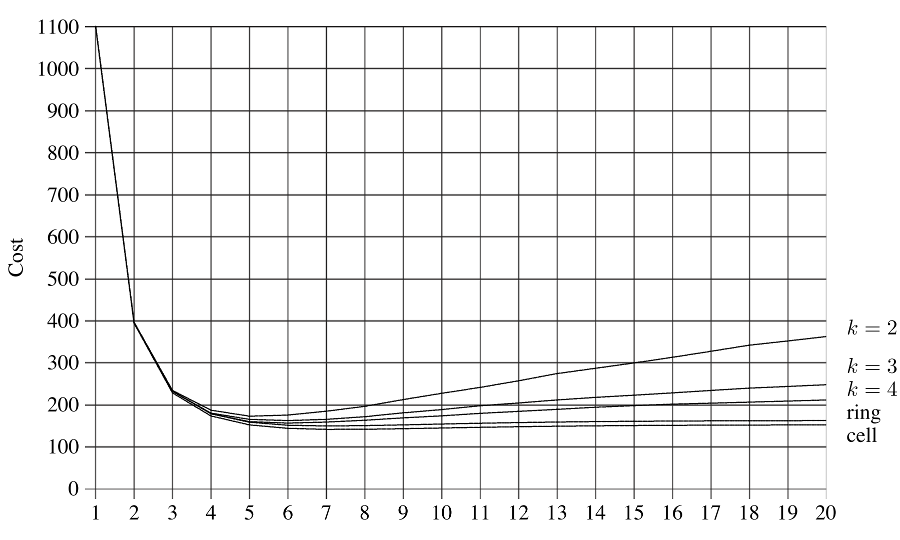

Fig. 7. Location management cost in DBLMS-CPLU with selective paging ($q = 4$).

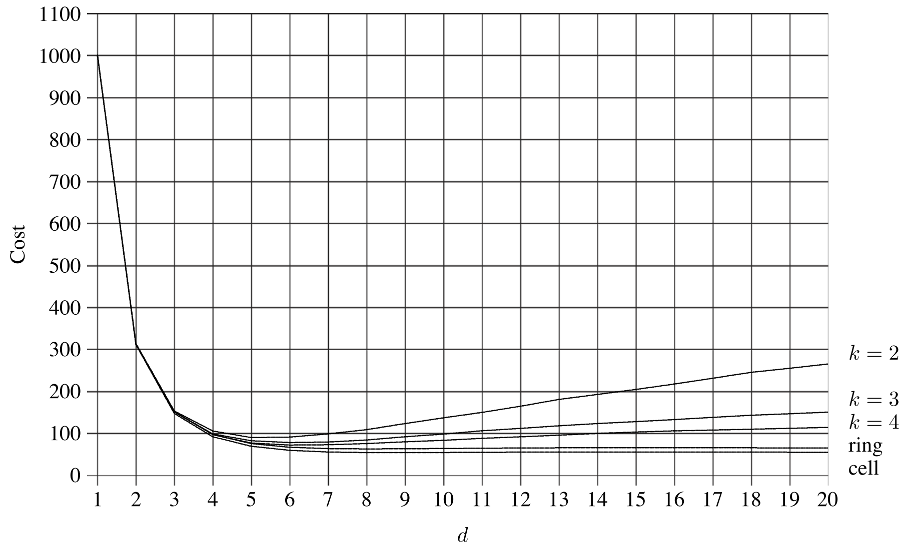

Fig. 8. Location management cost in DBLMS-CWLU with selective paging ($q = 4$).

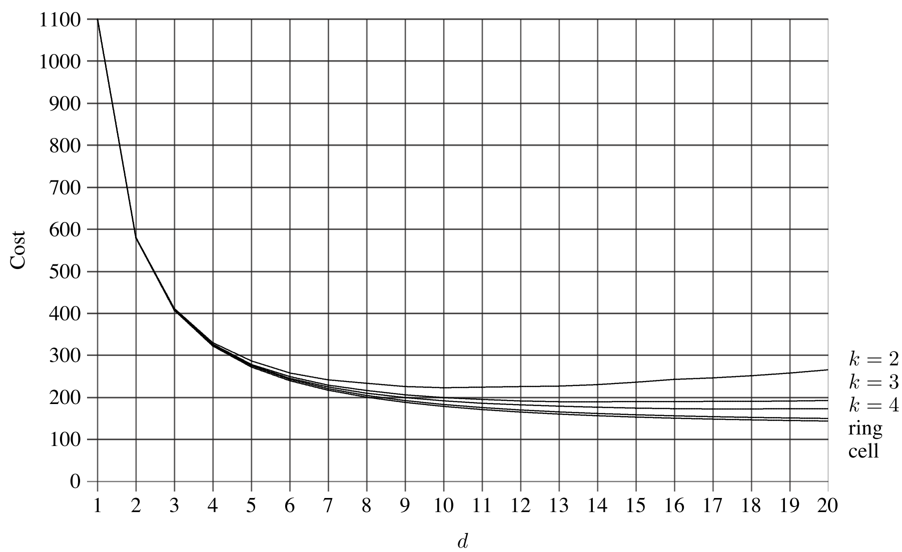

Fig. 9. Location management cost in MBLMS-CPLU with selective paging ($q = 3$).

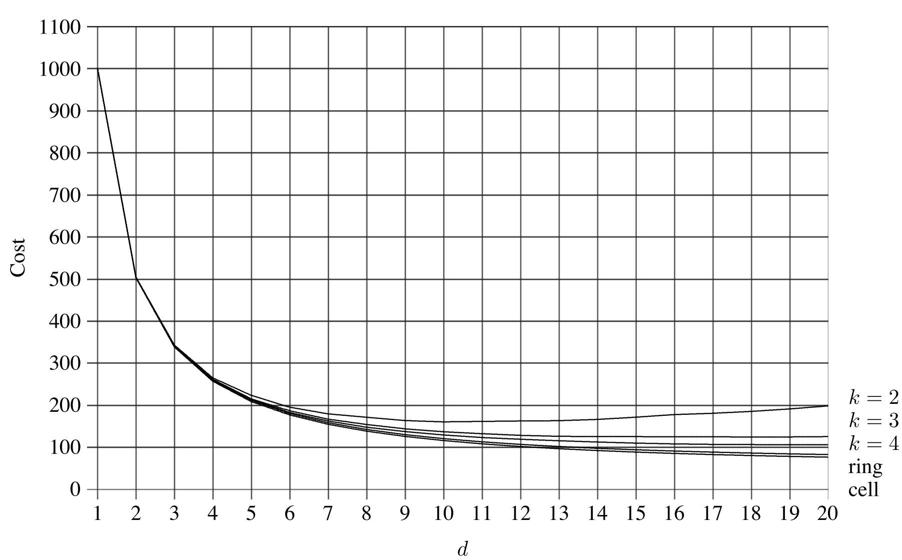

Fig. 10. Location management cost in MBLMS-CWLU with selective paging ($q = 3$).

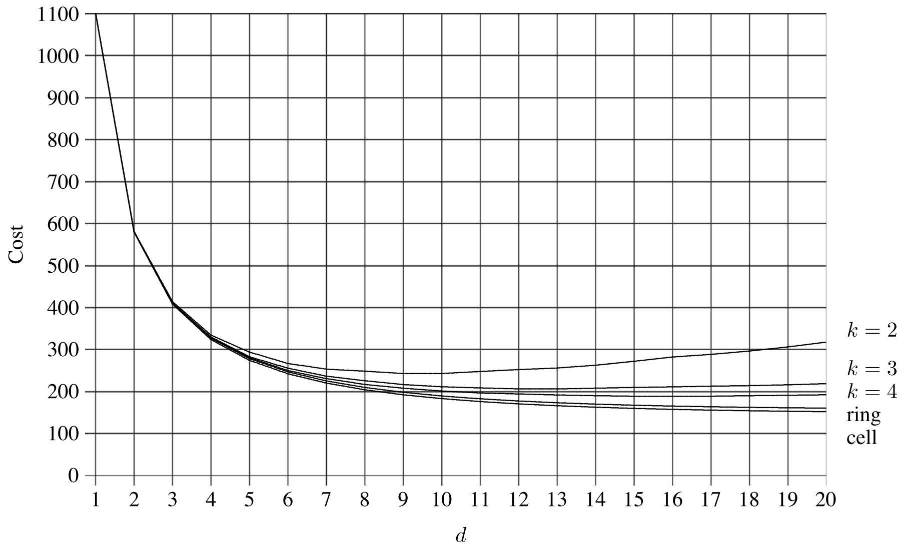

Fig. 11. Location management cost in MBLMS-CPLU with selective paging ($q = 4$).

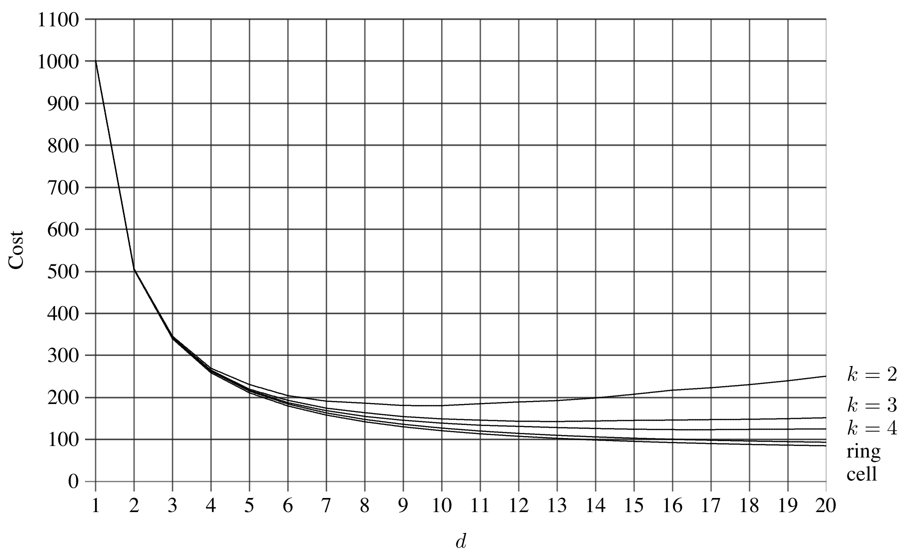

Fig. 12. Location management cost in MBLMS-CWLU with selective paging ($q = 4$).

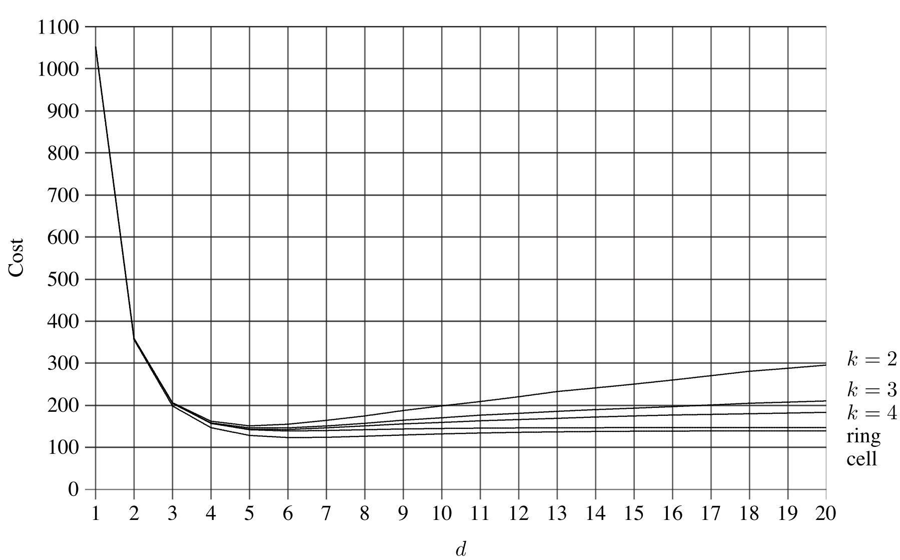

Fig. 13. Location management cost in TBLMS-CPLU with selective paging ($\tau = N(d)\boldsymbol{E}(T_s)$, $q = 3$).

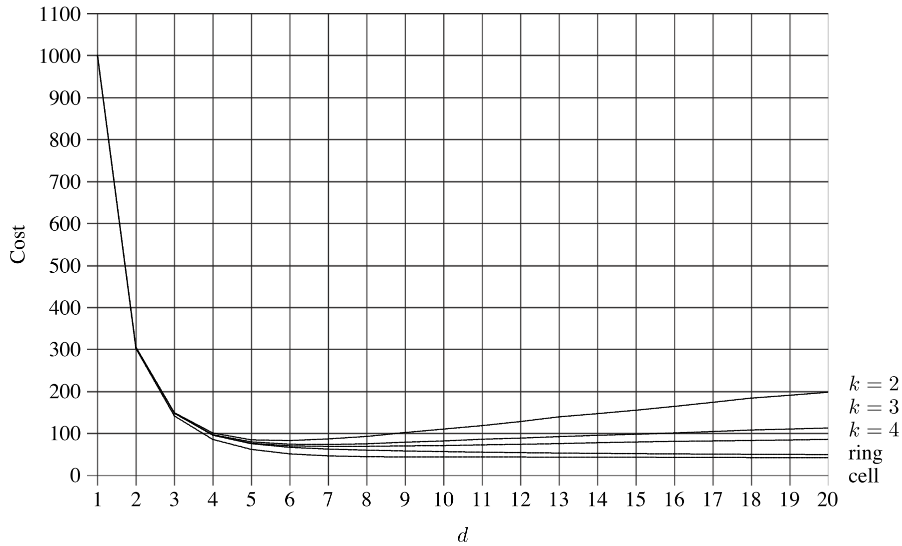

Fig. 14. Location management cost in TBLMS-CWLU with selective paging ($\tau = N(d)\boldsymbol{E}(T_s)$, $q = 3$).

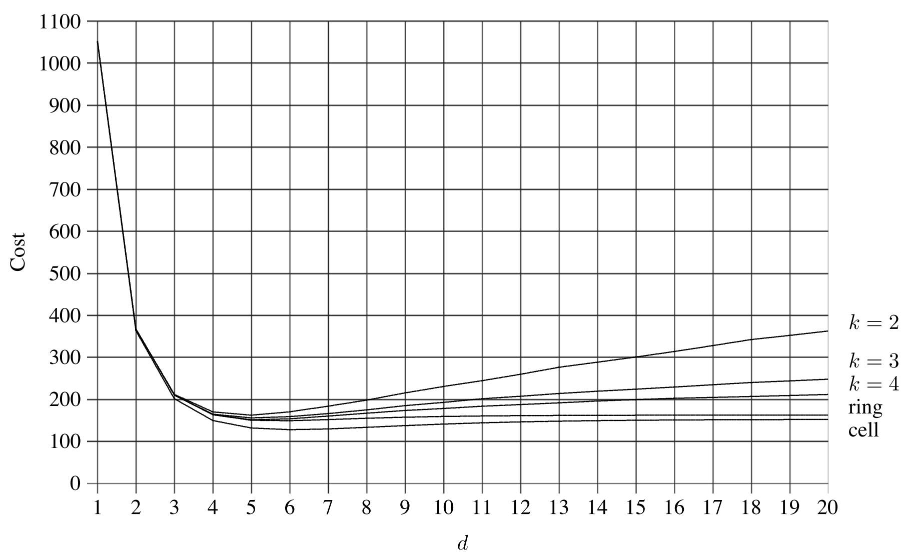

Fig. 15. Location management cost in TBLMS-CPLU with selective paging ($\tau = N(d)\boldsymbol{E}(T_s)$, $q = 4$).

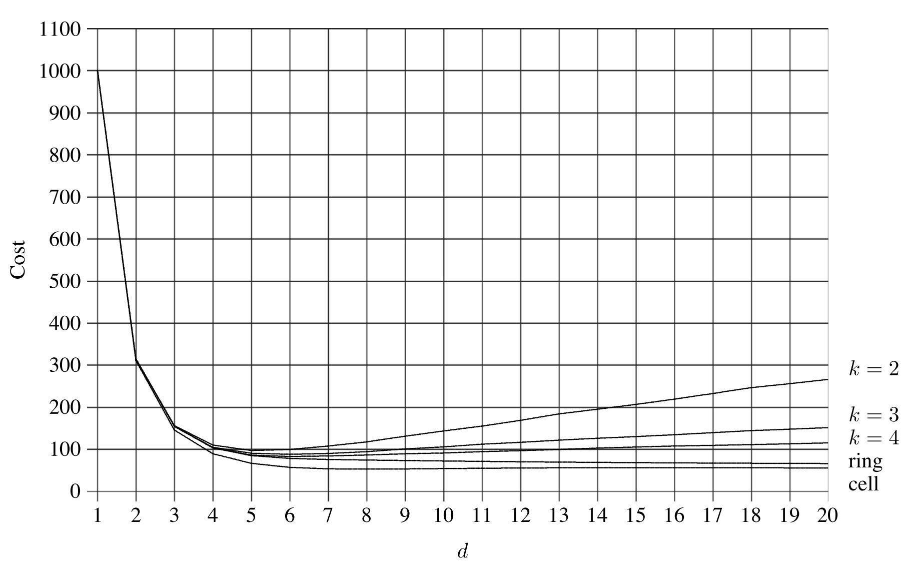

Fig. 16. Location management cost in TBLMS-CWLU with selective paging ($\tau = N(d)\boldsymbol{E}(T_s)$, $q = 4$).

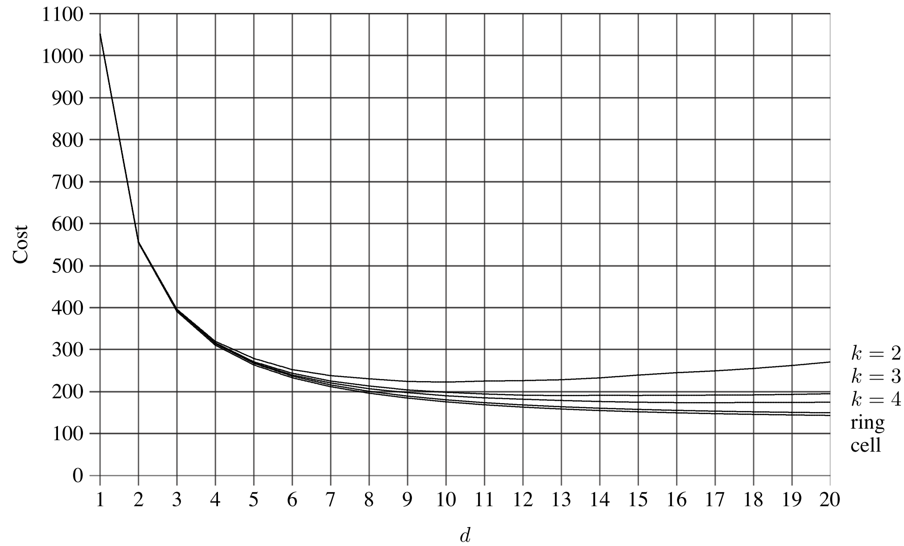

Fig. 17. Location management cost in TBLMS-CPLU with selective paging ($\tau = d\boldsymbol{E}(T_s)$, $q = 3$).

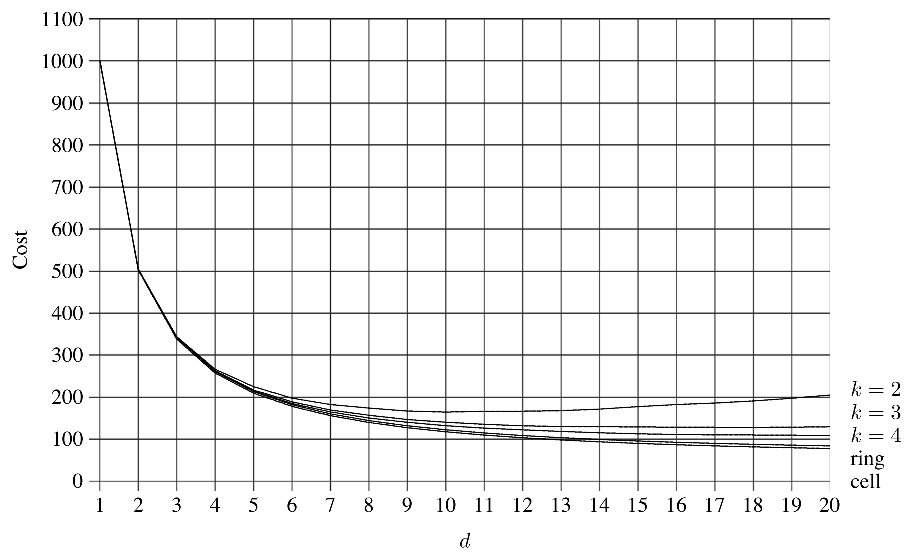

Fig. 18. Location management cost in TBLMS-CWLU with selective paging ($\tau = d\boldsymbol{E}(T_s)$, $q = 3$).

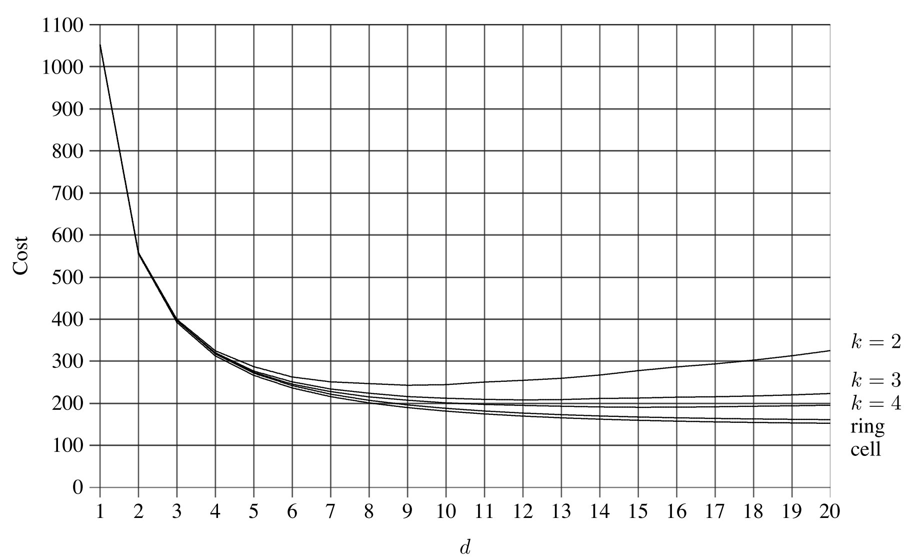

Fig. 19. Location management cost in TBLMS-CPLU with selective paging ($\tau = d\boldsymbol{E}(T_s)$, $q = 4$).

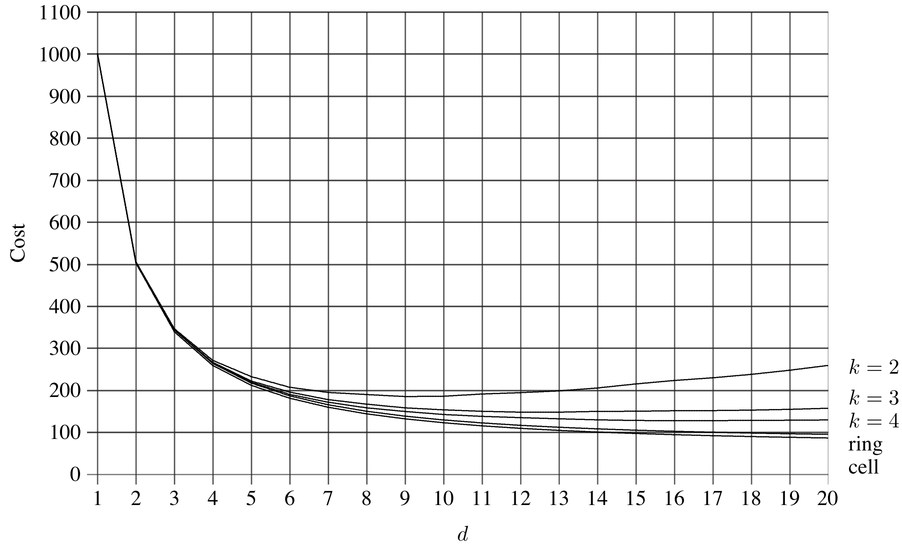

Fig. 20. Location management cost in TBLMS-CWLU with selective paging ($\tau = d\boldsymbol{E}(T_s)$, $q = 4$).

10. Concluding remarks

By using our previous results on random walks among rings of cell structures, we have analyzed the location distribution of a mobile terminal in a paging area when a phone call arrives, for both distance-based and movement-based location management schemes, and for two different call handling models, where the inter-call time and the cell residence time can have arbitrary probability distributions. Together with our earlier results on location distribution in time-based location management schemes, for several selective paging methods, including progressive paging methods, ring paging methods, and cell paging methods, we are able to obtain their expected costs of paging for distance-based, movement-based, and time-based location management schemes. We find that a progressive paging method with very small time delay can reduce the terminal paging cost dramatically, while further increasing the time delay does not result in noticeable reduction of terminal paging cost. Our work reported in this paper significantly extends our understanding of cost reduction and minimization of dynamic location management in wireless communication networks.

We would like to mention that our effective ways (Theorems 5, 6, 7, 9) to calculate the location distribution of a mobile terminal is applicable to any real-world scenario, as long as $\boldsymbol{P}[X_s' = j]$ or $\boldsymbol{P}[X_s(\gamma) = j]$, $j = 0, 1, 2, \ldots$, are available, through analysis or simulation or experimentation, for arbitrary probability distribution of the inter-call time and the cell residence time. Once the location distribution is available, all the selective paging methods in this paper can be applied. Our observations made in this paper are certainly valid for any location distribution in the real world. Due to space limitation, we do not pursue in this direction, and we believe that such effort brings no further insight.

The investigation in this paper is mainly applicable to 2G (second-generation) GSM (Global System for Mobile Communications) and 3G (third-generation) UMTS (Universal Mobile Telecommunications System) digital cellular networks used by mobile phones, which are currently the default global standards for mobile communications with over 90% market share, operating in over 219 countries and territories. With the emergence of 4G (fourth-generation) LTE (Long-Term Evolution) standard for wireless communication of high-speed data in mobile phones and data terminals and 5G (fifth-generation) mobile networks and wireless systems, we are facing new challenges in location management [19]. The application and extension of the analytical methods developed in this paper to these new generations of wireless communication networks are our further research directions.

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References

[1] http://en.wikipedia.org/wiki/Renewal_theory.
[2] A. Abutaleb, V.O.K. Li, Location update optimization in personal communication systems, Wirel. Netw. 3 (3) (1997) 205–216.
[3] A. Abutaleb, V.O.K. Li, Paging strategy optimization in personal communication systems, Wirel. Netw. 3 (3) (1997) 195–204.
[4] I.F. Akyildiz, J.S.M. Ho, Dynamic mobile user location update for wireless PCS networks, Wirel. Netw. 1 (1995) 187–196.
[5] I.F. Akyildiz, J.S.M. Ho, Y.-B. Lin, Movement-based location update and selective paging for PCS networks, IEEE/ACM Trans. Netw. 4 (1996) 629–638.
[6] I.F. Akyildiz, J. McNair, J.S.M. Ho, H. Uzunalioğlu, W. Wang, Mobility management in next-generation wireless systems, Proc. IEEE 87 (8) (1999) 1347–1384.
[7] I.F. Akyildiz, W. Wang, A dynamic location management scheme for next-generation multitier PCS systems, IEEE Trans. Wireless Commun. 1 (1) (2002) 178–189.
[8] E. Alba, J. García-Nieto, J. Taheri, A. Zomaya, New research in nature inspired algorithms for mobility management in GSM networks, Lecture Notes in Comput. Sci. 4974 (2008) 1–10.
[9] J.H. Baek, B.H. Ryu, Modeling and analysis of distance-based registration with implicit registration, ETRI J. 25 (6) (2003) 527–530.
[10] A. Bar-Noy, Y. Feng, M.J. Golin, Paging mobile users efficiently and optimally, in: Proc. of the 26th IEEE International Conference on Computer Communications, 2007, pp. 1910–1918.
[11] A. Bar-Noy, I. Kessler, M. Sidi, Mobile users: to update or not update ? Wirel. Netw. 1 (1995) 175–185.
[12] F.V. Baumann, I.G. Niemegeers, An evaluation of location management procedures, in: Proc. of IEEE International Conference on Universal Personal Communications, 1994, pp. 359–364.
[13] V.S. Bhadauria, S. Sharma, R. Patel, A comparative study of location management schemes: challenges and guidelines, Int. J. Comput. Sci. Eng. 3 (7) (2011) 2779–2785.
[14] T.X. Brown, S. Mohan, Mobility management for personal communications systems, IEEE Trans. Veh. Technol. 46 (2) (1997) 269–278.
[15] V. Casares-Giner, J. Mataix-Oltra, Global versus distance-based local mobility tracking strategies: a unified approach, IEEE Trans. Veh. Technol. 51 (3) (2002) 472–485.
[16] E. Cayirci, I.F. Akyildiz, User mobility pattern scheme for location update and paging in wireless systems, IEEE Trans. Mob. Comput. 1 (3) (2002) 236–247.
[17] D.R. Cox, On the number of renewals in a random interval, Biometrika 47 (3/4) (1960) 449–452.
[18] D.R. Cox, Renewal Theory, Methuen & Co. Ltd., London, 1962.
[19] T. Deng, X. Wang, P. Fan, K. Li, Modeling and performance analysis of tracking area list-based location management scheme in LTE networks, IEEE Trans. Veh. Technol. 65 (8) (2016) 6417–6431.
[20] P.G. Escalle, V.C. Giner, J.M. Oltra, Reducing location update and paging costs in a PCS network, IEEE Trans. Wireless Commun. 1 (1) (2002) 200–209.
[21] Y. Fang, General modeling and performance analysis for location management in wireless mobile networks, IEEE Trans. Comput. 51 (10) (2002) 1169–1181.

[22] Y. Fang, Movement-based mobility management and trade off analysis for wireless mobile networks, IEEE Trans. Comput. 52 (6) (2003) 791–803.
[23] Y. Fang, I. Chlamtac, Y.-B. Lin, Potable movement modeling for PCS networks, IEEE Trans. Veh. Technol. 49 (4) (2000) 1356–1363.
[24] W. Feller, An Introduction to Probability Theory and Its Applications, Vol. 1, third ed., John Wiley & Sons, New York, 1968.
[25] B. Furht, M. Ilyas (Eds.), Wireless Internet Handbook – Technologies, Standards, and Applications, CRC Press, Boca Raton, FL, 2003.
[26] S.G. Glisic, Advanced Wireless Networks – 4G Technologies, John Wiley & Sons, Chichester, England, 2006.
[27] J.S.M. Ho, I.F. Akyildiz, Mobile user location update and paging under delay constraints, Wirel. Netw. 1 (1995) 413–425.
[28] B. Krishnamachari, R.-H. Gau, S.B. Wicker, Z.J. Haas, Optimal sequential paging in cellular wireless networks, Wirel. Netw. 10 (2) (2004) 121–131.
[29] N.E. Kruijt, D. Sparreboom, F.C. Schoute, R. Prasad, Location management strategies for cellular mobile networks, IEE Electron. Commun. Eng. J. 10 (2) (1998) 64–72.
[30] K. Kyamakya, K. Jobmann, Location management in cellular networks: classification of the most important paradigms, realistic Simulation framework, and relative performance analysis, IEEE Trans. Veh. Technol. 54 (2) (2005) 687–708.
[31] G.Y. Lee, Y. Lee, Z.J. Haas, Hybrid location-update scheme for mobile networks, IEEE Trans. Veh. Technol. 58 (1) (2009) 338–348.
[32] K. Li, Cost analysis and minimization of movement-based location management schemes in wireless communication networks: a renewal process approach, Wirel. Netw. 17 (4) (2011) 1031–1053.
[33] K. Li, Analysis of distance-based location management in wireless communication networks, IEEE Trans. Parallel Distrib. Syst. 24 (2) (2013) 225–238.
[34] K. Li, Cost optimization for scalable communication in wireless networks with movement-based location management, in: S.U. Khan, A.Y. Zomaya, L. Wang (Eds.), Scalable Computing and Communications: Theory and Practice, John Wiley & Sons, 2013, pp. 185–208. (Chapter 10).
[35] K. Li, Analysis of cost and quality of service of time-based dynamic mobility management in wireless networks, Wirel. Netw. 20 (2) (2014) 261–288.
[36] J. Li, H. Kameda, K. Li, Optimal dynamic mobility management for PCS networks, IEEE/ACM Trans. Netw. 8 (3) (2000) 319–327.
[37] J. Li, Y. Pan, X. Jia, Analysis of dynamic location management for PCS networks, IEEE Trans. Veh. Technol. 51 (5) (2002) 1109–1119.
[38] Y.-B. Lin, Reducing location update cost in a PCS network, IEEE/ACM Trans. Netw. 5 (1) (1997) 25–33.
[39] G.L. Lyberopoulos, J.G. Markoulidakis, D.V. Polymeros, D.F. Tsirkas, E.D. Sykas, Intelligent paging strategies for third generation mobile telecommunication systems, IEEE Trans. Veh. Technol. 44 (3) (1995) 543–554.
[40] U. Madhow, M.L. Honig, K. Steiglitz, Optimization of wireless resources for personal communications mobility tracking, IEEE/ACM Trans. Netw. 3 (12) (1995) 698–707.
[41] C.K. Ng, H.W. Chan, Enhanced distance-based location management of mobile communication systems using a cell coordinates approach, IEEE Trans. Mob. Comput. 4 (1) (2005) 41–55.
[42] R.M. Rodríguez-Dagnino, H. Takagi, Counting handovers in a cellular mobile communication network: equilibrium renewal process approach, Perform. Eval. 52 (2003) 153–174.
[43] R.M. Rodríguez-Dagnino, H. Takagi, Movement-based location management for general cell residence times in wireless networks, IEEE Trans. Veh. Technol. 56 (5) (2007) 2713–2722.
[44] C. Rose, Minimizing the average cost of paging and registration: A timer-based method, Wirel. Netw. 2 (2) (1996) 109–116.
[45] C. Rose, State-based paging/registration: a greedy technique, IEEE Trans. Veh. Technol. 48 (1) (1999) 166–173.
[46] C. Rose, R. Yates, Minimizing the average cost of paging under delay constraints, Wirel. Netw. 1 (1995) 211–219.
[47] C. Rose, R. Yates, Ensemble polling strategies for increased paging capacity in mobile communication networks, Wirel. Netw. 3 (2) (1997) 159–167.
[48] S. Saha, M. Mukherjee, A.R. Paul, S. Neogy, TPRLM: time-based probabilistic relational location management scheme for wireless cellular networks, in: Proceedings of the 2nd International Conference on Communication Systems Software and Middleware, 2007, pp. 1–10.
[49] C.U. Saraydar, O.E. Kelly, C. Rose, One-dimensional location area design, IEEE Trans. Veh. Technol. 49 (5) (2000) 1626–1632.
[50] D. Sarddar, et al., A time based mobility management method for Leo satellite networks, Int. J. Comput. Appl. 42 (2) (2012) 33–40.
[51] R. Subrata1, A.Y. Zomaya, Dynamic location management for mobile computing, Telecommun. Syst. 22 (1–4) (2003) 169–187.
[52] S. Tabbane, Location management methods for third-generation mobile systems, IEEE Commun. Mag. 35 (1997) 72–84.
[53] J. Taheri, A.Y. Zomaya, A combined genetic-neural algorithm for mobility management, J. Math. Model. Algorithms 6 (3) (2007) 481–507.
[54] J. Taheri, A.Y. Zomaya, A simulated annealing approach for mobile location management, Comput. Commun. 30 (4) (2007) 714–730.
[55] J. Taheri, A.Y. Zomaya, Clustering techniques for dynamic location management in mobile computing, J. Parallel Distrib. Comput. 67 (4) (2007) 430–447.
[56] J. Taheri, A.Y. Zomaya, Bio-inspired algorithms for mobility management, in: Proceedings of the International Symposium on Parallel Architectures, Algorithms, and Networks, 2008, pp. 216–223.
[57] M. Verkama, Optimal paging – a search theory approach, in: Proceedings of IEEE International Conference on Universal Personal Communications, 1996, pp. 956–960.
[58] G. Wan, E. Lin, Cost reduction in location management using semi-realtime movement information, Wirel. Netw. 5 (1999) 245–256.
[59] W. Wang, I.F. Akyildiz, G.L. Stüber, An optimal paging scheme for minimizing signaling costs under delay bounds, IEEE Commun. Lett. 5 (2) (2001) 43–45.
[60] W. Wang, I.F. Akyildiz, G.L. Stüber, B.-Y. Chung, Effective paging schemes with delay bounds as QoS constraints in wireless systems, Wirel. Netw. 7 (2001) 455–466.
[61] X. Wang, P. Fan, J. Li, Y. Pan, Modeling and cost analysis of movement-based location management for PCS networks with HLR/VLR architecture, general location area and cell residence time distributions, IEEE Trans. Veh. Technol. 57 (6) (2008) 3815–3831.
[62] X. Wang, K. Li, R.-G. Cheng, P. Fan, X. Lei, R.Q. Hu, Cost analysis of a hybrid movement- and time-based location update scheme in cellular networks, IEEE Trans. Veh. Technol. 64 (11) (2015) 5314–5326.
[63] V.W.S. Wong, V.C.M. Leung, An adaptive distance-based location update algorithm for next-generation PCS networks, IEEE J. Sel. Areas Commun. 19 (10) (2001) 1942–1952.
[64] A. Yener, C. Rose, Highly mobile users and paging: optimal polling strategies, IEEE Trans. Veh. Technol. 47 (4) (1998) 1251–1257.
[65] Q. Zhang, S.K. Das, A. Rodriguez-Carrion, A 2-D random walk mobility model for WiMAX location update, Comput. Commun. 78 (2016) 86–96.
[66] Y. Zhang, J. Zheng, L. Zhang, Y. Chen, M. Ma, Modeling location management in wireless networks with generally distributed parameters, Comput. Commun. 29 (12) (2006) 2386–2395.
[67] Y.-h. Zhu, V.C.M. Leung, Derivation of moving distance distribution to enhance sequential paging in distance-based mobility management for PCS networks, IEEE Trans. Wireless Commun. 5 (11) (2006) 3029–3033.
[68] Y.-h. Zhu, V.C.M. Leung, Optimization of distance-based location management for PCS networks, IEEE Trans. Wireless Commun. 7 (9) (2008) 3507–3516.

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