Supplementary Material for
Analysis of Distance-Based Location Management in Wireless Communication Networks

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Appendix 1. Proofs of Theorems 1–4

Proof of Theorem 1. Since $X(t) < j$ if and only if $S_j > t$, we have

$$\boldsymbol{P}[X(t) < j] = \boldsymbol{P}[S_j > t] = 1 - F_{S_j}(t).$$

Hence, we get

$$\boldsymbol{P}[X(t) = j] = \boldsymbol{P}[X(t) < j+1] - \boldsymbol{P}[X(t) < j] = F_{S_j}(t) - F_{S_{j+1}}(t),$$

for all $j \geq 0$, with $F_{S_0}(t) = 1$. Then, we have

$$\boldsymbol{P}[X = j] = \int_0^\infty \boldsymbol{P}[X(t) = j] f_{T_c}(t)dt = \int_0^\infty \left(F_{S_j}(t) - F_{S_{j+1}}(t)\right) f_{T_c}(t)dt.$$

Note that

$$\begin{aligned}
\boldsymbol{E}(X) &= \sum_{j=1}^\infty j\boldsymbol{P}[X = j] \\
&= \sum_{j=1}^\infty j \int_0^\infty \left(F_{S_j}(t) - F_{S_{j+1}}(t)\right) f_{T_c}(t)dt \\
&= \int_0^\infty \left(\sum_{j=1}^\infty j \left(F_{S_j}(t) - F_{S_{j+1}}(t)\right)\right) f_{T_c}(t)dt \\
&= \int_0^\infty \left(\sum_{j=1}^\infty F_{S_j}(t)\right) f_{T_c}(t)dt,
\end{aligned}$$

which is exactly what given in the theorem. ∎

Proof of Theorem 2. It is clear that

$$f_T^*(s) = \sum_{i=1}^k w_i \left(\frac{\lambda_i}{s + \lambda_i}\right)^{\gamma_i},$$

and

$$f_{S_j}^*(s) = \left(\sum_{i=1}^k w_i \left(\frac{\lambda_i}{s + \lambda_i}\right)^{\gamma_i}\right)^j,$$

for all $j \geq 0$. To proceed with the Cauchy residual theorem, we write $f_{S_j}^*(s)$ as follows,

$$\begin{aligned}
f_{S_j}^*(s) &= \sum_{j_1+j_2+\cdots+j_k=j} \binom{j}{j_1, j_2, ..., j_k} \prod_{i=1}^k \left(w_i \left(\frac{\lambda_i}{s+\lambda_i}\right)^{\gamma_i}\right)^{j_i} \\
&= \sum_{j_1+j_2+\cdots+j_k=j} \binom{j}{j_1, j_2, ..., j_k} \prod_{i=1}^k (w_i \lambda_i^{\gamma_i})^{j_i} \prod_{i=1}^k \frac{1}{(s+\lambda_i)^{\gamma_i j_i}}.
\end{aligned}$$

By the linearity of Laplace transform, we obtain

$$F_{S_j}(t) = \sum_{j_1+j_2+\cdots+j_k=j} \binom{j}{j_1, j_2, ..., j_k} \left(\prod_{i=1}^{k} (w_i \lambda_i^{\gamma_i})^{j_i} \right) \left(\frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} \frac{e^{st}}{s} \prod_{i=1}^{k} \frac{1}{(s+\lambda_i)^{\gamma_i j_i}} ds \right),$$

and we can treat each term

$$\frac{e^{st}}{s} \prod_{i=1}^{k} \frac{1}{(s+\lambda_i)^{\gamma_i j_i}}$$

individually. The above function has a pole of order 1 at 0 with residual

$$\text{Res}_{j_1,j_2,...,j_k}(0) = \prod_{i=1}^{k} \frac{1}{\lambda_i^{\gamma_i j_i}},$$

and a pole of order $\gamma_i j_i$ at $-\lambda_i$ with residual

$$\text{Res}_{j_1,j_2,...,j_k}(-\lambda_i) = \frac{1}{(\gamma_i j_i - 1)!} \left(\frac{\partial}{\partial s} \right)^{\gamma_i j_i - 1} \left[\frac{e^{st}}{s} \prod_{i' \neq i} \frac{1}{(s+\lambda_{i'})^{\gamma_{i'} j_{i'}}} \right]_{s=-\lambda_i},$$

for all $1 \leq i \leq k$. Therefore, we have

$$F_{S_j}(t) = \sum_{j_1+j_2+\cdots+j_k=j} \binom{j}{j_1, j_2, ..., j_k} \prod_{i=1}^{k} (w_i \lambda_i^{\gamma_i})^{j_i} R_{j_1,j_2,...,j_k}(t),$$

where

$$\begin{aligned} R_{j_1,j_2,...,j_k}(t) &= \text{Res}_{j_1,j_2,...,j_k}(0) + \sum_{i=1}^{k} \text{Res}_{j_1,j_2,...,j_k}(-\lambda_i) \\ &= \prod_{i=1}^{k} \frac{1}{\lambda_i^{\gamma_i j_i}} + \sum_{i=1}^{k} \frac{1}{(\gamma_i j_i - 1)!} \left(\frac{\partial}{\partial s} \right)^{\gamma_i j_i - 1} \left[\frac{e^{st}}{s} \prod_{i' \neq i} \frac{1}{(s+\lambda_{i'})^{\gamma_{i'} j_{i'}}} \right]_{s=-\lambda_i}. \end{aligned}$$

The theorem is proven. ∎

Proof of Theorem 3. For any two independent random variables X and Y, we have

$$f^*_{X+Y}(s) = f^*_X(s) f^*_Y(s).$$

It is well known that the residual time T_1 has pdf

$$f_{T_1}(t) = \lambda_T(1 - F_T(t))$$

and

$$f^*_{T_1}(s) = \lambda_T \left(\frac{1 - f^*_T(s)}{s} \right),$$

(see [24], Eq. (5.10), p. 172). Hence, for an equilibrium renewal process, we have

$$f^*_{S_j}(s) = \prod_{i=1}^{j} f^*_{T_i}(s) = f^*_{T_1}(s) \prod_{i=2}^{j} f^*_{T_i}(s) = \lambda_T \left(\frac{1 - f^*_T(s)}{s} \right) (f^*_T(s))^{j-1},$$

and

$$F^*_{S_j}(s) = \frac{f^*_{S_j}(s)}{s} = \frac{\lambda_T}{s^2}(1 - f^*_T(s))(f^*_T(s))^{j-1},$$

for all $j \geq 1$, where we notice that $F^*_X(s) = f^*_X(s)/s$ for any random variable X. Therefore, we have

$$H^*(s) = \sum_{j=1}^{\infty} F^*_{S_j}(s) = \frac{\lambda_T}{s^2}(1 - f^*_T(s)) \sum_{j=1}^{\infty} (f^*_T(s))^{j-1} = \frac{\lambda_T}{s^2},$$

which implies that $H(t) = \lambda_T t$. By Theorem 1, the expected number of renewals in a random time interval of length T_c is

$$\boldsymbol{E}(X) = \int_0^{\infty} H(t) f_{T_c}(t) dt = \lambda_T \int_0^{\infty} t f_{T_c}(t) dt = \lambda_T \boldsymbol{E}(T_c) = \boldsymbol{E}(T_c)/\boldsymbol{E}(T) = \lambda_T/\lambda_{T_c}.$$

The theorem is proven. ∎

Proof of Theorem 4. The probability generating function (pgf) of $X(t)$ is defined as

$$G_{X(t)}(t, z) = \sum_{j=0}^{\infty} \boldsymbol{P}[X(t) = j] z^j.$$

That is,

$$G_{X(t)}(t, z) = \sum_{j=0}^{\infty} (F_{S_j}(t) - F_{S_{j+1}}(t)) z^j = 1 + \sum_{j=1}^{\infty} F_{S_j}(t) z^{j-1}(z - 1),$$

where we note that $F_{S_0}(t) = 1$. Applying Laplace transform to $G_{X(t)}(t, z)$, we get

$$G^*_{X(t)}(s, z) = \frac{1}{s} + \frac{1}{s} \sum_{j=1}^{\infty} z^{j-1}(z - 1) f^*_{S_j}(s),$$

where we notice that $F^*_T(s) = f^*_T(s)/s$ for any random variable T. It is clear that for a modified renewal process, we have

$$f^*_{S_j}(s) = \prod_{i=1}^{j} f^*_{T_i}(s) = f^*_{T_1}(s) \prod_{i=2}^{j} f^*_{T_i}(s) = f^*_{T_1}(s)(f^*_T(s))^{j-1},$$

and

$$F^*_{S_j}(s) = \frac{f^*_{T_1}(s)(f^*_T(s))^{j-1}}{s},$$

for all $j \geq 1$. It is easy to verify that in a modified renewal process,

$$\begin{aligned} G^*_{X(t)}(s, z) &= \frac{1}{s} + \frac{1}{s} \sum_{j=1}^{\infty} z^{j-1}(z - 1) f^*_{T_1}(s)(f^*_T(s))^{j-1} \\ &= \frac{1}{s} + \frac{f^*_{T_1}(s)(z - 1)}{s(1 - f^*_T(s) z)}. \end{aligned}$$

The pgf of X, the number of renewals in a random time interval T_c, is

$$G_X(z) = \sum_{j=0}^{\infty} \boldsymbol{P}[X = j] z^j = \int_0^{\infty} G_{X(t)}(t, z) f_{T_c}(t) dt.$$

The Laplace transform of $G_{X(t)}(t, z)$ is

$$G^*_{X(t)}(s, z) = \int_0^{\infty} G_{X(t)}(t, z) e^{-st} dt.$$

Notice that

$$\left(-\frac{\partial}{\partial s}\right)^{\gamma_c - 1} G^*_{X(t)}(s, z) = \int_0^{\infty} G_{X(t)}(t, z) t^{\gamma_c - 1} e^{-st} dt,$$

and

$$\frac{\lambda_c^{\gamma_c}}{(\gamma_c - 1)!} \left(-\frac{\partial}{\partial s}\right)^{\gamma_c - 1} G^*_{X(t)}(s, z) = \int_0^{\infty} G_{X(t)}(t, z) \frac{\lambda_c (\lambda_c t)^{\gamma_c - 1} e^{-st}}{(\gamma_c - 1)!} dt.$$

The last equation implies that if T_c has an Erlang distribution with parameters (γ_c, λ_c),

$$f_{T_c}(t) = \frac{\lambda_c (\lambda_c t)^{\gamma_c - 1} e^{-\lambda_c t}}{(\gamma_c - 1)!},$$

then we should have

$$G_X(z) = \frac{\lambda_c^{\gamma_c}}{(\gamma_c - 1)!} \left(-\frac{\partial}{\partial s}\right)^{\gamma_c - 1} [G^*_{X(t)}(s, z)]_{s=\lambda_c}.$$

For a modified renewal process, we get

$$\begin{aligned} G_X(z) &= \frac{\lambda_c^{\gamma_c}}{(\gamma_c - 1)!} \left(-\frac{\partial}{\partial s}\right)^{\gamma_c - 1} \left[\frac{1}{s} + \frac{f^*_{T_1}(s)(z-1)}{s(1 - f^*_T(s) z)}\right]_{s=\lambda_c} \\ &= 1 + \frac{\lambda_c^{\gamma_c}}{(\gamma_c - 1)!} \left(-\frac{\partial}{\partial s}\right)^{\gamma_c - 1} \left[\frac{f^*_{T_1}(s)(z-1)}{s(1 - f^*_T(s) z)}\right]_{s=\lambda_c}. \end{aligned}$$

The Erlang distribution of T_c can be easily generalized to a hyper-Erlang distribution with pdf

$$f_{T_c}(t) = \sum_{i=1}^{k_c} w_{c,i} \left(\frac{\lambda_{c,i} (\lambda_{c,i} t)^{\gamma_{c,i} - 1} e^{-\lambda_{c,i} t}}{(\gamma_{c,i} - 1)!}\right),$$

where $w_{c,1} + w_{c,2} + \cdots + w_{c,k_c} = 1$. For a modified renewal process, we get

$$G_X(z) = 1 + \sum_{i=1}^{k_c} w_{c,i} \left(\frac{\lambda_{c,i}^{\gamma_{c,i}}}{(\gamma_{c,i} - 1)!}\right) \left(-\frac{\partial}{\partial s}\right)^{\gamma_{c,i} - 1} \left[\frac{f^*_{T_1}(s)(z-1)}{s(1 - f^*_T(s) z)}\right]_{s=\lambda_{c,i}}.$$

Notice that

$$\boldsymbol{P}[X = j] = \frac{1}{j!} \cdot \frac{d^j}{dz^j} G_X(z)\Big|_{z=0},$$

for all $j \geq 0$. It is easy to verify that

$$G_X(0) = 1 - \sum_{i=1}^{k_c} w_{c,i} \left(\frac{\lambda_{c,i}^{\gamma_{c,i}}}{(\gamma_{c,i}-1)!} \right) \left(-\frac{\partial}{\partial s} \right)^{\gamma_{c,i}-1} \left[\frac{f_{T_1}^*(s)}{s} \right]_{s=\lambda_{c,i}},$$

and

$$\frac{1}{j!} \cdot \frac{d^j}{dz^j} G_X(z) \bigg|_{z=0} = \sum_{i=1}^{k_c} w_{c,i} \left(\frac{\lambda_{c,i}^{\gamma_{c,i}}}{(\gamma_{c,i}-1)!} \right) \left(-\frac{\partial}{\partial s} \right)^{\gamma_{c,i}-1} \left[\left(\frac{f_{T_1}^*(s)(1-f_T^*(s))}{s} \right) (f_T^*(s))^{j-1} \right]_{s=\lambda_{c,i}},$$

for all $j \geq 1$. ∎

Appendix 2. Numerical Values for $N(d)$

In Figure 1, we display $N(d)$, where $N(d)$ is calculated as

$$\sum_{n=d}^{n^*} n\phi_{0d}^{(n)},$$

with n^* sufficiently large such that

$$\sum_{n=d}^{n^*} \phi_{0d}^{(n)} > 1 - 10^{-10}.$$

For instance, when $d = 20$, we need to set $n^* = 6,000$.

As a comparison, we also show $N(d)$ obtained by the previous approach where $\delta_q = 0$. The curves are labeled with $q' = 3$ and $q' = 4$. It is observed that there is noticeable difference between our $N(d)$ and the one obtained by the previous approach.

Appendix 3. First-Passage Distribution

In Tables 1–2, we show the first-passage distribution $(\phi_{0d}^{(1)}, \phi_{0d}^{(2)}, ..., \phi_{0d}^{(n)}, ...)$ for K_d, where $1 \leq d \leq 10$ and $1 \leq n \leq 40$. The last line of each table gives the value $\phi_{0d}^{(1)} + \phi_{0d}^{(2)} + \cdots + \phi_{0d}^{(40)}$, i.e., the probability that a mobile terminal has reached ring d after crossing cell boundaries for 40 times.

Appendix 4. Numerical Data

We now present numerical data to show the impact of various parameters and performance optimization.

In Figures 2–5, we display the total cost of location management $D_{\text{CPLU}}(d)$ with $q = 3$, $D_{\text{CWLU}}(d)$ with $q = 3$, $D_{\text{CPLU}}(d)$ with $q = 4$, and $D_{\text{CWLU}}(d)$ with $q = 4$, and study the impact of Δ_u. The values of the variables are as follows: $\lambda_c = 1$, $\lambda_s = 40$, $\gamma_s = 2$,

Table 1: The First-Passage Distribution $(\phi_{0d}^{(1)}, \phi_{0d}^{(2)}, \phi_{0d}^{(3)}, ...)$ for K_d $(q = 3)$.

n	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	K_{10}
1	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
2	0.00000	0.50000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
3	0.00000	0.16667	0.20208	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
4	0.00000	0.13889	0.13472	0.07690	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
5	0.00000	0.07407	0.12757	0.07690	0.02836	0.00000	0.00000	0.00000	0.00000	0.00000
6	0.00000	0.04784	0.09898	0.08307	0.03781	0.01026	0.00000	0.00000	0.00000	0.00000
7	0.00000	0.02829	0.08233	0.07722	0.04645	0.01709	0.00366	0.00000	0.00000	0.00000
8	0.00000	0.01740	0.06629	0.07180	0.04932	0.02365	0.00732	0.00129	0.00000	0.00000
9	0.00000	0.01052	0.05408	0.06494	0.05034	0.02803	0.01129	0.00302	0.00045	0.00000
10	0.00000	0.00641	0.04386	0.05857	0.04954	0.03111	0.01473	0.00514	0.00121	0.00016
11	0.00000	0.00389	0.03566	0.05249	0.04791	0.03293	0.01764	0.00731	0.00226	0.00048
12	0.00000	0.00236	0.02896	0.04698	0.04573	0.03387	0.01995	0.00939	0.00347	0.00097
13	0.00000	0.00144	0.02353	0.04198	0.04330	0.03413	0.02172	0.01129	0.00476	0.00159
14	0.00000	0.00087	0.01912	0.03750	0.04078	0.03390	0.02300	0.01296	0.00606	0.00232
15	0.00000	0.00053	0.01553	0.03348	0.03826	0.03332	0.02388	0.01438	0.00731	0.00312
16	0.00000	0.00032	0.01262	0.02989	0.03581	0.03250	0.02442	0.01557	0.00848	0.00394
17	0.00000	0.00020	0.01025	0.02668	0.03346	0.03153	0.02468	0.01652	0.00955	0.00477
18	0.00000	0.00012	0.00833	0.02381	0.03122	0.03044	0.02473	0.01728	0.01051	0.00558
19	0.00000	0.00007	0.00677	0.02126	0.02911	0.02930	0.02460	0.01785	0.01135	0.00636
20	0.00000	0.00004	0.00550	0.01897	0.02713	0.02813	0.02433	0.01827	0.01209	0.00709
21	0.00000	0.00003	0.00447	0.01693	0.02527	0.02696	0.02396	0.01855	0.01271	0.00777
22	0.00000	0.00002	0.00363	0.01511	0.02354	0.02579	0.02351	0.01871	0.01323	0.00839
23	0.00000	0.00001	0.00295	0.01349	0.02192	0.02465	0.02300	0.01877	0.01366	0.00895
24	0.00000	0.00001	0.00240	0.01204	0.02040	0.02354	0.02244	0.01875	0.01400	0.00945
25	0.00000	0.00000	0.00195	0.01074	0.01899	0.02246	0.02186	0.01865	0.01427	0.00990
26	0.00000	0.00000	0.00158	0.00959	0.01768	0.02141	0.02126	0.01850	0.01447	0.01029
27	0.00000	0.00000	0.00128	0.00856	0.01646	0.02041	0.02064	0.01830	0.01461	0.01063
28	0.00000	0.00000	0.00104	0.00764	0.01532	0.01945	0.02002	0.01807	0.01470	0.01092
29	0.00000	0.00000	0.00085	0.00682	0.01426	0.01853	0.01940	0.01780	0.01474	0.01116
30	0.00000	0.00000	0.00069	0.00609	0.01327	0.01765	0.01879	0.01750	0.01473	0.01136
31	0.00000	0.00000	0.00056	0.00543	0.01235	0.01681	0.01818	0.01719	0.01470	0.01153
32	0.00000	0.00000	0.00045	0.00485	0.01150	0.01600	0.01758	0.01686	0.01463	0.01166
33	0.00000	0.00000	0.00037	0.00433	0.01070	0.01524	0.01700	0.01652	0.01453	0.01176
34	0.00000	0.00000	0.00030	0.00386	0.00996	0.01451	0.01642	0.01617	0.01441	0.01183
35	0.00000	0.00000	0.00024	0.00345	0.00927	0.01381	0.01587	0.01581	0.01427	0.01187
36	0.00000	0.00000	0.00020	0.00308	0.00863	0.01314	0.01532	0.01546	0.01411	0.01189
37	0.00000	0.00000	0.00016	0.00275	0.00803	0.01251	0.01480	0.01510	0.01394	0.01188
38	0.00000	0.00000	0.00013	0.00245	0.00748	0.01191	0.01429	0.01474	0.01376	0.01186
39	0.00000	0.00000	0.00011	0.00219	0.00696	0.01134	0.01379	0.01438	0.01357	0.01182
40	0.00000	0.00000	0.00009	0.00195	0.00648	0.01079	0.01331	0.01403	0.01337	0.01177
	1.00000	1.00000	0.99963	0.98379	0.91300	0.78710	0.63739	0.49013	0.35991	0.25307

Table 2: The First-Passage Distribution $(\phi_{0d}^{(1)}, \phi_{0d}^{(2)}, \phi_{0d}^{(3)}, ...)$ for K_d $(q = 4)$.

n	K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	K_{10}
1	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
2	0.00000	0.50000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
3	0.00000	0.18750	0.20563	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
4	0.00000	0.13281	0.14137	0.08208	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
5	0.00000	0.07324	0.12720	0.08037	0.03227	0.00000	0.00000	0.00000	0.00000	0.00000
6	0.00000	0.04407	0.10005	0.08465	0.04067	0.01257	0.00000	0.00000	0.00000	0.00000
7	0.00000	0.02568	0.08182	0.07810	0.04891	0.01930	0.00486	0.00000	0.00000	0.00000
8	0.00000	0.01514	0.06589	0.07186	0.05097	0.02602	0.00879	0.00187	0.00000	0.00000
9	0.00000	0.00889	0.05333	0.06472	0.05137	0.03002	0.01310	0.00389	0.00072	0.00000
10	0.00000	0.00522	0.04308	0.05811	0.05008	0.03276	0.01652	0.00634	0.00168	0.00028
11	0.00000	0.00307	0.03483	0.05195	0.04808	0.03419	0.01936	0.00865	0.00298	0.00072
12	0.00000	0.00180	0.02815	0.04639	0.04563	0.03479	0.02148	0.01082	0.00437	0.00137
13	0.00000	0.00106	0.02275	0.04139	0.04303	0.03475	0.02305	0.01271	0.00580	0.00214
14	0.00000	0.00062	0.01839	0.03691	0.04039	0.03427	0.02412	0.01432	0.00718	0.00301
15	0.00000	0.00037	0.01486	0.03292	0.03780	0.03349	0.02479	0.01565	0.00847	0.00391
16	0.00000	0.00022	0.01201	0.02935	0.03531	0.03251	0.02513	0.01672	0.00964	0.00481
17	0.00000	0.00013	0.00971	0.02617	0.03294	0.03141	0.02522	0.01755	0.01068	0.00570
18	0.00000	0.00007	0.00785	0.02333	0.03070	0.03023	0.02511	0.01818	0.01159	0.00654
19	0.00000	0.00004	0.00634	0.02080	0.02860	0.02902	0.02485	0.01862	0.01237	0.00732
20	0.00000	0.00003	0.00513	0.01854	0.02663	0.02780	0.02447	0.01892	0.01303	0.00805
21	0.00000	0.00002	0.00414	0.01653	0.02479	0.02658	0.02400	0.01908	0.01357	0.00870
22	0.00000	0.00001	0.00335	0.01474	0.02307	0.02539	0.02347	0.01914	0.01402	0.00929
23	0.00000	0.00001	0.00271	0.01314	0.02147	0.02424	0.02289	0.01911	0.01436	0.00981
24	0.00000	0.00000	0.00219	0.01171	0.01998	0.02312	0.02228	0.01900	0.01463	0.01026
25	0.00000	0.00000	0.00177	0.01044	0.01859	0.02204	0.02165	0.01883	0.01482	0.01066
26	0.00000	0.00000	0.00143	0.00931	0.01730	0.02100	0.02101	0.01861	0.01494	0.01100
27	0.00000	0.00000	0.00116	0.00830	0.01610	0.02000	0.02037	0.01835	0.01501	0.01128
28	0.00000	0.00000	0.00093	0.00740	0.01498	0.01905	0.01973	0.01806	0.01504	0.01152
29	0.00000	0.00000	0.00075	0.00660	0.01394	0.01814	0.01909	0.01775	0.01501	0.01171
30	0.00000	0.00000	0.00061	0.00588	0.01297	0.01727	0.01847	0.01741	0.01496	0.01186
31	0.00000	0.00000	0.00049	0.00524	0.01207	0.01644	0.01785	0.01707	0.01487	0.01197
32	0.00000	0.00000	0.00040	0.00467	0.01123	0.01565	0.01725	0.01671	0.01475	0.01205
33	0.00000	0.00000	0.00032	0.00417	0.01045	0.01489	0.01666	0.01634	0.01461	0.01210
34	0.00000	0.00000	0.00026	0.00371	0.00972	0.01417	0.01609	0.01597	0.01445	0.01213
35	0.00000	0.00000	0.00021	0.00331	0.00904	0.01349	0.01554	0.01560	0.01428	0.01213
36	0.00000	0.00000	0.00017	0.00295	0.00841	0.01284	0.01500	0.01523	0.01409	0.01211
37	0.00000	0.00000	0.00014	0.00263	0.00783	0.01222	0.01447	0.01486	0.01389	0.01207
38	0.00000	0.00000	0.00011	0.00235	0.00728	0.01163	0.01397	0.01449	0.01369	0.01201
39	0.00000	0.00000	0.00009	0.00209	0.00678	0.01107	0.01348	0.01413	0.01347	0.01194
40	0.00000	0.00000	0.00007	0.00187	0.00631	0.01053	0.01300	0.01377	0.01325	0.01186
	1.00000	1.00000	0.99969	0.98468	0.91569	0.79289	0.64712	0.50375	0.37622	0.27031

which result in $\rho = 0.05$ and $C_{T_s} = 0.824$, and $\Delta_p = 1$. The parameter Δ_u is set as $\Delta_u = 10, 20, 40, 80$. The parameter d is in the range $1 \leq d \leq 20$.

In Figures 6–9, we display the total cost of location management $D_{\text{CPLU}}(d)$ with $q = 3$, $D_{\text{CWLU}}(d)$ with $q = 3$, $D_{\text{CPLU}}(d)$ with $q = 4$, and $D_{\text{CWLU}}(d)$ with $q = 4$, and study the impact of λ_s. The values of the variables are as follows: $\lambda_c = 1$, $\lambda_s = 50.00, 25.00, 12.50, 6.25$, $\gamma_s = 1$, which result in $\rho = 0.02, 0.04, 0.08, 0.16$, and $C_{T_s} = 1.060$. We also set $\Delta_p = 1$ and $\Delta_u = 30$. The parameter d is in the range $1 \leq d \leq 20$.

In Figures 10–13, we display the total cost of location management $D_{\text{CPLU}}(d)$ with $q = 3$, $D_{\text{CWLU}}(d)$ with $q = 3$, $D_{\text{CPLU}}(d)$ with $q = 4$, and $D_{\text{CWLU}}(d)$ with $q = 4$, and study the impact of γ_s. The values of the variables are as follows: $\lambda_c = 1$, $\lambda_s = 20$, $\gamma_s = 0.5, 1.0, 2.0, 4.0$, which result in $\rho = 0.025, 0.050, 0.100, 0.200$, and $C_{T_s} = 1.419, 1.060, 0.824, 0.676$. We also set $\Delta_p = 1$ and $\Delta_u = 35$. The parameter d is in the range $1 \leq d \leq 20$.

In Figures 14–15, we show d^* in DBLMS-CWLU with $q = 3$ and DBLMS-CWLU with $q = 4$ respectively, as a function of ρ, where $0 < \rho \leq 1$. We also set $\Delta_p = 1$ and $\Delta_u = 10, 50, 100$. The data in the figures are obtained by using our closed-form approximation of $N(d)$ with $\alpha = 0.6$ and verified to be correct.

We have the following observations.

- When d is small, the total cost of location management is dominated by location update cost, which is determined by Δ_u and ρ. An increased Δ_u will increase the total cost of location management noticeably. Similarly, a decreased ρ, which is caused by an increased λ_s or a decreased γ_s, will increase the total cost of location management noticeably.

- A DBLMS operated under the CWLU model has lower location update cost than a DBLMS operated under the CPLU model. Although this is not very obvious from Theorems 11 and 12, it is intuitively acceptable.

- When d is large, the total cost of location management is dominated by terminal paging cost, which is determined by Δ_p, q, and d. Although not demonstrated in the figures, we can conclude that an increased Δ_p will increase the total cost of location management noticeably. Also, the different values of $q = 3$ and $q = 4$ give rise to noticeable difference in the total cost of location management.

- There is an optimal value of d^* which minimizes $D_{\text{CPLU}}(d)$ or $D_{\text{CWLU}}(d)$. The optimal value of d^* which minimizes $D_{\text{CPLU}}(d)$ or $D_{\text{CWLU}}(d)$ depends on q, ρ, Δ_p, and Δ_u. In particular, d^* is an increasing function of Δ_u and a decreasing function of q, ρ, and Δ_p.

Finally, we would like to mention that all our parameters are chosen for the purpose of demonstrating the impact of various parameters and performance optimization. Changing the parameters will affect the optimal distance threshold that minimizes the total location management cost; however, it does not affect our general observations.

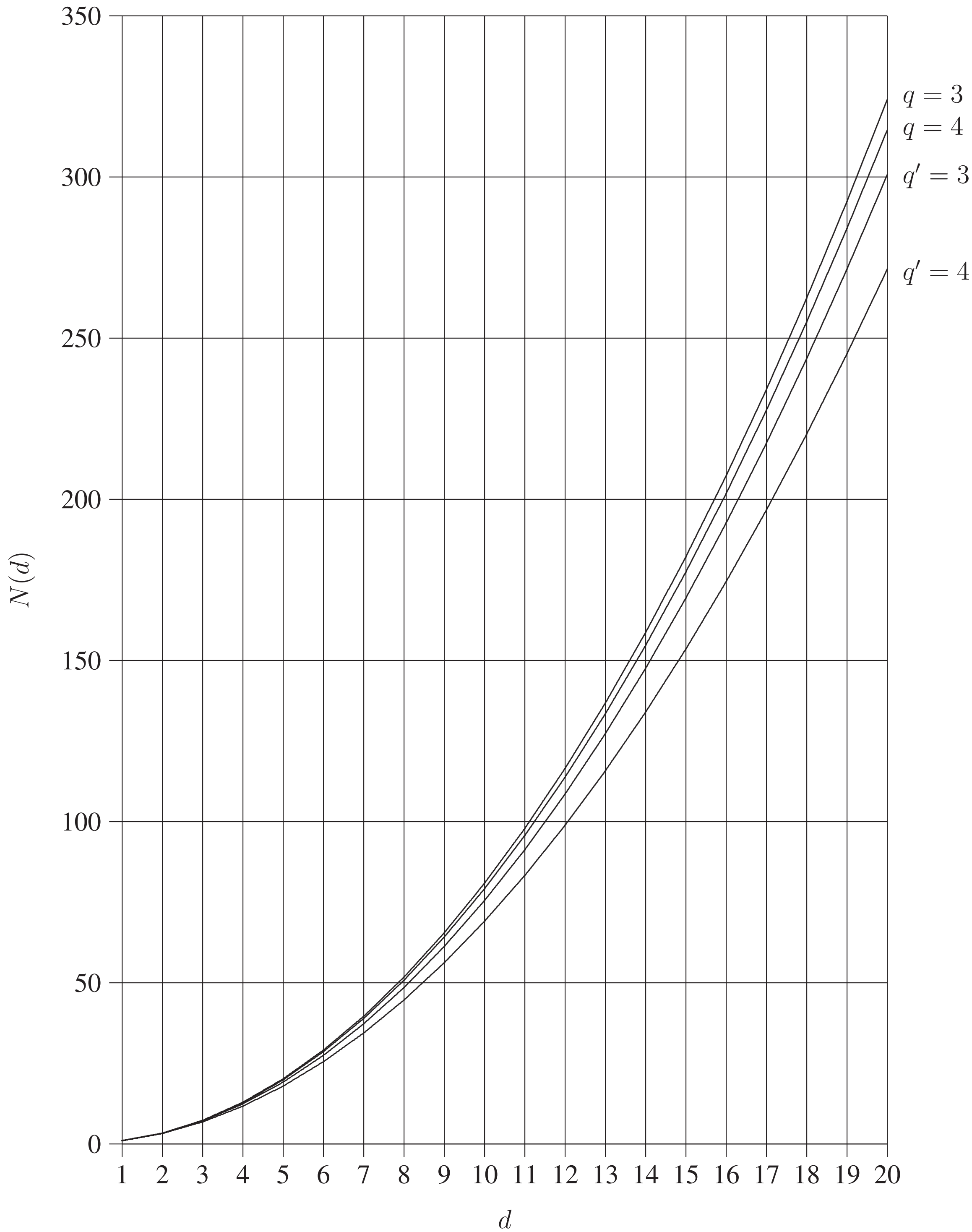

Figure 1: The expected number of steps $N(d)$ to reach ring d.

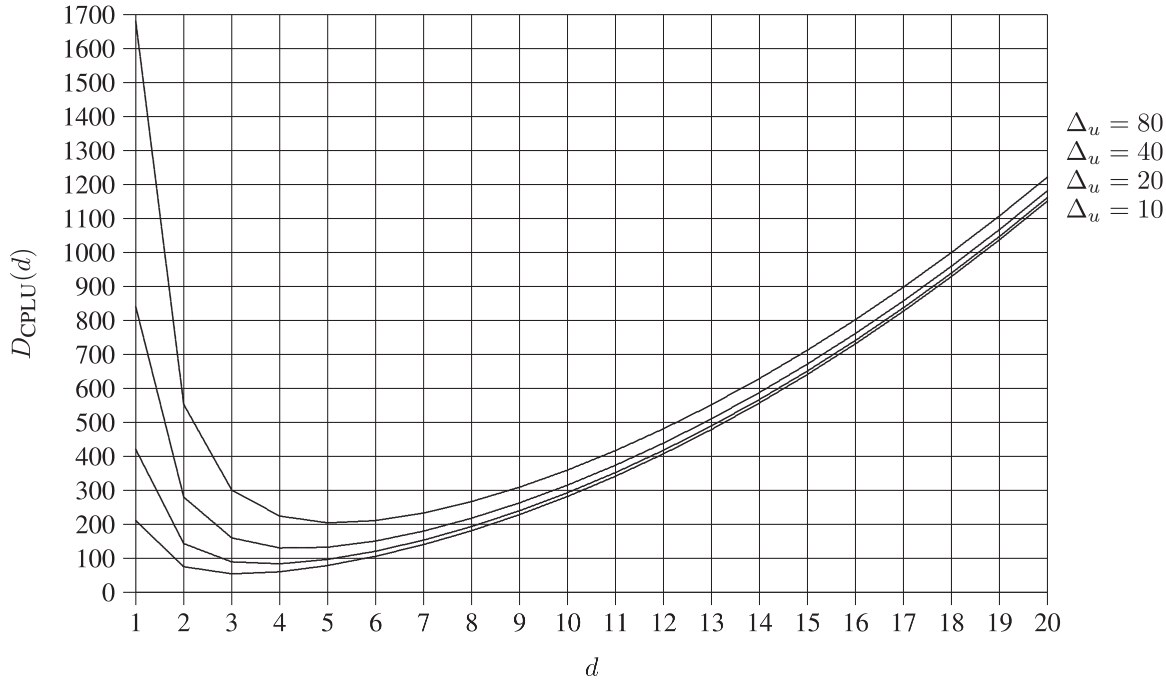

Figure 2: Location management cost in DBLMS-CPLU ($q = 3$, varying Δ_u).

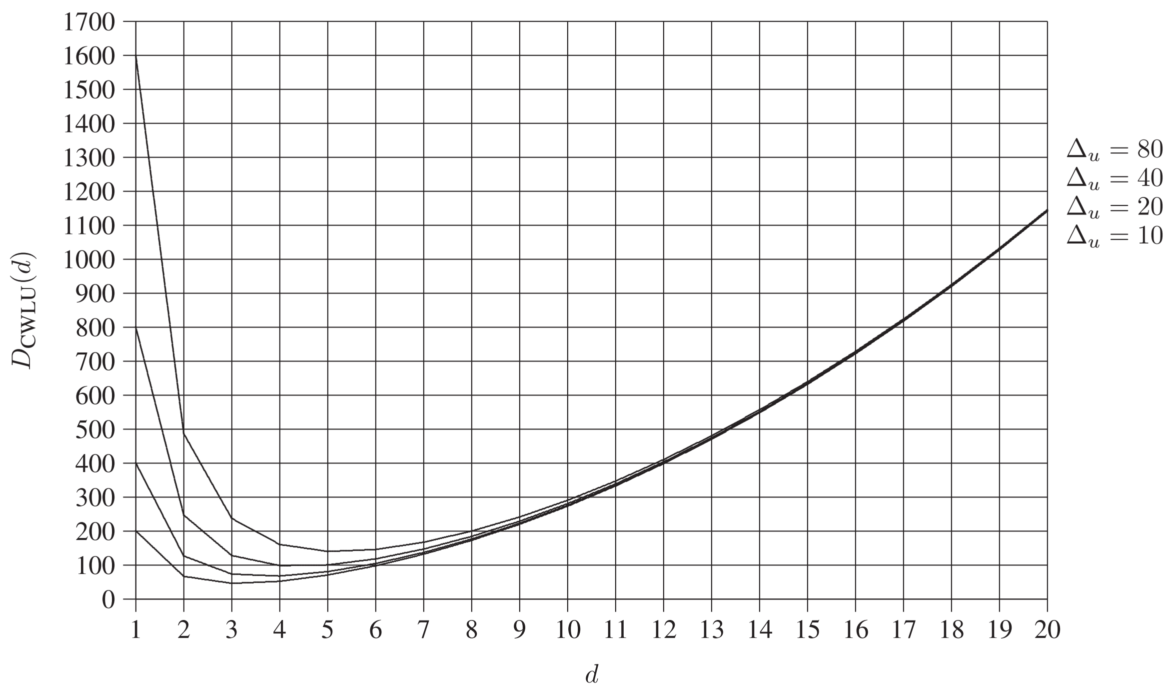

Figure 3: Location management cost in DBLMS-CWLU ($q = 3$, varying Δ_u).

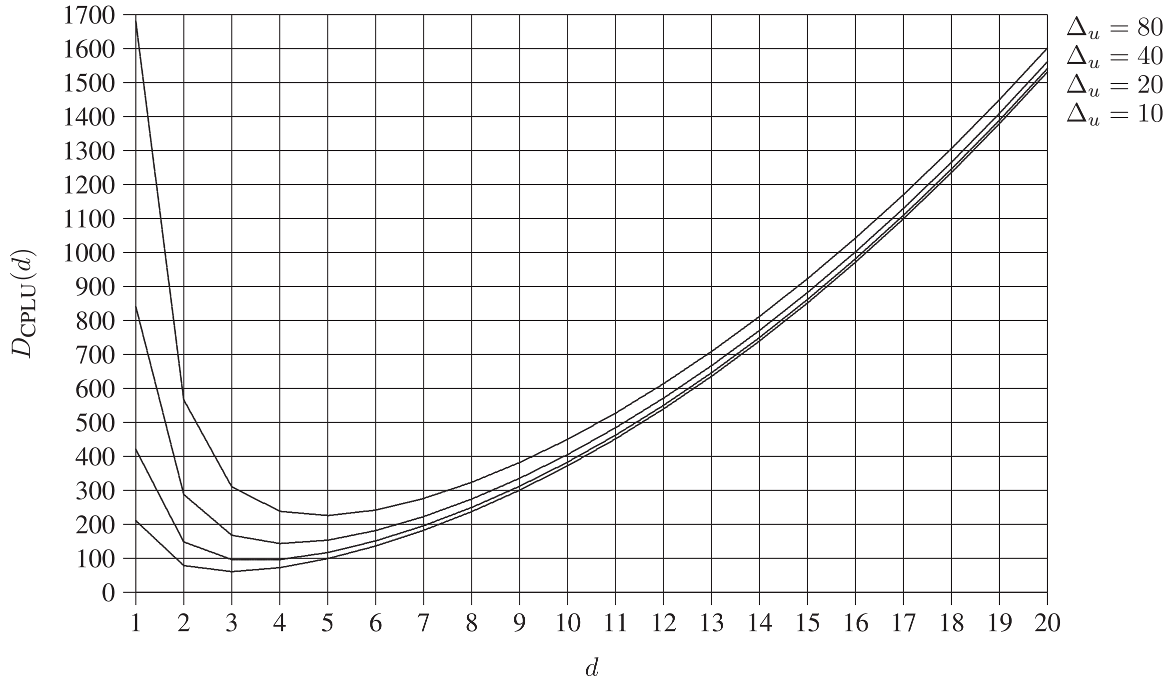

Figure 4: Location management cost in DBLMS-CPLU ($q = 4$, varying Δ_u).

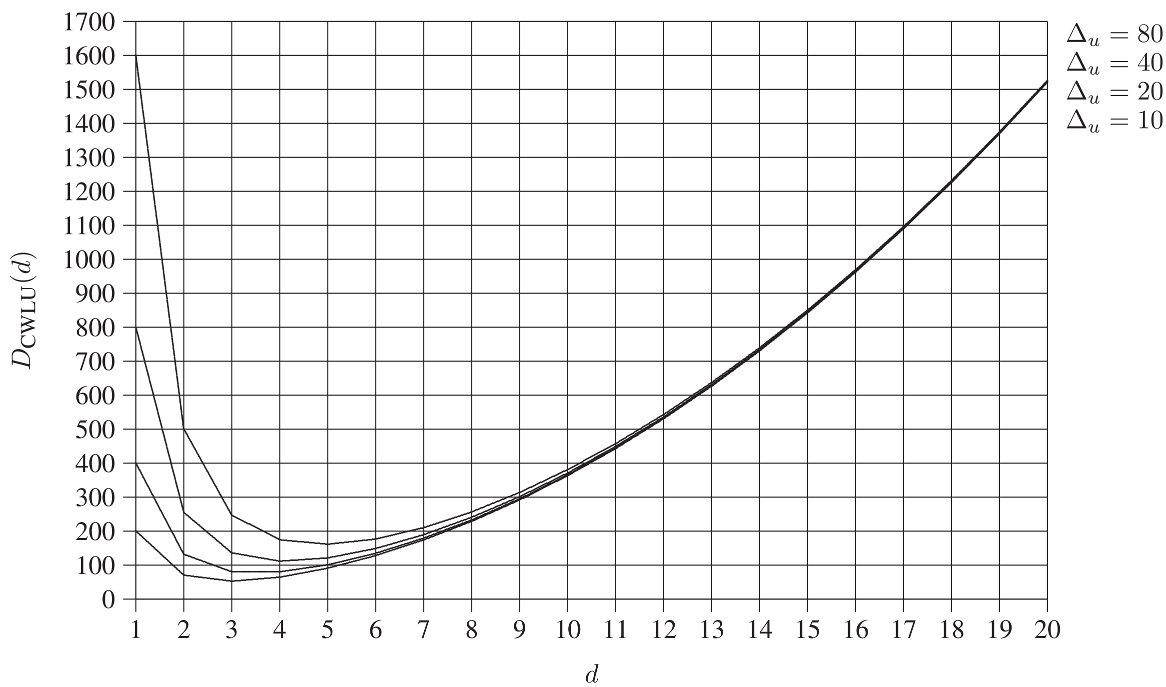

Figure 5: Location management cost in DBLMS-CWLU ($q = 4$, varying Δ_u).

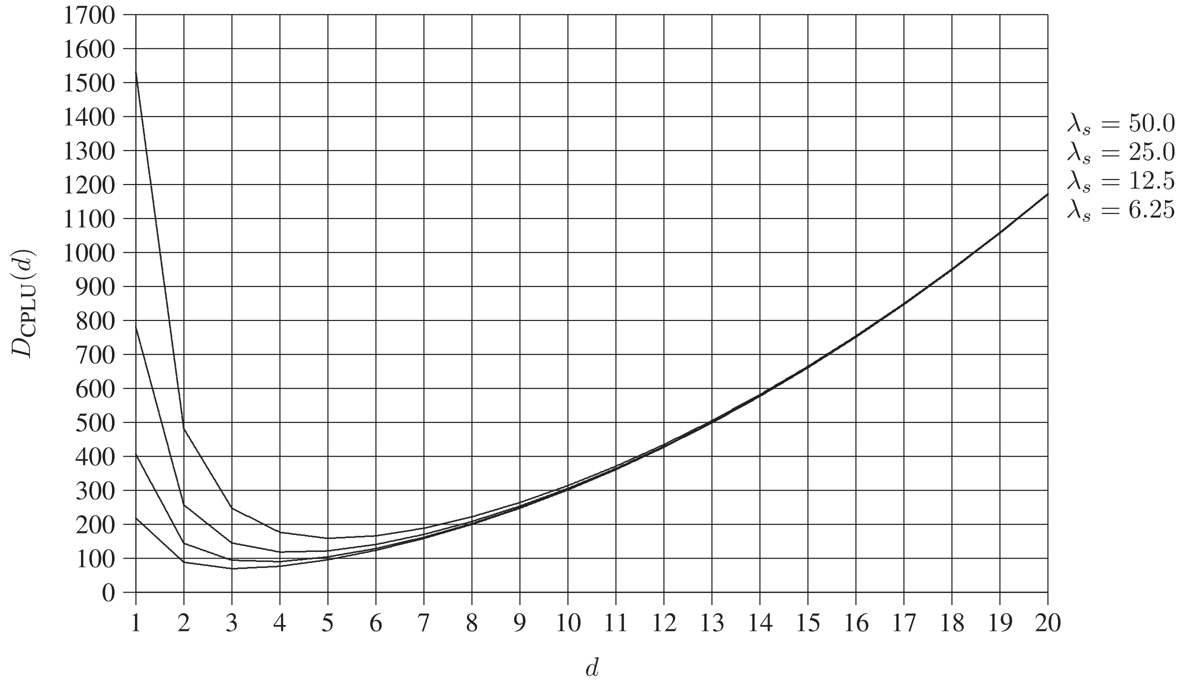

Figure 6: Location management cost in DBLMS-CPLU ($q = 3$, varying λ_s).

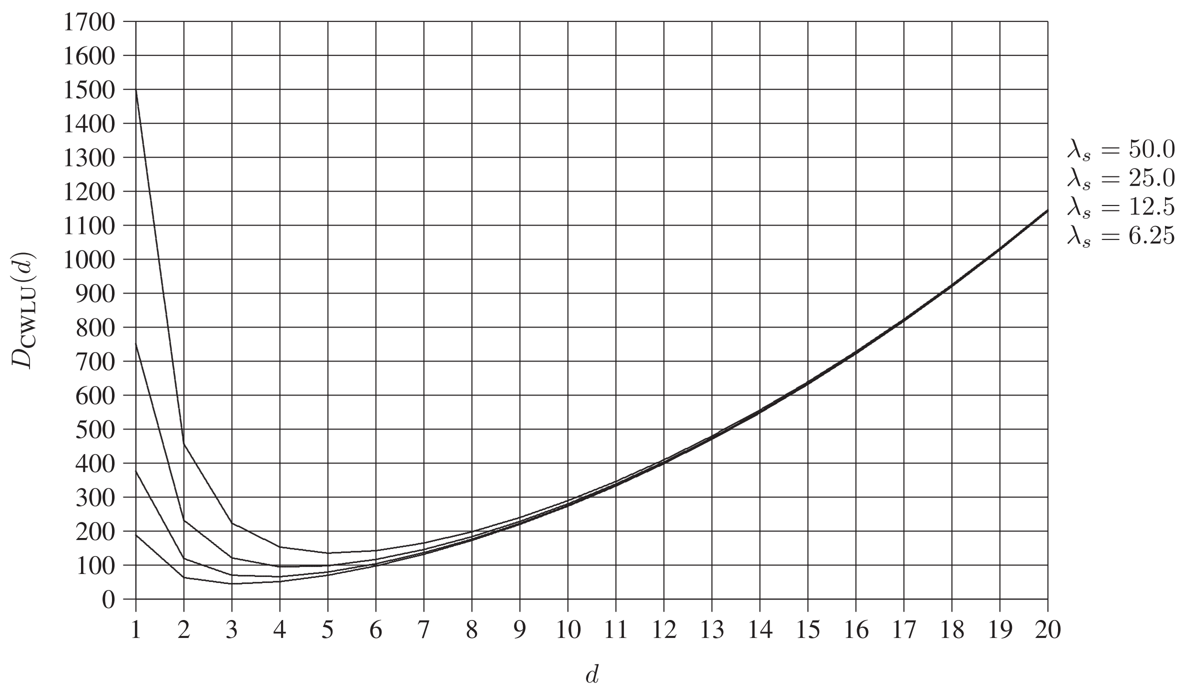

Figure 7: Location management cost in DBLMS-CWLU ($q = 3$, varying λ_s).

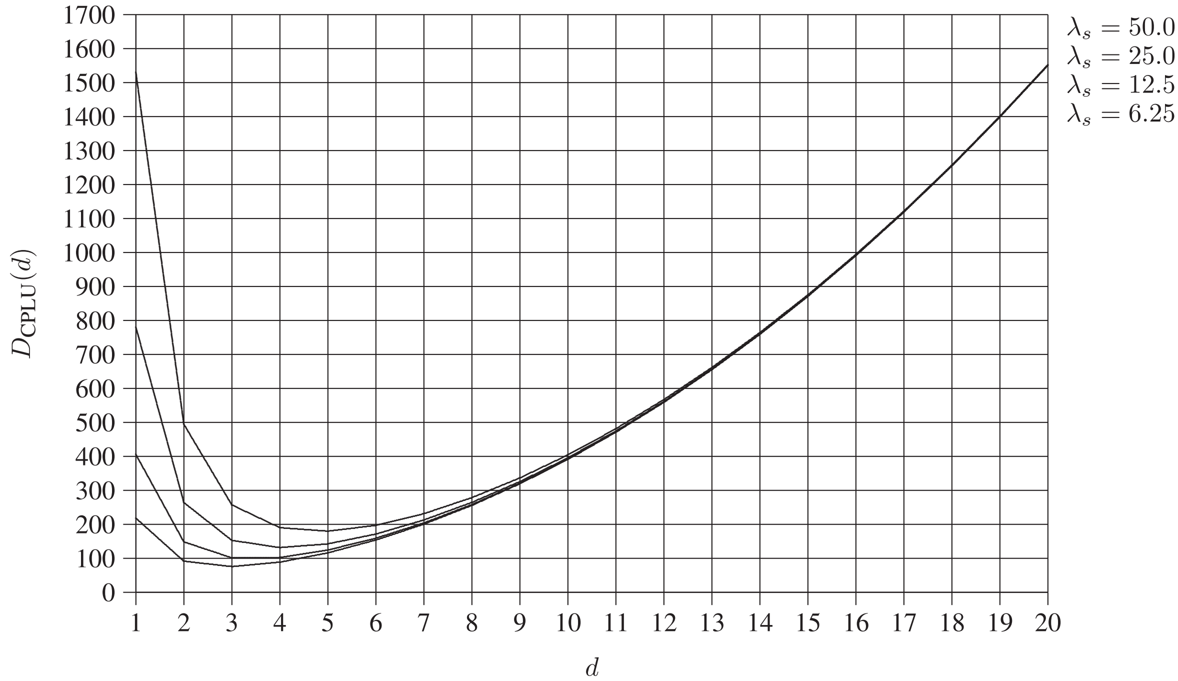

Figure 8: Location management cost in DBLMS-CPLU ($q = 4$, varying λ_s).

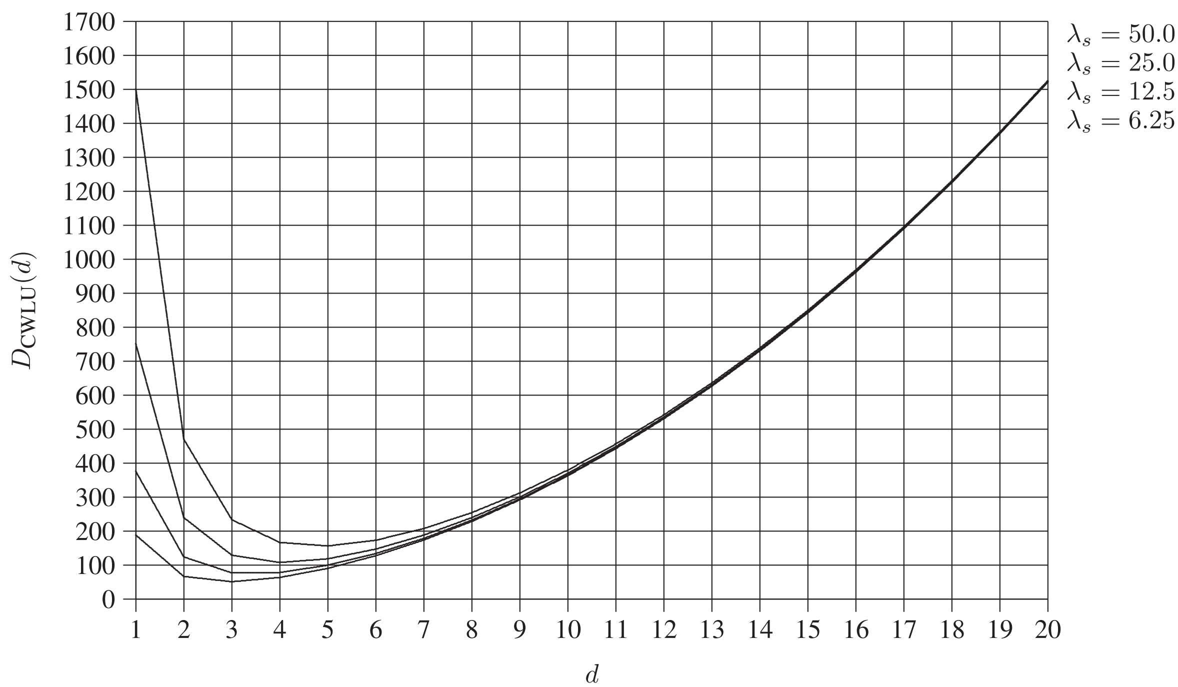

Figure 9: Location management cost in DBLMS-CWLU ($q = 4$, varying λ_s).

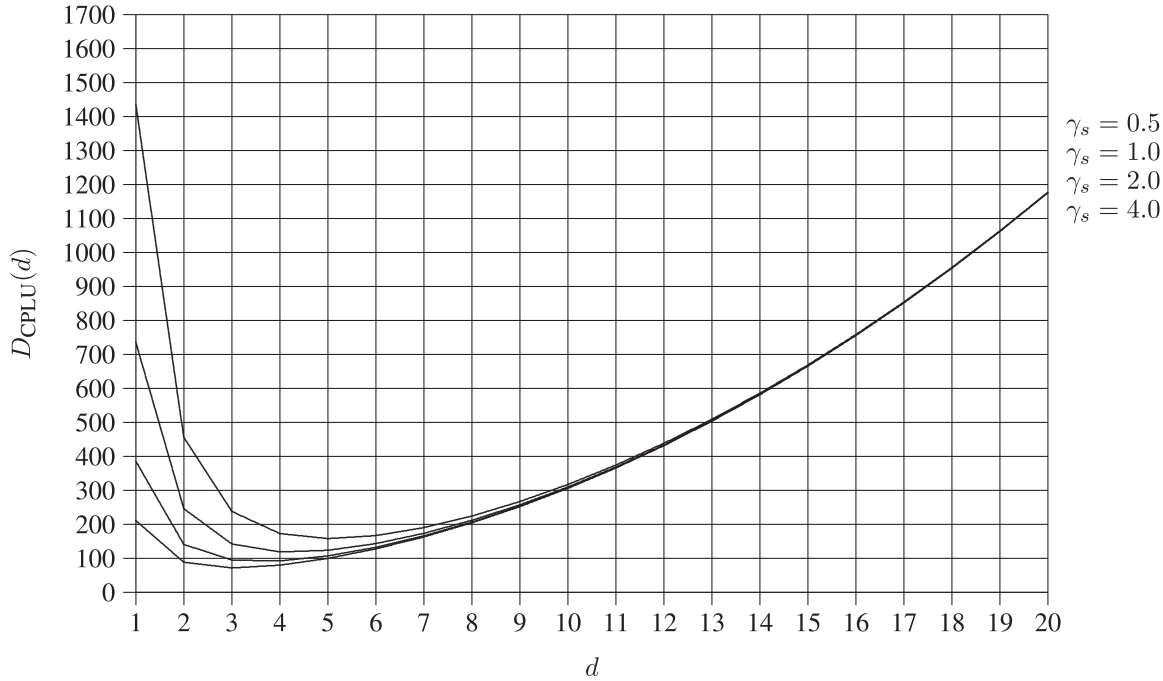

Figure 10: Location management cost in DBLMS-CPLU ($q = 3$, varying γ_s).

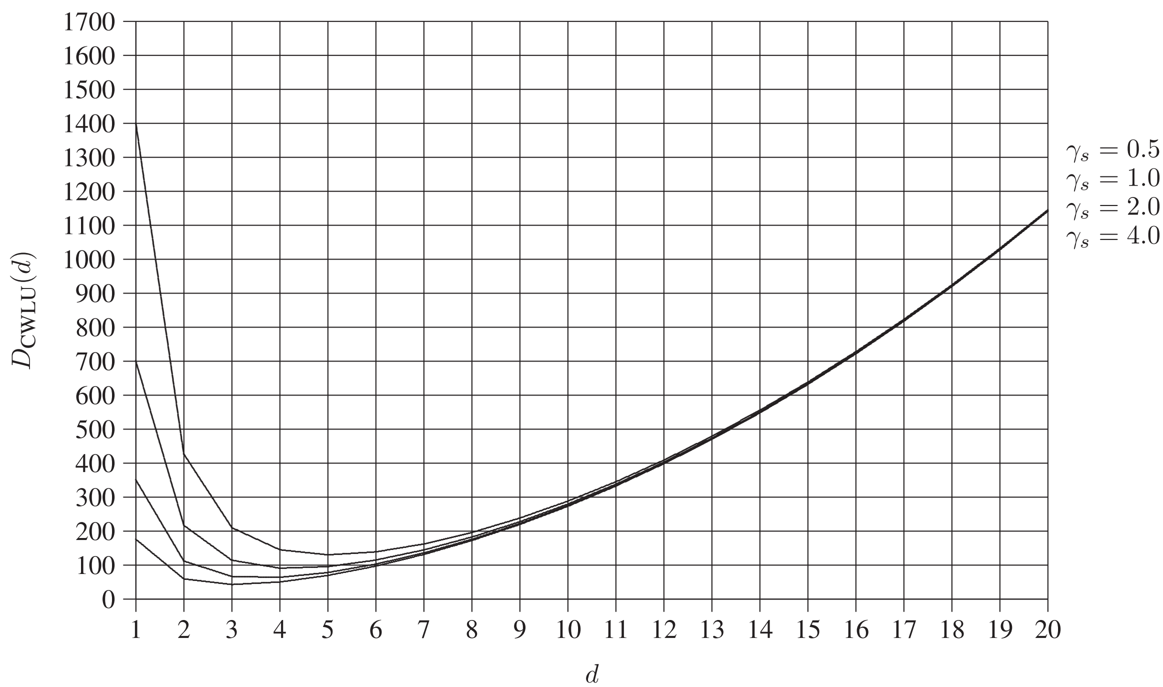

Figure 11: Location management cost in DBLMS-CWLU ($q = 3$, varying γ_s).

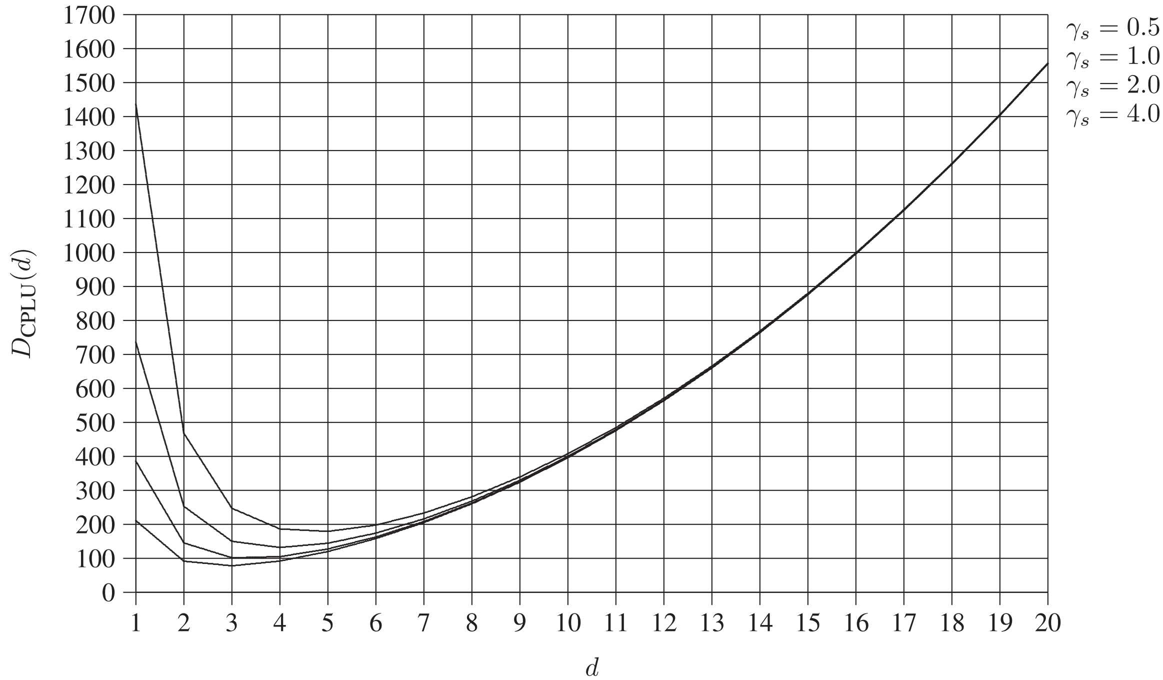

Figure 12: Location management cost in DBLMS-CPLU ($q = 4$, varying γ_s).

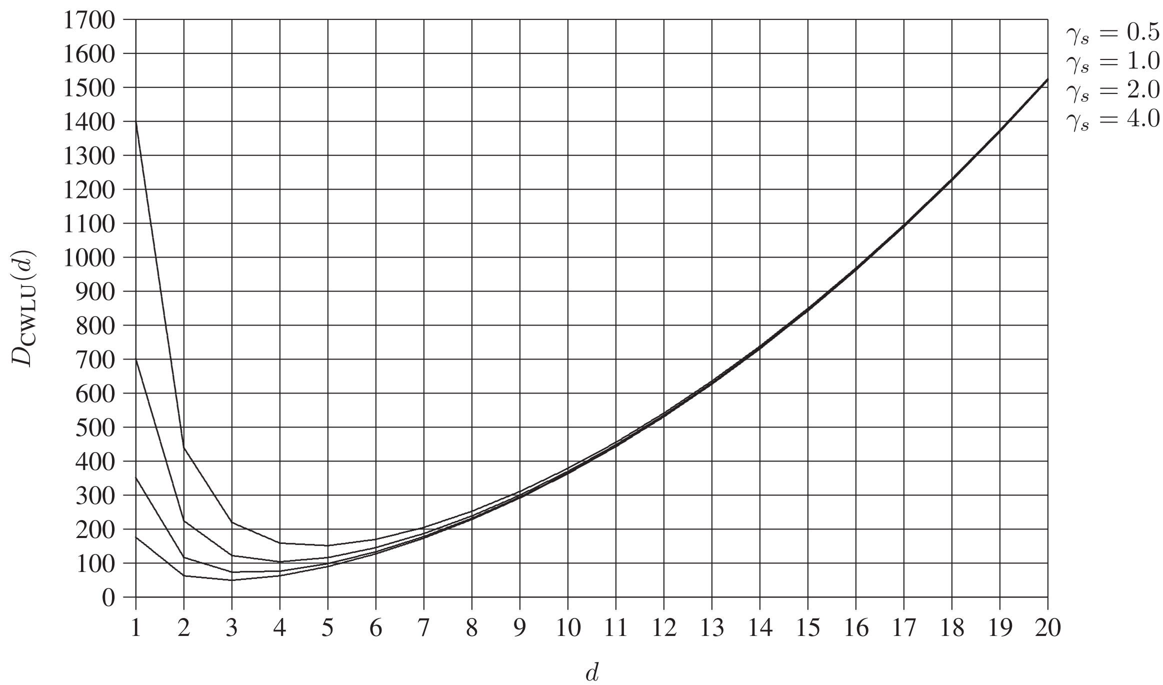

Figure 13: Location management cost in DBLMS-CWLU ($q = 4$, varying γ_s).

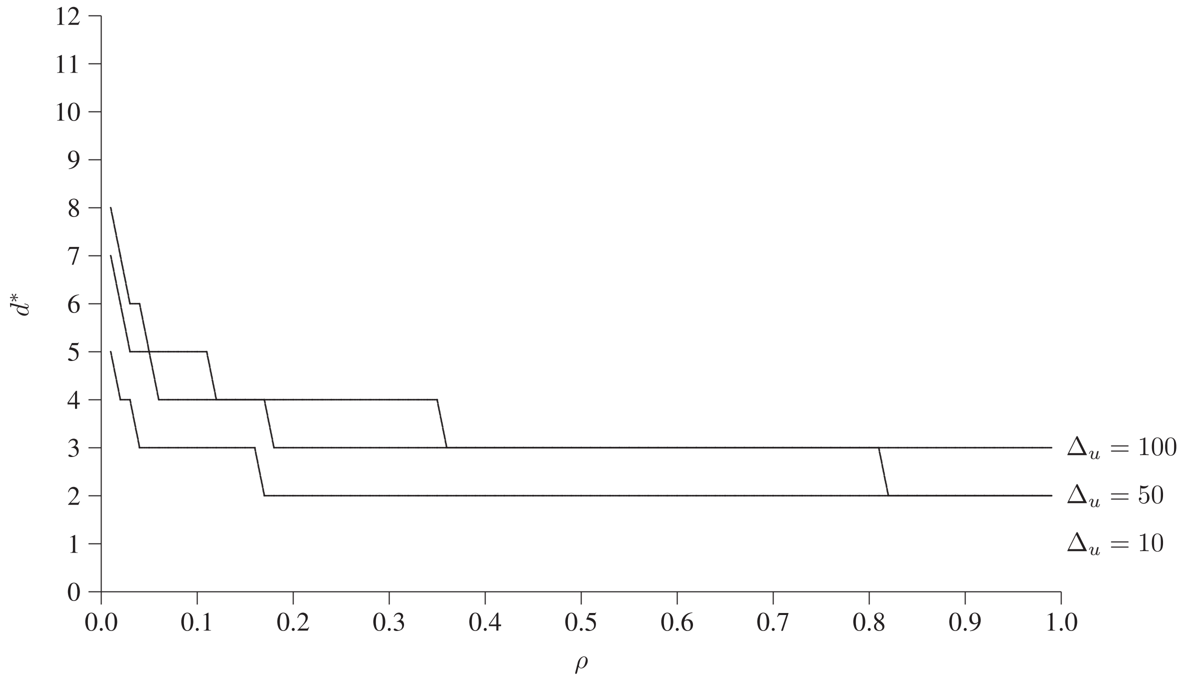

Figure 14: Optimal value d^* minimizing $D_{\text{CWLU}}(d)$ in DBLMS-CWLU ($q = 3$).

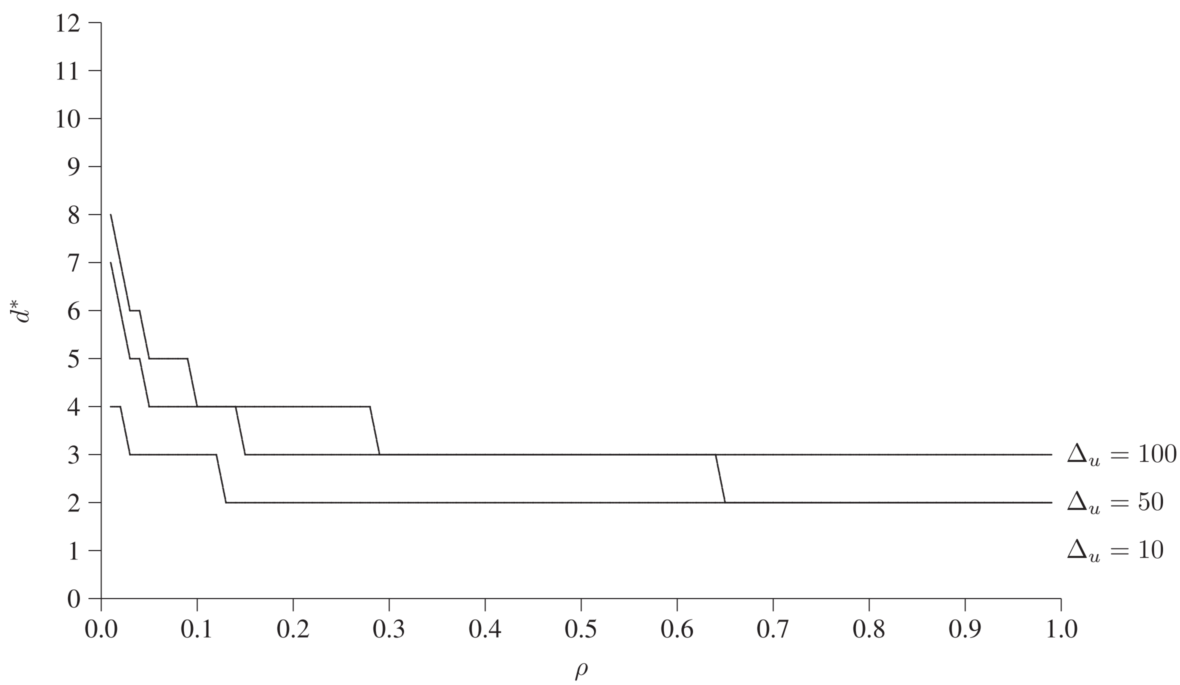

Figure 15: Optimal value d^* minimizing $D_{\text{CWLU}}(d)$ in DBLMS-CWLU ($q = 4$).