Supplementary Material for Computation Offloading Strategy Optimization with Multiple Heterogeneous Servers in Mobile Edge Computing

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1 Notations and Definitions

A summary of the notations used in this paper is provided in Table 1. The symbols are listed in the order introduced in the paper.

2 Power-Performance Tradeoff

2.1 Minimization of Average Response Time

In this section, we show the power-performance tradeoff and the impact of various parameters for the constant-speed model in minimization of average response time with average power consumption constraint.

In Figure 1, we examine the impact of the speed of data communication on the average response time of all offloadable and non-offloadable tasks generated on the UE for the constant-speed model. We show T as a function of P^* for $c_i = c + 0.5(i-1)$ MB/second, where $c = 10.0, 15.0, 20.0, 25.0, 30.0$ MD/second.

In Figure 2, we examine the impact of the amount of data communication on the average response time of all offloadable and non-offloadable tasks generated on the UE for the constant-speed model. We show T as a function of P^* for $\overline{d} = 0.6, 0.7, 0.8, 0.9, 1.0$ MD.

In Figure 3, we examine the impact of the energy consumption of data communication on the average response time of all offloadable and non-offloadable tasks generated on the UE for the constant-speed model. We show T as a function of P^* for $J = 0.02, 0.04, 0.06, 0.08, 0.10$ Joules.

Our observations are similar to those for the idle-speed model in the main paper.

2.2 Minimization of Average Power Consumption

In this section, we show the power-performance tradeoff and the impact of various parameters for the constant-speed model in minimization of average power consumption with average response time constraint.

In Figure 4, we examine the impact of the speed of data communication on the average power consumption of all offloadable and non-offloadable tasks generated on the UE for the constant-speed model. We show P as a function of T^* for $c_i = c + 0.5(i-1)$ MB/second, where $c = 6.0, 7.0, 8.0, 9.0, 10.0$ MD/second.

In Figure 5, we examine the impact of the amount of data communication on the average power consumption of all offloadable and non-offloadable tasks generated on the UE for the constant-speed model. We show P as a function of T^* for $\overline{d} = 1.0, 1.1, 1.2, 1.3, 1.4$ MD.

Algorithm 7: Minimize Cost-Performance Ratio

Input: $p_1, p_2, ..., p_n, \hat{\lambda}_0, \dot{\lambda}, \overline{r_0}, \overline{r_0^2}, \overline{r}, \overline{r^2}, \overline{d}, \overline{d^2}, \xi, \alpha, P_s, J$, and $\hat{\lambda}_i, \overline{r_i}, \overline{r_i^2}, s_i, c_i$, where $1 \le i \le n$.
Output: $(\tilde{\lambda}_1, \tilde{\lambda}_2, ..., \tilde{\lambda}_n)$ and the minimized R.

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Initialize the search interval of P* as (lb, ub);
while (the length of the search interval is ≥ ϵ) do
    P* ← the middle point of the search interval;
    Calculate T1 with P* by using Algorithm 5;
    Calculate T2 with P* + Δ by using Algorithm 5;
    Calculate R1 = P*T1;
    Calculate R2 = (P* + Δ)T2;
    Calculate ∂R/∂P* = (R2 − R1)/Δ;
    if (∂R/∂P* < 0) then
        Change the search interval to the right half;
    else
        Change the search interval to the left half;
    end if
end do;
P* ← the middle point of the search interval;
Calculate T by using Algorithm 5;
return (λ̃1, λ̃2, ..., λ̃n) and R = P*T.
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In Figure 6, we examine the impact of the energy consumption of data communication on the average power consumption of all offloadable and non-offloadable tasks

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Manuscript received Month Day, 2017; revised Month Day, 2018.

TABLE 1
Summary of Notations and Definitions

Notation	Definition
n	the number of MECs
MEC_i	the ith mobile edge cloud
p_i	the probability that MEC_i is preferred for offloading when a new offloadable task is generated
$\check{\lambda}$	$\hat{\lambda}_0 + \dot{\lambda}$: the arrival rate of a Poisson stream of offloadable/non-offloadable computation tasks on the UE
$\hat{\lambda}_0$	the arrival rate of a Poisson stream of non-offloadable computation tasks on the UE
$\dot{\lambda}$	the arrival rate of a Poisson stream of offloadable computation tasks on the UE
$\dot{\lambda}_i$	$p_i\dot{\lambda}$, $\tilde{\lambda}_i + \ddot{\lambda}_i$: the arrival rate of the ith substream generated when MEC_i is preferred for offloading
$\tilde{\lambda}_i$	the arrival rate of the sub-substream of tasks offloaded to MEC_i and processed remotely in MEC_i
$\ddot{\lambda}_i$	the arrival rate of the sub-substream of offloadable tasks processed locally in the UE
λ_0	$\hat{\lambda}_0 + \tilde{\lambda}_0$: the total arrival rate of computation tasks that are processed locally in the UE
$\tilde{\lambda}_0$	$\ddot{\lambda}_1 + \ddot{\lambda}_2 + \cdots + \ddot{\lambda}_n$: the total arrival rate of offloadable computation tasks processed locally in the UE
$\tilde{\lambda}$	$\tilde{\lambda}_1 + \tilde{\lambda}_2 + \cdots + \tilde{\lambda}_n$: the total arrival rate of computation tasks that are offloaded to the n MECs
$\hat{\lambda}_i$	the arrival rate of a Poisson stream of computation tasks on MEC_i
λ_i	$\hat{\lambda}_i + \tilde{\lambda}_i$: the total arrival rate of computation tasks that are processed by MEC_i
r_0	the random variable of the execution requirements of non-offloadable computation tasks generated on the UE
$\overline{r_0}, \overline{r_0^2}$	the mean and the second moment of r_0
r	the random variable of the execution requirements of offloadable computation tasks generated on the UE
$\bar{r}, \overline{r^2}$	the mean and the second moment of r
r_i	the random variable of the execution requirements of computation tasks preloaded on MEC_i
$\overline{r_i}, \overline{r_i^2}$	the mean and the second moment of r_i
d	the random variable of the amount of data communicated between the UE and the MECs for offloadable tasks
$\bar{d}, \overline{d^2}$	the mean and the second moment of d
s_0	the execution speed of the UE
s_i	the execution speed of MEC_i
c_i	the communication speed between the UE and MEC_i
P_d	dynamic power consumption of the UE
ξ, α	parameters to calculate dynamic power consumption of the UE
P_s	static power consumption of the UE
P_i	the transmission power of the UE for MEC_i
β_i	a combined quantity which summarizes various factors of the communication channel between the UE and MEC_i
J	the average energy consumption to complete data transmission for one task
P	average power consumption of the UE
x_0	the random variable of execution times of all tasks on the UE
$\overline{x_0}, \overline{x_0^2}$	the mean and the second moment of x_0
ρ_0	the utilization of the server in the UE
W_0	the average waiting time of the tasks on the UE
T_0	the average response time of the tasks on the UE
x_i	the random variable of execution times of all tasks on MEC_i
$\overline{x_i}, \overline{x_i^2}$	the mean and the second moment of x_i
ρ_i	the utilization of the server in MEC_i
W_i	the average waiting time of the tasks on MEC_i
T_i	the average response time of offloaded tasks on MEC_i
T	the average response time of all offloadable and non-offloadable tasks generated on the UE
R	cost-performance ratio (i.e., the power-time product $R = PT$)
P^*	power constraint
T^*	performance constraint
ϕ	a Lagrange multiplier

generated on the UE for the constant-speed model. We show P as a function of T^* for $J = 0.10, 0.12, 0.14, 0.16, 0.18$ Joules.

Our observations are similar to those for the idle-speed model in the main paper.

3 Minimization of Cost-Performance Ratio

In this section, we solve the problem of minimization of the cost-performance ratio.

3.1 A Numerical Algorithm

Our optimization problem to minimize the cost-performance ratio can be solved by using the algorithms in Section 6.1. Our numerical method is given in Algorithm 7. The algorithm uses the classical bisection method based on the observation that $R = P^*T$ is a convex function, and $\partial R/\partial P^*$ is an increasing function of P^*. The overall time complexity of Algorithm 7 is $O(n(\log(I/\epsilon))^4)$.

3.2 Numerical Examples and Data

In this section, we demonstrate numerical examples and data.

We use the same UE and MEC parameter setting in Section 6.2.

In Tables 2–3, we show numerical data for minimizing the cost-performance ratio for the idle-speed model and

TABLE 2
Numerical Data for Minimizing Cost-Performance Ratio (Idle-Speed Model)

	0	1	2	3	4	5	6	7
p_i	—	0.0828571	0.1028571	0.1228571	0.1428571	0.1628571	0.1828571	0.2028571
$\dot{\lambda}_i$	—	0.3728571	0.4628571	0.5528571	0.6428571	0.7328571	0.8228571	0.9128571
$\hat{\lambda}_i$	1.0000000	1.5000000	1.4500000	1.4000000	1.3500000	1.3000000	1.2500000	1.2000000
$\overline{r_i}$	0.5000000	1.0000000	1.0500000	1.1000000	1.1500000	1.2000000	1.2500000	1.3000000
$\overline{r_i^2}$	0.4000000	1.3500000	1.5435000	1.7545000	1.9837500	2.2320000	2.5000000	2.7885000
s_i	1.1106917	2.5000000	2.6000000	2.7000000	2.8000000	2.9000000	3.0000000	3.1000000
$\overline{r}/s_i$	1.3505098	0.6000000	0.5769231	0.5555556	0.5357143	0.5172414	0.5000000	0.4838710
$\overline{r_i}/s_i$	0.4501699	0.4000000	0.4038462	0.4074074	0.4107143	0.4137931	0.4166667	0.4193548
$\hat{\lambda}_i(\overline{r_i}/s_i)$	0.4501699	0.6000000	0.5855769	0.5703704	0.5544643	0.5379310	0.5208333	0.5032258
$\tilde{\lambda}_i^*$	—	0.3728571	0.4628571	0.5528571	0.6428571	0.7328571	0.8228571	0.8858407
c_i	—	10.0000000	10.5000000	11.0000000	11.5000000	12.0000000	12.5000000	13.0000000
$\overline{d}/c_i$	—	0.1000000	0.0952381	0.0909091	0.0869565	0.0833333	0.0800000	0.0769231
$\tilde{\lambda}_i$	0.2743130	0.3728571	0.4628571	0.5528571	0.6329354	0.6819533	0.7337829	0.7884440
λ_i	1.2743130	1.8728571	1.9128571	1.9528571	1.9829354	1.9819533	1.9837829	1.9884440
ρ_i	0.8206323	0.8237143	0.8526099	0.8775132	0.8935368	0.8906655	0.8877248	0.8847309
T_i	3.4073789	2.6903135	3.5453376	4.9879970	6.9418570	6.8302205	6.7171257	6.6028032

TABLE 3
Numerical Data for Minimizing Cost-Performance Ratio (Constant-Speed Model)

	0	1	2	3	4	5	6	7
p_i	—	0.0828571	0.1028571	0.1228571	0.1428571	0.1628571	0.1828571	0.2028571
$\dot{\lambda}_i$	—	0.3728571	0.4628571	0.5528571	0.6428571	0.7328571	0.8228571	0.9128571
$\hat{\lambda}_i$	1.0000000	1.5000000	1.4500000	1.4000000	1.3500000	1.3000000	1.2500000	1.2000000
$\overline{r_i}$	0.5000000	1.0000000	1.0500000	1.1000000	1.1500000	1.2000000	1.2500000	1.3000000
$\overline{r_i^2}$	0.4000000	1.3500000	1.5435000	1.7545000	1.9837500	2.2320000	2.5000000	2.7885000
s_i	1.0412716	2.5000000	2.6000000	2.7000000	2.8000000	2.9000000	3.0000000	3.1000000
$\overline{r}/s_i$	1.4405463	0.6000000	0.5769231	0.5555556	0.5357143	0.5172414	0.5000000	0.4838710
$\overline{r_i}/s_i$	0.4801821	0.4000000	0.4038462	0.4074074	0.4107143	0.4137931	0.4166667	0.4193548
$\hat{\lambda}_i(\overline{r_i}/s_i)$	0.4801821	0.6000000	0.5855769	0.5703704	0.5544643	0.5379310	0.5208333	0.5032258
$\tilde{\lambda}_i^*$	—	0.3728571	0.4628571	0.5528571	0.6428571	0.7328571	0.8228571	0.8858407
c_i	—	10.0000000	10.5000000	11.0000000	11.5000000	12.0000000	12.5000000	13.0000000
$\overline{d}/c_i$	—	0.1000000	0.0952381	0.0909091	0.0869565	0.0833333	0.0800000	0.0769231
$\tilde{\lambda}_i$	0.2600204	0.3728571	0.4628571	0.5528571	0.6362170	0.6854266	0.7374521	0.7923125
λ_i	1.2600204	1.8728571	1.9128571	1.9528571	1.9862170	1.9854266	1.9874521	1.9923125
ρ_i	0.8547535	0.8237143	0.8526099	0.8775132	0.8952948	0.8924620	0.8895594	0.8866028
T_i	4.4249947	2.6903135	3.5453376	4.9879970	7.2195229	7.1035733	6.9860850	6.8673000

the constant-speed model respectively. For the idle-speed model, we get $P = 4.1092012$ Watts, $T = 5.2600537$ seconds, $R = 21.6146195$, and $\tilde{\lambda} = 4.2256870$ tasks/second. For the constant-speed model, we get $P = 4.1174908$ Watts, $T = 5.6422605$ seconds, $R = 23.2319559$, and $\tilde{\lambda} = 4.2399796$ tasks/second. As expected, the constant-speed model has higher cost-performance ratio than the idle-speed model.

From both Tables 2 and 3, we make the following observations. (1) Lower indexed MECs receive all the offloadable tasks designated to them, due to their relatively low $\dot{\lambda}_i$. (2) Higher indexed MECs do not receive all the offloadable tasks designated to them, and the remaining offloadable tasks are processed by the UE itself, due to their relatively high $\dot{\lambda}_i$. (3) Compared with the idle-speed model, the constant-speed model results in reduced s_0 and $\tilde{\lambda}_0$, increased $\tilde{\lambda}$, increased T_i for all $i = 0, 4, 5, 6, 7$, increased T, reduced P, and increased R.

3.3 Power-Time Product

In this section, we show the power-time product and the impact of various parameters.

In Figures 7 and 10, we examine the impact of the speed of data communication on the cost-performance ratio of all offloadable and non-offloadable tasks generated on the UE for both power consumption models. We show R as a function of P^* for $c_i = c + 0.5(i-1)$ MB/second, where $c = 10.0, 15.0, 20.0, 25.0, 30.0$ MD/second.

In Figures 8 and 11, we examine the impact of the amount of data communication on the cost-performance ratio of all offloadable and non-offloadable tasks generated on the UE for both power consumption models. We show R as a function of P^* for $\overline{d} = 0.6, 0.7, 0.8, 0.9, 1.0$ MD.

In Figures 9 and 12, we examine the impact of the energy consumption of data communication on the cost-performance ratio of all offloadable and non-offloadable tasks generated on the UE for both power consumption models. We show R as a function of P^* for $J = 0.02, 0.04, 0.06, 0.08, 0.10$ Joules.

We have the following observations. (1) These figures all demonstrate that the power-time product has an optimal value, i.e., increasing P^* reduces the cost-performance ratio R; however, beyond certain value, R increases. (2) Figures 7, 8, 10, and 11 show that for the same power constraint, increasing the speed of data communication or decreasing the amount of data communication results in noticeable decrement in the cost-performance ratio. The reason is that the processing times of offloaded tasks on all the MECs are reduced. (3) However, Figures 9 and 12 show that decreasing the energy consumption of data communication only increases the speed of the UE and slightly decreases the cost-performance ratio.

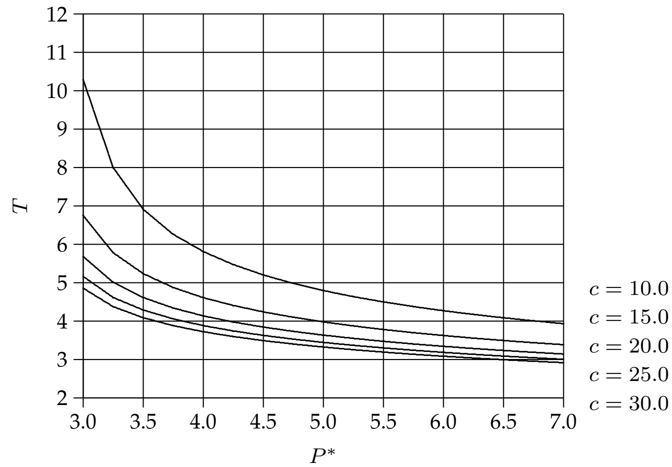

Fig. 1. The average response time T vs. the average power consumption P^* (varying c_i, constant-speed model).

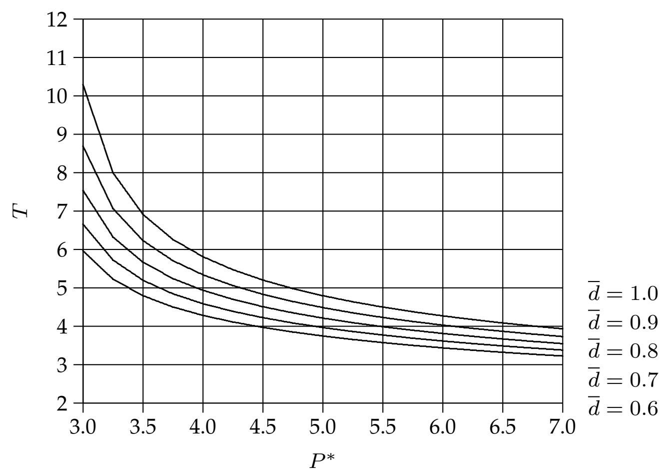

Fig. 2. The average response time T vs. the average power consumption P^* (varying $\overline{d}$, constant-speed model).

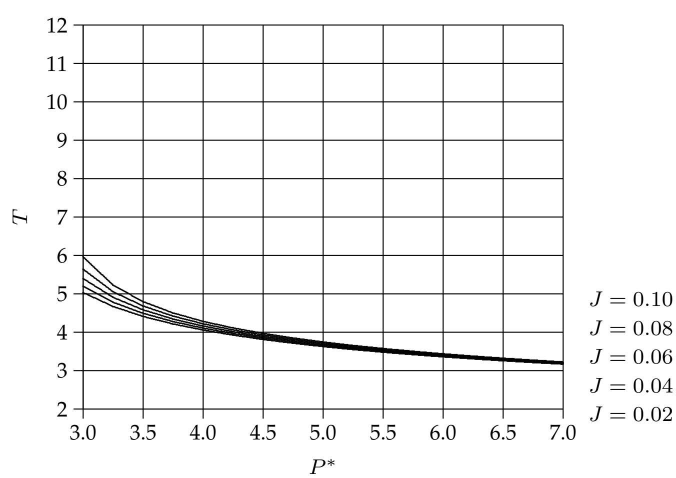

Fig. 3. The average response time T vs. the average power consumption P^* (varying J, constant-speed model).

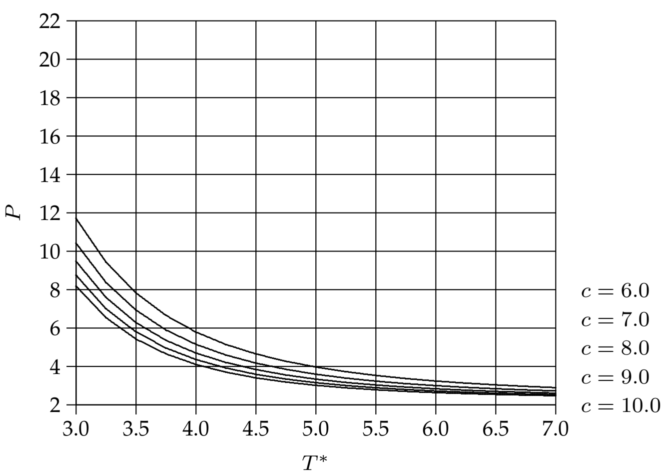

Fig. 4. The average power consumption P vs. the average response time T^* (varying c_i, constant-speed model).

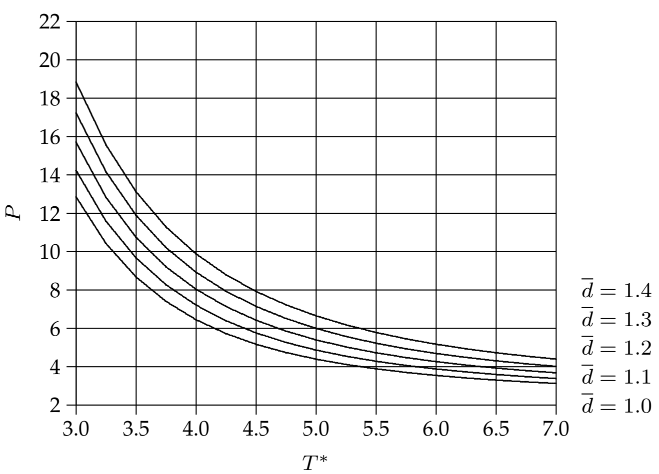

Fig. 5. The average power consumption P vs. the average response time T^* (varying $\overline{d}$, constant-speed model).

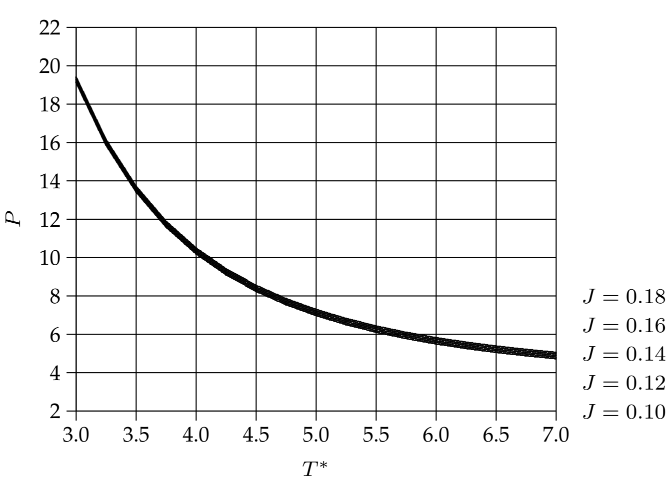

Fig. 6. The average power consumption P vs. the average response time T^* (varying J, constant-speed model).

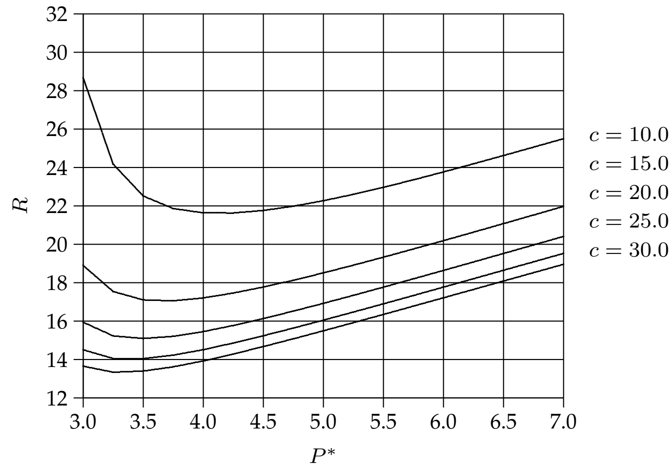

Fig. 7. The cost-performance ratio R vs. the average power consumption P^* (varying c_i, idle-speed model).

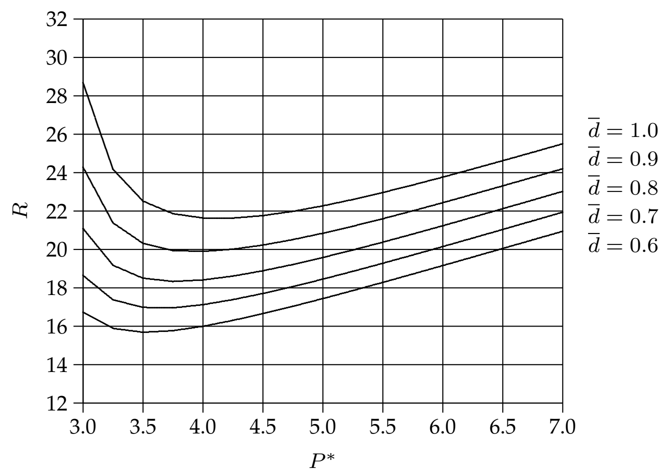

Fig. 8. The cost-performance ratio R vs. the average power consumption P^* (varying $\overline{d}$, idle-speed model).

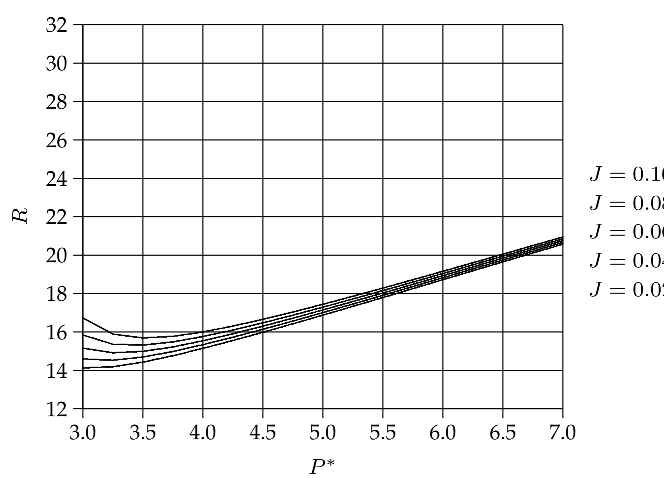

Fig. 9. The cost-performance ratio R vs. the average power consumption P^* (varying J, idle-speed model).

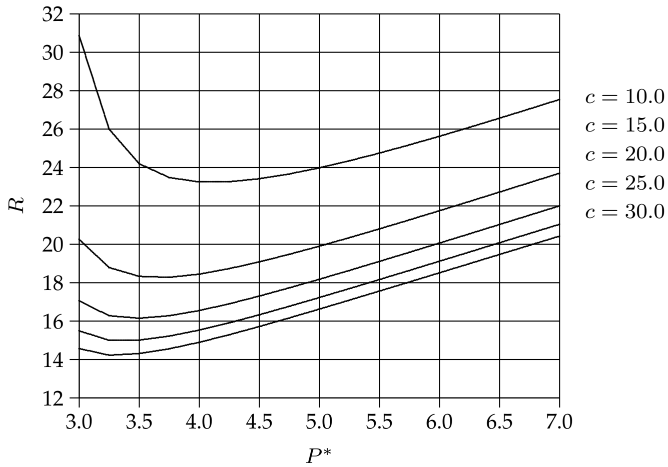

Fig. 10. The cost-performance ratio R vs. the average power consumption P^* (varying c_i, constant-speed model).

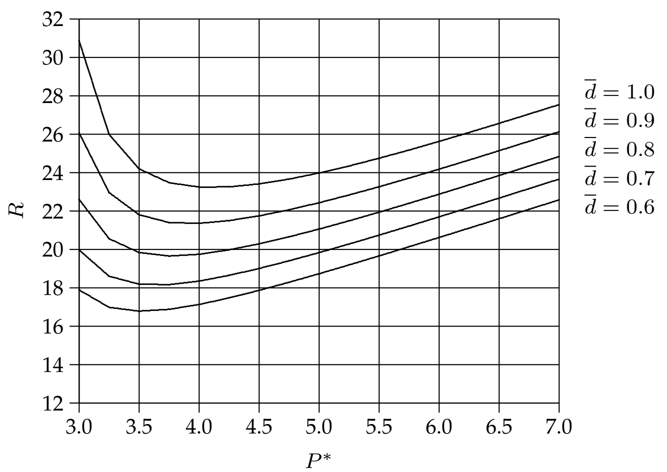

Fig. 11. The cost-performance ratio R vs. the average power consumption P^* (varying $\overline{d}$, constant-speed model).

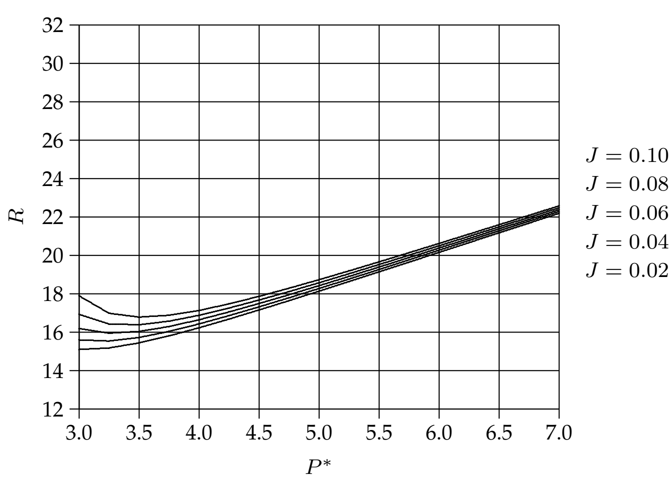

Fig. 12. The cost-performance ratio R vs. the average power consumption P^* (varying J, constant-speed model).