Minimizing Redundancy to Satisfy Reliability Requirement for a Parallel Application on **Heterogeneous Service-Oriented Systems**

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Abstract—Reliability is widely identified as an increasingly relevant issue in heterogeneous service-oriented systems because 6 processor failure affects the quality of service to users. Replication-based fault-tolerance is a common approach to satisfy application's reliability requirement. This study solves the problem of minimizing redundancy to satisfy reliability requirement for a directed acyclic 8 graph (DAG)-based parallel application on heterogeneous service-oriented systems. We first propose the enough replication for redundancy minimization (ERRM) algorithm to satisfy application's reliability requirement, and then propose heuristic replication for redundancy minimization (HRRM) to satisfy application's reliability requirement with low time complexity. Experimental results on real and randomly generated parallel applications at different scales, parallelism, and heterogeneity verify that ERRM can generate least redundancy followed by HRRM, and the state-of-the-art MaxRe and RR algorithm. In addition, HRRM implements approximate minimum redundancy with a short computation time.

Index Terms—Fault-tolerance, heterogeneous service-oriented systems, quality of service, reliability requirement, replication 15

INTRODUCTION 1 16

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1.1 Background 17

LOUD-BASED service is a new service-based resource 18 sharing paradigm [1], [2]. In X as a service (XaaS) 19 20 clouds, resources as services (e.g., infrastructure, platform and software as a service) are sold to applications such as 21 22 scientific and big data analysis workflows [1], [3], [4], [5], [6]. Meanwhile, cloud computing systems become more het-23 erogeneous as old, slow machines are continuously 24 replaced with new, fast ones. Heterogeneous computing 25 systems consist of diverse sets of processors interconnected 26 with a high-speed network, and are applied in business-crit-27 ical, mission-critical, and safety-critical scenarios to achieve 28 operational goals [7]. Applications in the system are increas-29 ingly parallel and the tasks in an application have obvious 30 data dependencies and precedence constraints [1], [8], [9], 31 [10], [11]. Examples of parallel applications are Gaussian 32

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elimination and fast Fourier transform [9]. A parallel appli- 33 cation with precedence constrained tasks at a high level is 34 described by a directed acyclic graph (DAG) [1], [8], [9], 35 [10], [11], where nodes represent tasks, and edges represent 36 communication messages between tasks. Such application 37 is usually called DAG-based parallel application [12].

The current cloud-based service systems are actually 39 heterogeneous service-oriented systems where resource 40 management is a considerable challenge owing to the vari- 41 ous configurations or capacities of the hardware or software 42 [13]. The processing capacity of processors in heterogeneous 43 service-oriented systems has been developed to provide 44 powerful cloud-based services, whereas failures of process- 45 ors will affect the reliability of systems and quality of 46 service (QoS) for users [2]. Reliability is defined as the pro- 47 bability of a schedule successfully completing its execution, 48 and it has been widely identified as an increasingly relevant 49 issue in service-oriented computing systems [2], [14], [15], 50 [16], [17], [18]. 51

Fault-tolerance by primary-backup replication, which 52 means that a primary task will have zero, one, or multiple 53 backup tasks, is an important reliability enhancement mech- 54 anism. In the primary-backup replication scheme, the pri- 55 mary and all the backups are called replicas. Although 56 replication-based fault-tolerance is an important reliability 57 enhancement mechanism [14], [15], [19], [20], [21], any 58 application cannot be 100 percent reliable in practice. There- 59 fore, if an application can satisfy its specified reliability 60 requirement (also named reliability goal or reliability assur- 61 ance in some studies), then it is considered to be reliable. 62 For example, assume that the application's reliability 63 requirement is 0.9, only if the application's reliability 64 exceeds 0.9, will the application be reliable. Specifically, 65

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Fig. 1. Pareto optima and pareto curve for a bicriteria minimization problem [19].

reliability requirement has been defined in some reliability 66 related standards (e.g., IEC 61508 [22] and ISO 9000 [23]), 67 68 and it is one of the most important QoS in cloud and services computing systems [14], [15]. Therefore, reliability 69 70 requirement must be satisfied from standards and QoS per-71 spectives. However, as pointed out in [2], many cloud-based services failed to fulfill their reliability requirements due to 72 processor failures in practice. 73

74 1.2 Motivation

Users and resource providers are the two types of roles with 75 different requirements for service-oriented systems [24]. For 76 77 users, satisfying application's reliability requirement is one of the most important QoS requirements, for which replication-78 based fault-tolerance is a common approach. For resource 79 providers, minimizing resource redundancy caused by repli-80 cation is one of the most important concerns [14], [15]. How-81 ever, adding more replicas (including primary and backups) 82 could increase both reliability and redundancy for a parallel 83 application. Therefore, both criteria (low redundancy and 84 85 high reliability, short schedule length and high reliability) are conflicting, and optimizing them is a bi-criteria optima prob-86 lem [19]. In Fig. 1, each point x^1-x^7 represents a solution of a 87 bicriteria minimization problem [19]. The points x^1, x^2, x^3, x^4 , 88 and x^5 are Pareto optima [25]; the points x^1 and x^5 are weak 89 optima, whereas the points x^2 , x^3 , and x^4 are strong optima. 90 The set of all Pareto optima is called the Pareto curve [19]. 91 Many studies have dealt specifically with the bi-criteria 92 (i.e., minimizing schedule length and maximizing reliability) 93 problem to obtain such an approximate Pareto curve for a 94 DAG-based parallel application [19], [20], [21], [26], [27], [28]. 95 In [26], [27], [28], the approaches increase reliability by effi-96 cient task scheduling without using replication. In [19], [20], 97 [21], the approaches presented replicate tasks to increase 98 99 reliability.

However, for heterogeneous service-oriented systems,
 resolving the above bi-criteria is not strictly required for the
 following reasons:

- (1) Clouds allow flexible and dynamic resource alloca tions based on a pay-as-you-go scheme [29], where
 users pay only for the reliability requirement they
 apply and will not pay additional fees for the reli ability that surpasses their reliability requirement.
- (2) The application cannot be 100 percent reliable as
 mentioned earlier. The most common component of
 service-level agreement (SLA) between resource pro viders and the users is that the services (reliability

requirement in this study) should be provided to the 112 users as agreed upon in the contract [30]. Therefore, 113 satisfying application's reliability requirement is the 114 service level objective. 115

In summary, considering the actual demand, the theoretical bi-objective optimization problem could be degradated 117 to a constrained single-objective optimization problem in 118 most cases. In other words, reliability is not the higher the 119 better, but as long as you can satisfy the reliability requirement from a practical perspective. Therefore, the reliability 121 problem of service-oriented systems is mainly to satisfy 122 application's reliability requirement while still reducing the resource as far as possible. 124

The approaches related to our work are [14] and [15], in 125 which the authors presented the MaxRe and RR algorithms 126 to minimize redundancy of a parallel application to satisfy 127 application's reliability requirement on heterogeneous distributed systems. The main procedures of the MaxRe and 129 RR are follows: 130

- The reliability requirement of the application is trans- 131 ferred to the sub-reliability requirements of the tasks. 132 In this way, as long as the sub-reliability requirement 133 of each task can be satisfied, the application's reliabil- 134 ity requirement can be satisfied, such that a heuristic 135 replication can be used in the following. 136
- MaxRe and RR iteratively assign the replicas of each 137 task to the processors with maximum reliability val- 138 ues until the sub-reliability requirement of the task is 139 satisfied.

However, the essential limitation of MaxRe and RR is 141 that the sub-reliability requirements of tasks are too high, 142 thereby causing them need unnecessary redundancy to 143 satisfy the sub-reliability requirements. 144

1.3 Our Contributions

Similar to the state-of-the-art MaxRe and RR, this study 146 aims to implement redundancy minimization to satisfy 147 application's reliability requirement for a parallel application on heterogeneous service-oriented distributed systems. 149 Our contributions comparing to the MaxRe and RR are summarized as follows: 151

- We present the just enough replication for redun- 152 dancy minimization (ERRM) algorithm to satisfy 153 application's reliability requirement by two-stage 154 replications. The first stage involves obtaining the 155 lower bound on redundancy (i.e., the minimum 156 required number of replicas) for each task; the second 157 stage is iteratively selecting the available replicas and 158 corresponding processors with the maximum reliability values until application's reliability requirement is satisfied.
- (2) To overcome the high time complexity of ERRM 162 algorithm, we propose the heuristic replication 163 for redundancy minimization (HRRM) algorithm to 164 deal with large-scale parallel applications. Similar to 165 the MaxRe and RR algorithms, HRRM first transfers 166 the reliability requirement of the application to the 167 sub-reliability requirements of the tasks. Then, 168 HRRM iteratively assign the replicas of each task to 169 the processors with maximum reliability values until 170

the sub-reliability requirement of the task is satisfied.
The main improvement of HRRM over MaxRe and
RR is that it can obtain lower sub-reliability requirements for most tasks, such that HRRM generates less
redundancy than MaxRe and RR.

(3) Experimental results on real and randomly generated parallel applications at different scales, parallelism degrees, and heterogeneity degrees validate that ERRM can generate the least redundancy followed by HRRM, the state-of-the-art MaxRe and RR algorithm.
In addition, HRRM implements approximate minimum redundancy with a short computation time.

The rest of this paper is organized as follows. Section 2 reviews related research. Section 3 presents the reliability modeling and problem statement. Section 4 explains the state-of-the-art MaxRe and RR algorithms. Sections 5 and 6 proposed the ERRM and HRRM algorithms, respectively. Section 7 verifies the ERRM and HRRM algorithms. Section 8 concludes this study.

190 2 RELATED WORK

The widely-accepted reliability model was presented by 191 192 Shatz and Wang [31], where each hardware component 193 (processor) is characterized by a constant failure rate per time unit λ and the reliability during the interval of time *t* 194 is $e^{-\lambda t}$. That is, the failure occurrence follows a constant 195 parameter Poisson law [31]. This law is also known as the 196 exponential distribution model [19]. This section mainly 197 reviews the related research on reliability and fault-198 tolerance of DAG-based parallel applications. 199

Two main types of primary-backup replication app-200 roaches exist in current: active replication [14], [15], [20], [21] 201 and passive replication [32], [33], [34], [35]. For the active rep-202 lication scheme, each task is simultaneously replicated on 203 several processors, and the task will succeed if at least one 204 of them does not fail. For the passive scheme, whenever 205 a processor fails, the task will be rescheduled to proceed on a 206 backup processor. When a processor crashes, it is subse-207 quently restarted to continue from the checkpoint just as if 208 no failure had occurred; such scheme is called checkpoint 209 and restart scheme, and can be considered as an improved 210 version of the passive scheme [14], [15]. Meanwhile, accord-211 ing to the number of the backups, three types of primary-212 213 backup replication approaches exist; single backup for each primary, fixed ε backups for each primary, and quantitative 214 backups for each primary. 215

The single backup for each primary approach is a simple 216 method. Main representative methods include efficient 217 fault-tolerant reliability cost driven (eFRCD) [33], efficient 218 fault-tolerant reliability driven (eFRD) [34], and minimum 219 completion time with less replication cost (MCT-LRC) [35] 220 et al. Regarding their limitations, first, these approaches 221 assume that no more than one failure happens at one 222 moment; they are too ideal to tolerate potential multiple fail-223 ures. Second, although passive replication also supports 224 multiple backups for each primary [32], it is unsuitable 225 for service-oriented applications; the reason is that once a 226 processor failure is detected, the scheduler should resched-227 ule the task located on the failed processor, and reassign it 228 to a new processor, such that the QoS for the application is 229 uncertain. 230

The fixed ε backups for each primary approach is an 231 active replication approach, and is suitable for service- 232 oriented systems because it can directly shield the failed 233 tasks in performing, and the failure recovery time is almost 234 close to zero [19], [20], [21]. In [19], the authors presented 235 bicriteria scheduling heuristic (BSH) to minimize the sched- 236 ule length of the application while taking the failure rate as 237 a constraint; BSH can generate a Pareto curve of non- 238 dominated solutions, among which the user can choose the 239 compromise that fits his requirements best. However, the 240 time complexity of BSH is as high as $O(n \times 2^u)$, where *n* is 241 the number of replicas and u is the number of processors. In 242 [20], Benoit et al. presented the fault-tolerant scheduling 243 algorithm (FTSA) for a parallel application on heteroge- 244 neous systems to minimize the schedule length given a 245 fixed number of failures supported in the system based on 246 the active replication scheme. In [21], Benoit et al. further 247 designed a new scheduling algorithm to minimize schedule 248 length under both throughput and reliability constraints for 249 a parallel application on heterogeneous systems based on 250 the active replication scheme. The main problem in [20], 251 [21] is that they need ε backups for each task with high 252 redundancy to satisfy application's reliability requirement. 253 Although application's reliability requirement can be satis- 254 fied by using active replication scheme, high redundancy 255 causes high resource cost to resource providers. 256

Considering that fixed ε backups for each primary 257 approach has high redundancy, recent studies begun to 258 explore quantitative backups for each task approach to satisfy 259 application's reliability requirement [14], [15]. Quantitative 260 backups means different primaries have different numbers 261 of backups, and the quantitative approach has lower resource 262 cost than the fixed ε backups for each task based on active 263 replication [14]. In [14] and [15], the authors proposed fault- 264 tolerant scheduling algorithms MaxRe and RR; both MaxRe 265 and RR incorporate reliability analysis into the active replica- 266 tion and exploit a dynamic number of backups for different 267 tasks by considering each task's sub-reliability requirement. 268 As discussed in Section 1.2, both MaxRe and RR have limita- 269 tions in calculating the sub-reliability requirements of tasks. 270 In [15], the authors also presented the DRR algorithm that 271 extends RR by further considering the deadline requirement 272 of a parallel application; however, we are only interested in 273 satisfying reliability requirement in this study. 274

3 RELIABILITY MODELING AND PROBLEM STATEMENT

Table 1 gives the important notations and their definitions277as used in this study.278

3.1 Application Model

Let $U = \{u_1, u_2, \ldots, u_{|U|}\}$ represent a set of heterogeneous 280 processors, where |U| is the size of set U. In this study, for 281 any set X, |X| is used to denote size. A development life 282 cycle of a service-oriented system usually involves the anal-283 ysis, design, implementation, and testing phases. In this 284 study, we focus on the design phase. Therefore, we assume 285 that the processor and application parameter values are 286 known in the design phase, because these values have been 287 already calculated in the analysis phase. 288

TABLE 1 Important Notations in this Study

Notation	Definition
$c_{i,j}$	Communication time between the tasks n_i and n_j
$w_{i,k}$	Execution time of the task n_i on the processor u_k
$\overline{w_i}$	Average execution time of the task n_i
$rank_{\mathrm{u}}(n_i)$	Upward rank value of the task n_i
X	Size of the set <i>X</i>
λ_k	Constant failure rate per time unit of the processor u_k
num_i	Number of replicas of the task n_i
NR(G)	Total number of the replicas of the application G
$lb(n_i)$	Lower bound on number of replicas of the task n_i
n_i^x	x th replica of the task n_i
$u_{pr(n_i^x)}$	Assigned processor of the replica n_i^x
$R(n_i, u_k)$	Reliability of the task n_i on the processor u_k
$R(n_i)$	Reliability of the task n_i
R(G)	Reliability of the application G
$R_{\rm seq}(G)$	Reliability requirement of the application G
$R_{\rm seq}(n_i)$	Sub-reliability requirement of the task n_i
$R_{\rm up_seq}(n_i)$	Upper bound on reliability requirement of the task n_{i}

289 As mentioned earlier, a parallel application running on processors is represented by a DAG G=(N, W, M, C) with 290 known values. 291

N represents a set of nodes in G, and each node (1)292 293 $n_i \in N$ is a task with different execution time values on different processors. In addition, task executions 294 of a given application are assumed to be non-295 preemptive which is possible in many systems [8], 296 [14]. $pred(n_i)$ is the set of immediate predecessor 297 tasks of n_i , while $succ(n_i)$ is the set of immediate suc-298 cessor tasks of n_i . Tasks without predecessor tasks 299 are denoted by n_{entry} ; and tasks with no successor 300 tasks are denoted by n_{exit} . If an application has multi-301 ple entry or multiple exit tasks, then a dummy entry 302 or exit task with zero-weight dependencies is added 303 304 to the graph. W is an $|N| \times |U|$ matrix in which $w_{i,k}$ denotes the execution time of n_i running on u_k . 305

(2)*M* is a set of communication edges, and each edge 306 $m_{i,j} \in M$ represents a communication from n_i to n_j . 307 Accordingly, $c_{i,j} \in C$ represents the communication 308



TABLE 2 Execution Time Values of Tasks on Different Processors of the Motivating Parallel Application [9], [10], [11]

Task	u_1	u_2	u_3
$\overline{n_1}$	14	16	9
n_2	13	19	18
$\overline{n_3}$	11	13	19
n_4	13	8	17
n_5	12	13	10
n_6	13	16	9
n_7	7	15	11
n_8	5	11	14
n_9	18	12	20
n_{10}	21	7	16

time of $m_{i,j}$ if n_i and n_j are assigned to different 309 processors because two tasks with immediate pre- 310 cedence constraints need to exchange messages. 311 When both tasks n_i to n_j are allocated to the same 312 processor, $c_{i,i}$ becomes zero because we assume 313 that the intra-processor communication cost is neg- 314 ligible [14], [15]. 315

Fig. 2 shows a motivating parallel application with tasks 316 and messages [9], [10], [11], [12]. The example shows 317 10 tasks executed on 3 processors $\{u_1, u_2, u_3\}$. The weight 318 18 of the edge between n_1 and n_2 represents communication 319 time, denoted by $c_{1,2}$ if n_1 and n_2 are not assigned to the 320 same processor. 321

Table 2 is the execution time matrix $|N| \times |U|$ of tasks on 322 different processors of the motivating parallel application. 323 For example, the weight 14 of n_1 and u_1 in Table 2 repre- 324 sents execution time of n_1 on u_1 , denoted by $w_{1,1}$ =14. We 325 can see that the same task has different execution time val- 326 ues on different processors due to the heterogeneity of the 327 processors.

The motivating example will be used to explain the MaxRe, 329 RR, and the proposed LBR, ERRM, and HRRM algorithms in 330 the paper. 331

3.2 Reliability Model

There are two major types of failures: transient failure 333 (also called random hardware failure) and permanent 334 failure. Once a permanent failure occurs, the processor 335 cannot be restored unless by replacement. The transient 336 failure appears for a short time and disappear without 337 damage to processors. Therefore, this paper mainly takes 338 the transient failures into account for our research. In 339 general, the occurrence of transient failure for a task in a 340 DAG-based application follows the Poisson distribution 341 [14], [15], [19], [31], [36]. The reliability of an event in unit 342 time t is denoted by 343

$$R(t) = e^{-\lambda t},$$

where λ is the constant failure rate per time unit for a proces-346 sor. We use λ_k to represent the constant failure rate per time 347 unit of the processor u_k . The reliability of n_i executed on u_k 348 in its execution time is denoted by 349

Fig. 2. Motivating example of a DAG-based parallel application with 10 tasks [9], [10], [11], [12].

$$R(n_i, u_k) = e^{-\lambda_k w_{i,k}},\tag{1}$$

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and the failure probability for n_i without using the active replication is

$$1 - R(n_i, u_k) = 1 - e^{-\lambda_k w_{i,k}}.$$
(2)

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However, each task has a number of replicas with the active replication. We define num_i $(num_i \leq |U|)$ as the number of replicas of n_i . Hence, the replica set of n_i is $\{n_i^1, n_i^2, \ldots, n_i^{num_i}\}$ where n_i^1 is the primary and others are backups. As long as one replica of n_i is successfully completed, then we can recognize that there is no occurrence of failure for n_i , and the reliability of n_i is updated to

$$R(n_i) = 1 - \prod_{x=1}^{num_i} \left(1 - R\left(n_i^x, u_{pr(n_i^x)}\right) \right),$$
(3)

where $u_{pr(n_i^x)}$ represents the assigned processor of n_i^x . The difference between $R(n_i, u_k)$ and $R(n_i)$ is below: $R(n_i, u_k)$ is the value before task replication, whereas $R(n_i)$ is the value after task replication.

The reliability of the parallel application with precedence-constrained tasks should be [14]

$$R(G) = \prod_{n_i \in N} R(n_i).$$
(4)

In [15], both communication and computation failures 374 are considered; however, some communication networks 375 themselves provide fault-tolerance. For instance, routing 376 information protocol (RIP) and open shortest path first 377 (OSPF) are designed to reroute packets to ensure that they 378 reach their destination [37]. Therefore, similar to some 379 380 studies [14], [35], [38], this study only considers processor 381 failure and excludes communication failure (i.e., the communication is assumed to be reliable in this study). In addi-382 tion, we mainly focus on the redundancy minimization 383 of tasks, which is not directly related to communication. 384

385 3.3 Problem Statement

As discussed in Section 1, any application cannot be 386 100 percent reliable, but if the system can satisfy 387 388 application's reliability requirement, then the application is considered reliable. The problem addressed in this study 389 390 can be formally described as follows. Assume that we are given a parallel application G and a heterogeneous proces-391 392 sor set U. The problem is to assign replicas and corresponding processors for each task, while minimizing the number 393 of replicas and ensuring that the obtained reliability of the 394 application R(G) satisfies the application's reliability 395 requirement $R_{seq}(G)$. The formal description is to find the 396 replicas and processor assignments of all tasks to minimize 397

400 subject to

$$R(G) = \prod_{n_i \in N} R(n_i) \ge R_{\mathrm{req}}(G),$$

 $NR(G) = \sum_{n_i \in N} num_i,$

(5)

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4 for
$$\forall i : 1 \leq i \leq |N|$$
.

TABLE 3 Upward Rank Values for Tasks of the Motivating Parallel Application

Task	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9	n_{10}
$rank_{\mathrm{u}}(n_i)$	108	77	80	80	69	63.3	42.7	35.7	44.3	14.7

4 STATE-OF-THE-ART APPROACHES

4.1 Task Prioritizing

A fault-tolerant scheduling algorithm generally consists of 407 three steps: 1) task prioritizing, 2) processor selection, and 408 3) task execution. Therefore, we should first compute the 409 task priority before processor selection. Similar to state-of-410 the-art studies [14], [15], this study uses the famous upward 411 rank value ($rank_u$) of a task (Eq. (6)) as the task priority stan-412 dard. In this case, the tasks are ordered by descending order 413 of $rank_u$, which are obtained by Eq. (6) [9], as follows: 414

$$rank_{\mathbf{u}}(n_i) = \overline{w_i} + \max_{n_j \in succ(n_i)} \{c_{i,j} + rank_{\mathbf{u}}(n_j)\},$$
(6)
416

in which $\overline{w_i}$ represents the average execution time of task n_i 417 and is calculated as follows: 418

$$\overline{w_i} = \left(\sum_{k=1}^{|U|} w_{i,k}\right) / |U|.$$

Table 3 shows the upward rank values of all the tasks in 422 Fig. 2. Note that only if all the predecessors of n_i have been 423 assigned, will n_i prepare to be assigned. Assume that two 424 tasks n_i and n_j satisfy $rank_u(n_i) > rank_u(n_j)$; if no prece-425 dence constraint exists between n_i and n_j , n_i does not neces-426 sarily take precedence for n_j to be assigned. Therefore, the 427 task assignment order in G is $\{n_1, n_3, n_4, n_2, n_5, n_6, n_9, n_7, 428$ $n_8, n_{10}\}$.

4.2 Existing MaxRe Algorithm

As the application reliability is the product of all the 431 task reliability values, such problem is usually solved by 432 transferring application's reliability requirement to the subreliability requirements of tasks [14], [15], [39]. In the MaxRe 434 algorithm [14], the sub-reliability requirement for each task 435 is calculated by 436

$$R_{\rm req}(n_i) = \sqrt[|N|]{R_{\rm req}(G)}.$$
(7)
438

If the sub-reliability requirement of each task can be satis- 439 fied by active replication below 440

$$R(n_i) \geqslant R_{\text{req}}(n_i),$$
442

then obviously the application's reliability requirement can 443 be satisfied. Therefore, the main idea of the MaxRe algo- 444 rithm is to iteratively select the replica n_i^x and processor 445 $u_{pr(n_i^x)}$ with the maximum $R(n_i^x, u_{pr(n_i^x)})$ until the actual reli- 446 ability value is larger than or equal to the sub-reliability 447 requirement of the task, namely, 448

$$R(n_i) \ge R_{\rm req}(n_i). \tag{450}$$

Moreover, this policy was also employed by the authors 452 in [39]. 453

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TABLE 4 Task Assignment of the Motivating Parallel Application Using the MaxRe Algorithm

n_i	$R_{\rm req}(n_i)$	$R(n_i, u_1)$	$R(n_i, u_2)$	$R(n_i, u_3)$	num_i	$R(n_i)$
n_1	0.99383156	0.98609754	0.97628571	0.98393051	2	0.99977659
n_3	0.99383156	0.98906028	0.98068890	0.96637821	2	0.99978874
n_4	0.99383156	0.98708414	0.98807171	0.96986344	2	0.99984594
n_2	0.99383156	0.98708414	0.97190229	0.96811926	2	0.99963709
n_5	0.99383156	0.98807171	0.98068890	0.98216103	2	0.99978721
n_6	0.99383156	0.98708414	0.97628571	0.98393051	2	0.99979245
n_9	0.99383156	0.98216103	0.98216103	0.96464029	2	0.99968177
n_7	0.99383156	0.99302444	0.97775124	0.98039473	2	0.99986324
n_8	0.99383156	0.99501248	0.98363538	0.97511487	1	0.99501248
n_{10}	0.99383156	0.97921896	0.98955493	0.97161077	2	0.99978294
		NR(G) =	= 19, R(G) =	0.99298048		

TABLE 5 Task Assignment of the Motivating Parallel Application Using the RR Algorithm

n_i	$R_{ m req}(n_i)$	$R(n_i, u_1)$	$R(n_i, u_2)$	$R(n_i, u_3)$	num_i	$R(n_i)$			
n_1	0.99383156	0.98609754	0.97628571	0.98393051	2	0.99977659			
n_3	0.99317319	0.98906028	0.98068890	0.96637821	2	0.99978874			
n_4	0.99234932	0.98708414	0.98807171	0.96986344	2	0.99984594			
n_2	0.99128298	0.98708414	0.97190229	0.96811926	2	0.99963709			
n_5	0.98989744	0.98807171	0.98068890	0.98216103	2	0.99978721			
n_6	0.98793125	0.98708414	0.97628571	0.98393051	2	0.99979245			
n_9	0.98498801	0.98216103	0.98216103	0.96464029	2	0.99968177			
n_7	0.98013824	0.99302444	0.97775124	0.98039473	1	0.99302444			
n_8	0.97511487	0.99501248	0.98363538	0.97511487	1	0.99501248			
n_{10}	0.97161077	0.97921896	0.98955493	0.97161077	1	0.98955493			
	NR(G) = 17, R(G) = 0.97609982								

Example 1. Assume that the constant failure rates for three processors are $\lambda_1 = 0.0010$, $\lambda_2 = 0.0015$, and $\lambda_3 = 0.0018$, respectively. Moreover, assume that the reliability requirement of the parallel application in Fig. 2 is $R_{\text{seq}}(G) = 0.94$. Note that the above values are not the representatives of a real deployment, but are used to explain the example clearly.

Table 4 shows the task assignment for each task of the 461 motivating parallel application using the MaxRe algorithm. 462 Each row shows the selected processors (denoted with bold 463 text) and corresponding reliability values. For example, the 464 sub-reliability requirement of n_1 is $R_{\rm reg}(n_1) = \sqrt[10]{0.94} =$ 465 0.99383156; to satisfy the sub-reliability requirement, MaxRe 466 selects the processors u_1 and u_3 with the maximum and sec-467 468 ond maximum reliability values, respectively (i.e., $num_1 =$ 2). Then, the actual reliability value of n_1 is 0.99977659, which 469 470 is calculated by Eq. (3). The remaining tasks use the same pattern with n_1 . Finally, the number of replicas are 19 and 471 the actual reliability value of the application *G* is 0.99298048, 472 which are calculated by Eqs. (5) and (4), respectively. 473

474 4.3 Existing RR Algorithm

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Obviously, the main limitation of the MaxRe algorithm is 475 that the sub-reliability requirements of all tasks are equal 476 and high, such that it needs more replicas with extra 477 redundancy to satisfy the sub-reliability requirement of 478 each task. To solve such problem, the authors presented 479 the RR algorithm to lower down the sub-reliability require-480 ment of tasks while still satisfying the application's reliabil-481 ity requirement [15] as follows. 482

First, the sub-reliability requirement for the entry task isstill calculated by

$$R_{\rm req}(n_1) = \sqrt[|N|]{R_{\rm req}(G)}.$$

Second, for the rest of tasks (i.e., non-entry tasks), unlike
prior MaxRe algorithm [14], sub-reliability requirements in
the RR algorithm are calculated continuously based on the
actual reliability achieved by previous allocations

$$_{\text{req}}(n_{seq(j)}) = \sqrt[|N|-j+1]{\frac{R_{\text{req}}(G)}{\prod_{x=1}^{j-1} R(n_{seq(x)})}},$$
(8)

where $n_{seq(j)}$ represents the *j*th assigned task. Clearly, such 494 single improvement can reduce the sub-reliability require-495 ments of non-entry tasks.

Example 2. The same parameter values ($\lambda_1 = 0.0010$, 497 $\lambda_2 = 0.0015, \lambda_3 = 0.0018$, and $R_{
m seq}(G) = 0.94$) with Exam- 498 ple 1 are used. Table 5 shows the task assignment for 499 each task of the motivating parallel application using the 500 RR algorithm. Each row shows the selected processors 501 (denoted with bold text) and corresponding reliability 502 values. The sub-reliability requirement and task assign- 503 ment of n_1 using the RR algorithm is similar to the MaxRe 504 algorithm. However, the remaining tasks are different. 505 For example, as the actual reliability value for n_1 is 506 0.99977659, then the sub-reliability requirement for n_3 507 should be $\sqrt[9]{\frac{0.94}{0.99977659}} = 0.99317319$. When assigning n_7 508 and n_8 , the sub-reliability requirements are reduced to 509 0.98013824 and 0.97161077, respectively. That is, only one 510 replica for each of n_7 and n_8 will be able to satisfy individ- 511 ual sub-reliability requirements. Finally, the number of 512 replicas and the actual reliability value of the application 513 G are 17 and 0.97609982 (calculated by Eqs. (5) and (4)), 514 respectively, which still satisfy application's reliability 515 requirement, but their values are less than those obtained 516 with the MaxRe algorithm. 517

5 ENOUGH REPLICATION FOR REDUNDANCY MINIMIZATION

Although the RR algorithm can reduce the sub-reliability 520 requirements of tasks, the reduction ranges of tasks near the 521 entry task are much lower than those of the tasks near the exit 522 task. That is, the actual sub-reliability requirements show 523 unfairness among tasks, such that the RR algorithm still 524 requires unnecessary redundancy to satisfy application's 525 reliability requirement. To further reduce redundancy, we 526 first present good enough replication approach in this section, 527 and then propose a heuristic replication approach in the next 528 section. 529

5.1 Lower Bound on Redundancy

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Considering that application reliability is the product of all 531 task reliability values, the reliability value of each task should 532 be higher than or equal to $R_{req}(G)$; otherwise, if one task has 533

 $R(n_i) < R_{req}(G)$, then no matter how many replicas for any other tasks, $R_{req}(G)$ cannot be satisfied. Therefore, the lower bound on reliability requirement of the task n_i is

$$R_{\text{lb_req}}(n_i) = R_{\text{req}}(G). \tag{9}$$

In this way, there should be a lower bound on the numberof replicas for each task that satisfies

$$R(n_i) \ge R_{\text{lb_reg}}(n_i).$$

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In other words, we can determine the lower bound on the number of replicas $lb(n_i)$ for task n_i to satisfy

$$1 - \prod_{x=1}^{lb(n_i)} \left(1 - R(n_i^x, u_{pr(n_i^x)})\right) \ge R_{lb_req}(n_i), \tag{10}$$

548 according to Eq. (3).

We use the following steps to select the replica and the corresponding processor with the minimum number of replicas.

- (1) Calculate the $R(n_i, u_k)$ of each task on all available processors (if a replica of n_i has been assigned to the processor, then this processor is unavailable for n_i ; otherwise, it is available for n_i).
- 556 (2) To minimize the number of replicas, select the rep-557 lica n_i^x of the task n_i and the corresponding processor 558 $u_{pr(n^x)}$ with the maximum $R(n_i^x, u_{pr(n^x)})$.

(3) Repeat Steps (1) and (2) until Eq. (10) is satisfied.

560 5.2 The LBR Algorithm

561 On the basis of the above steps, we propose the lower 562 bound on redundancy (LBR) algorithm (Algorithm 1) to 563 generate the lower bound on the number of the replicas of 564 each task.

565 Algorithm 1. The LBR Algorithm

Input: $G = (N, W, M, C), U, R_{reg}(G)$ 566 **Output:** R(G), NR(G) and its related values 567 1: for (i = 1; i < = |N|; i + +) do 568 $R_{\text{lb_req}}(n_i) \leftarrow R(G);$ 569 2: 3: $num_i = 0;$ 570 571 4: $R(n_i) = 0$; // initial value is 0 while $(R(n_i) < R_{\text{lb}_{\text{req}}}(n_i))$ do 572 5: Calculate $R(n_i, u_k)$ for the task n_i on all available pro-573 6: cessors using Eq. (1); 574 Select replica n_i^x and the processor $u_{pr(n_i^x)}$ with the max-7: 575 imum reliability value $R(n_i^x, u_{pr(n_i^x)})$; 576 8: $num_i++;$ 577 9: Calculate $R(n_i)$ using Eq. (3); 578 579 10: end while 11: Calculate NR(G) using Eq. (5); 580 12: Calculate R(G) using Eq. (4); 581

582 13: end for

The core idea of the LBR algorithm is that each task iteratively selects the replica and available processor with the maximum reliability value $R(n_i^x, u_{pr(n_i^x)})$ for each task until the task's lower bound on reliability requirement is satisfied. The details are explained as follows:

TABLE 6 Task Assignment of the Motivating Parallel Application Using the LBR Algorithm

n_i	$R_{\rm req}(n_i)$	$R(n_i, u_1)$	$R(n_i, u_2)$	$R(n_i, u_3)$	num_i	$R(n_i)$			
n_1	0.94	0.98609754	0.97628571	0.98393051	1	0.98609754			
n_3	0.94	0.98906028	0.98068890	0.96637821	1	0.98906028			
n_4	0.94	0.98708414	0.98807171	0.96986344	1	0.98807171			
n_2	0.94	0.98708414	0.97190229	0.96811926	1	0.98708414			
n_5	0.94	0.98807171	0.98068890	0.98216103	1	0.98807171			
n_6	0.94	0.98708414	0.97628571	0.98393051	1	0.98708414			
n_9	0.94	0.98216103	0.98216103	0.96464029	1	0.98216103			
n_7	0.94	0.99302444	0.97775124	0.98039473	1	0.99302444			
n_8	0.94	0.99501248	0.98363538	0.97511487	1	0.99501248			
n_{10}	0.94	0.97921896	0.98955493	0.97161077	1	0.98955493			
	NR(G) = 10, R(G) = 0.89092057								

- In Line 2, LBR has obtained the lower bound on reliability requirement of the current task before it prepares to be assigned.
- (2) In Lines 5-10, LBR iteratively selects the replica and 591 available processor for each task with the maximum 592 reliability value until the task's lower bound on 593 reliability requirement is satisfied. Specifically, the 594 following details are made: 1) Line 5 compares the 595 actual reliability value and lower bound on reliability 596 requirement of the current task; 2) Lines 6-7 calculate 597 and select the replica and available processor with the 598 maximum reliability value for the current task; and 599 3) Line 9 calculates the actual reliability value of the 600 current task.
- (3) In Lines 11-12, LBR calculates the final number of 602 replicas and the actual reliability value of the appli-603 cation, respectively. 604

5.3 Time Complexity of the LBR Algorithm

The time complexity of the LBR algorithm is analyzed as 606 follows: 607

- (1) Calculating the reliability of the application must tra- 608 verse all tasks, which can be done within O(|N|) time 609 (Lines 1-13).
- (2) The total number of replicas for each task must be $_{611}$ lower or equal to the number of processors, which $_{612}$ can be done within O(|U|) time (Lines 5-10). $_{613}$
- (3) Selecting the replica and available processor with the 614 maximum reliability value for the current task must 615 traverse all processors, which can be done in 616 O(log|U|) time (Line 7).

Thus, the time complexity of the LBR algorithm is 618 $O(|N| \times |U| \times \log |U|)$. 619

5.4 Example of the LBR Algorithm

Example 3. The same parameter values ($\lambda_1 = 0.0010$, $\lambda_2 = 621$ 0.0015, $\lambda_3 = 0.0018$, and $R_{seq}(G) = 0.94$) with aforemen-622 tioned examples are used. Table 6 lists the replicas, selected 623 processor, and reliability value of each task (denoted with 624 bold text). We find that the reliability value of each task is 625 higher than the application's reliability requirement of 626 0.94. However, the current obtained reliability value of the 627

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parallel application is only R(G) = 0.89092057 (calculated by Eq. (4)), which is much lower than 0.94 (application's reliability requirement). Hence, application's reliability requirement is not satisfied by merely using the LBR algorithm.

633 5.5 Enough Replication

Considering that all the tasks merely satisfy $R(n_i) \ge R_{\text{lb_req}}(n_i)$ by using the LBR algorithm (Algorithm 1), we should add more new replicas for tasks to satisfy application's reliability requirement. However, choosing the remaining replicas is a complex work, because different replicas of different tasks may cause different reliability values on different processors.

Given that the current number of replicas for n_i is $h = num_i$ and the application reliability is R(G), if a new replica n_i^{h+1} is assigned to the processor $u_k = u_{pr(n_i^{h+1})}$ for n_i , then the number of replicas is changed to h + 1 and the new task reliability is changed to

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$$R_{\text{new}}(n_i) = 1 - \prod_{x=1}^{n+1} \left(1 - R(n_i^x, u_{pr(n_i^x)})\right).$$
(11)

Then, the application reliability is enhanced because of the reliability enhancement of n_i and is changed to

$$R^{i}(G) = R_{\text{new}}(n_{i}) \times \prod_{n_{j} \in N, i \neq j} R(n_{j}).$$
(12)

(13)

To minimize the number of replicas for each task, we use the following steps to obtain enough minimum redundancy of the application.

(1) Each available task (if the replicas of a task have been assigned to all the processors, then this task is unavailable; otherwise, a task is available) is assumed to be replicated once on an available processor with the maximum $R(n_i, u_k)$ (Eq. (1)), and the new task sub-reliability is changed to $R_{\text{new}}(n_i)$ (Eq. (11)).

(2) Calculate the application reliability $R^{i}(G)$ because of the reliability enhancement of each task (Eq. (12)).

(3) Select the replica n_i^x and corresponding processor $u_{pr(n_i^x)}$ that generate the maximum $R^i(G)$ from the generated replicas in Step 2), namely,

$$R^{i}(G) = \max \Big\{ R^{1}(G), R^{2}(G), \dots, R^{|N|}(G) \Big\}$$

(4) Repeat Steps (1), (2), and (3) until application's reli-ability requirement (Eq. (4)) is satisfied.

672 5.6 The ERRM Algorithm

In this section, we propose the ERRM algorithm to minimizeredundancy to satisfy application's reliability requirement,and describe the steps in Algorithm 2.

The core idea of the ERRM algorithm is that all the tasks are first assumed to be replicated once on an available processor with the maximum reliability values; then ERRM selects the replica n_s^x and corresponding processor $u_{pr(n_s^x)}$ that generate the maximum application reliability value $R^s(G)$ until application's reliability requirement is satisfied in the iterative replication process. The details are explained as follows:

- (1) In Line 1, ERRM calls the LBR algorithm (Algorithm 1) $_{683}$ to obtain the initial reliability R(G) and related values. $_{684}$
- (2) In Lines 2-11, ERRM iteratively selects the replica 685 and available processor that generate the maximum 686 application reliability value until application's reli- 687 ability requirement is satisfied. Specifically, the fol- 688 lowing details are made: 1) Line 2 compares the 689 actual reliability value and the reliability require- 690 ment of the application; 2) Lines 3-7 pre-replicate all 691 tasks once on an available processor with the maxi- 692 mum reliability values; 3) Line 8 selects the replica 693 and corresponding processor that generate the maxi- 694 mum application reliability value; and 4) Line 10 695 updates the application's reliability value.
- (3) In Line 13, ERRM calculates the final number of rep- 697 licas of the application. 698

Algorithm 2. The ERRM Algorithm70Input: $G = (N, W, M, C), U, R_{req}(G)$ 70Output: $R(G), NR(G)$ and its related values701: Call the LBR algorithm (Algorithm 1) to obtain the initial 7070
Input: $G = (N, W, M, C), U, R_{req}(G)$ 70 Output: $R(G), NR(G)$ and its related values 70 1: Call the LBR algorithm (Algorithm 1) to obtain the initial 70 70 $M^{1/2}$ $R(G)$ 70
Output: $R(G)$, $NR(G)$ and its related values 70 1: Call the LBR algorithm (Algorithm 1) to obtain the initial 70 R(G) and $R(G)$ and $R(G$
1: Call the LBR algorithm (Algorithm 1) to obtain the initial 70
-1 $(1, 1, 1)$ $D(C)$ $(1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1$
reliability $R(G)$ and related values; 70
2: while $(R(G) < R_{reg}(G))$ do 70
3: for $(i = 1; i < = N ; i + +)$ do 70
4: Pre-replicated the replica of n_i on an available proces- 70
sor with the maximum reliability value $R(n_i, u_k)$; 70
5: Update the task's sub-reliability value to $R_{\text{new}}(n_i)$ 70
(Eq. (11)); 77
6: Calculate the application reliability $R^i(G)$ after the reli- 7
ability enhancement of n_i (Eq. (12)); 75
7: end for 7:
8: Select the replica n_s^x and corresponding processor $u_{pr(n_s^x)}$ 7
that generate the maximum application reliability value 7
$R^{s}(G)$ (Eq. (13)); 77
9: $num_i + +;$ 77
10: $R(n_i) \leftarrow R_{\text{new}}(n_i);$ 72
11: $R(G) \leftarrow R^i(G);$ 72
12: end while 72
13: Calculate $NR(G)$ using Eq. (5); 72

5.7 Time Complexity of the ERRM Algorithm

The time complexity of the ERRM algorithm is analyzed as 723 follows: 724

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- (1) The maximum number of iterative replication process 725 is $|N| \times |U|$, which can be done within $O(|N| \times |U|)$ 726 time (Lines 2-12). 727
- (2) Each task must be assumed to be replicated once on 728 an available processor, which can be done in O(|N|) 729 time (Lines 3-7). 730
- (3) Selecting the replica and available processor with the 731 maximum reliability value must traverse all process-732 ors, which can be done in $O(\log|U|)$ time (Line 4). 733
- (4) Updating the task's new sub-reliability value can be 734 done in O(|U|) time (Line 5). 735
- (5) Calculating the application's new reliability value 736 can be done in O(|N|) time (Line 6). 737
- (6) Obtaining the maximum application reliability value 738 can be done in O(|N|) time (Line 8). 739

Considering that (3), (4), and (5) are not nested in the 740 algorithm, the time complexity of the ERRM algorithm is 741 $O(|N|^2 \times |U|^2 + |N|^3 \times |U|).$ 742

TABLE 7 Selected Processor and Reliability Pairs (Denoted with Underline Text) of Each Task in Each Step of the Motivating Parallel Application Using the ERRM Algorithm

Step	n_1	n_2	n_3	n_4	n_5	n_6	n_7	n_8	n_9	n_{10}
(1)	$(u_3, 0.9033)$	$(u_2, 0.9023)$	$(u_2, 0.9006)$	$(u_1, 0.9015)$	$(u_3, 0.9015)$	$(u_3, 0.9024)$	(<i>u</i> ₃ , 0.89714)	$(u_2, 0.8953)$	$(u_2, 0.9068)$	$(p_1, 0.9001)$
(2)	$(u_3, 0.9194)$	$(u_2, 0.9183)$	$(u_2, 0.9166)$	$(u_1, 0.9176)$	$(u_3, 0.9176)$	$(u_3, 0.9185)$	$(u_2, 0.9131)$	$(u_2, 0.9113)$	$(u_3, 0.9071)$	$(u_1, 0.9162)$
(3)	$(u_2, 0.9196)$	$(u_2, 0.9311)$	$(u_2, 0.9294)$	$(u_1, 0.9303)$	$(u_3, 0.9303)$	$(u_3, 0.9312)$	$(u_3, 0.9257)$	$(u_2, 0.9239)$	$(u_3, 0.9197)$	$(u_1, 0.9289)$
(4)	$(u_2, 0.9314)$	$(u_2, 0.9431)$	$(u_2, 0.9413)$	$(u_1, 0.9423)$	$(u_3, 0.9423)$	$(u_2, 0.9314)$	$(u_3, 0.9376)$	$(u_2, 0.9358)$	$(u_3, 0.9315)$	$(u_1, 0.9409)$

Considering that the time complexity of the LBR algo-743 rithm (i.e., $O(|N| \times |U| \times \log |U|)$) is less than that of the 744 ERRM algorithm, using the LBR algorithm in advance can 745 improve the efficiency of the replication compared with 746 only using the ERRM algorithm. The reason is that the LBR 747 748 algorithm can obtain an initial reliability value greater than zero, such that the number of iterative process of the ERRM 749 750 algorithm can be reduced. Considering the motivating example, the reliability value obtained is 0.89092057, shown 751 in Table 6, then the initial reliability value is not 0, but 752 0.89092057. Compared to starting from 0, 0.89092057 is close 753 to the actual reliability requirement of 0.93. 754

755 5.8 Example of the ERRM Algorithm

Example 4. The same parameter values ($\lambda_1 = 0.0010$, $\lambda_2 =$ 756 0.0015, $\lambda_3 = 0.0018$, and $R_{seq}(G) = 0.94$) with aforemen-757 tioned examples are used. Table 7 lists the selected proces-758 sor and reliability pairs of each task in each step by using 759 Algorithm 2, where the underlined values indicate those 760 that have the maximum $R_{\text{new}}(n_i)$ (Eq. (11)) and $R^{\text{s}}(G)$ 761 (Eq. (13)), and the replica is selected to enhance the reli-762 763 ability of the application in each step. For example, in Step (1), n_9 and u_2 are selected, because they can generate the 764 maximum value of 0.9068. In Step (4), the reliability value 765 is larger than or equal to application's reliability require-766 ment 0.94. Hence, application's reliability requirement is 767 satisfied, and the replication process successfully ends. 768

Table 8 lists the final replicas, selected processor, and reliability value for each task of the parallel application in Fig. 2. We find that the final reliability value of each task is larger than or equal to 0.94. Moreover, the current reliability value is R(G) = 0.94307237 (calculated by Eq. (4)), which is larger than 0.94. Hence, application's reliability requirement is satisfied, and the application proves

TABLE 8 Task Assignment of the Application in Fig. 2 Using the ERRM Algorithm

n_i	$R(n_i, u_1)$	$R(n_i, u_2)$	$R(n_i, u_3)$	num_i	$R(n_i)$
$\overline{n_1}$	0.98609754	0.97628571	0.98393051	2	0.99977659
n_3	0.98906028	0.98068890	0.96637821	1	0.98906028
n_4	0.98708414	0.98807171	0.96986344	1	0.98807171
n_2	0.98708414	0.97190229	0.99963709	2	0.98708414
n_5	0.98807171	0.98068890	0.98216103	1	0.98807171
n_6	0.98708414	0.97628571	0.98393051	2	0.99979245
n_9	0.98216103	0.98216103	0.96464029	2	0.99968177
n_7	0.99302444	0.97775124	0.98039473	1	0.99302444
n_8	0.99501248	0.98363538	0.97511487	1	0.99501248
n_{10}	0.97921896	0.98955493	0.97161077	1	0.98955493
	N	R(G) = 14, R	C(G) = 0.9430	7237	

reliable in this situation. Meanwhile, the final resource 776 consumption is NR(G) = 14 (Calculated by Eq. (5)). 777

6 HEURISTIC REPLICATION FOR REDUNDANCY MINIMIZATION

Although the ERRM algorithm can implement enough 780 redundancy minimization, it has high time complexity and 781 thereby it is time-consuming for a large-scale parallel application. To reduce the redundancy of a large-scale parallel 783 application within an acceptable computation time, this section presents a heuristic algorithm. 785

6.1 Upper Bound on Reliability Requirement

Although the RR algorithm can achieve more redundancy 787 reduction than the MaxRe algorithm by recalculating the sub-788 reliability requirement, the redundancy reduction ranges of 789 the tasks near the entry task is much lower than those of the 790 tasks near the exit task (see Table 5). The main reason for the discrepancy is that unfair sub-reliability requirements among tasks are generated. In fact, the tasks that are after $n_{seq(x)}$'s 793 allocations (i.e., unassigned tasks) can also be presupposed as assigned tasks with known reliability values. 795

We find that all the sub-reliability requirements of tasks 796 using the RR algorithm do not exceed 0.99383156 (see 797 Table 5), which is the sub-reliability requirement of each 798 task using the MaxRe algorithm (see Table 4). Thus, we let 799 $|N| \sqrt{R_{\text{req}}(G)}$ be the upper bound on task's reliability require 800 ment, namely, 801

$$R_{\text{up}_\text{req}}(n_i) = \sqrt[|N|]{R_{\text{req}}(G)}.$$
 (14) 803
804

Then, we have the following heuristic strategy: assume 805 that the task to be assigned is $n_{seq(j)}$ ($n_{seq(j)}$ represents the *j*th 806 assigned task as mentioned earlier), then $\{n_{seq(1)}, n_{seq(2)}, \ldots, 807, n_{seq(j-1)}\}$ represents the task set with assigned tasks, and 808 $\{n_{seq(j+1)}, n_{seq(j+2)}, \ldots, n_{seq(|N|)}\}$ represents the task set with 809 unassigned tasks. To ensure that the reliability of the applica-810 tion is satisfied at each task assignment, we presuppose that 811 each task in $\{n_{seq(j+1)}, n_{seq(j+2)}, \ldots, n_{seq(|N|)}\}$ is assigned to the 812 processor with reliability value on upper bound (Eq. (14)). 813 Hence, when assigning $n_{seq(j)}$, application's reliability 814 requirement is 815

$$R_{\text{req}}(G) = \prod_{x=1}^{j-1} R(n_{seq(x)}) \times R_{\text{req}}(n_{seq(j)}) \times \prod_{y=j+1}^{|N|} R_{\text{up_req}}(n_{seq(y)}).$$
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Then, the sub-reliability requirement for the task $n_{seq(j)}$ 818 should be 820

$$R_{\text{req}}(n_{seq(j)}) = \frac{R_{\text{req}}(G)}{\prod_{x=1}^{j-1} R(n_{seq(x)}) \times \prod_{y=j+1}^{|N|} R_{\text{up_req}}(n_{seq(y)})}.$$
 (15) 822
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6.2 The HRRM Algorithm 824

On the basis of the aforementioned new sub-reliability 825 requirement calculation for each task (Eq. (15)), we present 826 the heuristic algorithm HRRM described in Algorithm 3 to 827 minimize redundancy and satisfy application's reliability 828 requirement. 829

830 Algorithm 3. The HRRM Algorithm

Input: $G = (N, W, M, C), U, R_{req}(G)$ 831

- **Output:**R(G), NR(G) and its related values 832
- 1: Order tasks according to a descending order of $rank_u(n_i, u_k)$ 833 834 using Eq. (6);
- 835 2: for (j = 1; j < = |N|; j + +) do
- Calculate $R(n_{seq(j)})$ using Eq. (3); 3: 836
- $4 \cdot$ 837 $num_{seq(j)} = 0;$
- $R(n_{seq(i)}) = 0; //$ initial value is 0 5: 838 839 6:
- Calculate $R_{req}(n_{seq(j)})$ using Eq. (15); 7: 840
 - while $(R(n_{seq(j)}) < R_{req}(n_{seq(j)}))$ do

Calculate $R(n_{seq(j)}, u_k)$ for the task $n_{seq(j)}$ on all each 8: 841 available processor using Eq. (1); 842

- Select replica $n_{seq(j)}^{x}$ and the processor $u_{pr(n_{seq(j)}^{x})}$ with the 843 9: maximum $R(n_{seq(j)}^x, u_{pr(n_{seq(j)}^x)});$ 844
- 845 10: $num_{seq(j)}++;$
- 846 11: Calculate $R(n_{seq(j)})$ using Eq. (3);
- 847 12: end while
- 13: end for 848
- 849 14: Calculate NR(G) using Eq. (5);
- 15: Calculate R(G) using Eq. (4); 850

The core idea of HRRM is that the reliability requirement 851 of the application is transferred to the sub-reliability 852 requirement of each task. Each task just iteratively selects 853 the replica and available processor with the maximum reli-854 ability value until its sub-reliability requirement is satisfied. 855 The details are explained as follows: 856

In Line 6, HRRM has obtained the reliability require-857 (1)ment of the current task before it prepares to be 858 859 assigned.

In Lines 7-12, HRRM iteratively selects the replica (2)860 861 and available processor with the maximum reliability value for the current task until its sub-862 reliability requirement is satisfied. Specifically, the 863 following details are made: 1) Line 7 compares 864 the actual reliability value and sub-reliability 865 requirement of the current task; 2) Lines 8-9 calcu-866 late and select the replica and available processor 867 with the maximum reliability value for the current 868 task; and 3) Line 11 calculates the actual reliability 869 value of the current task. 870

(3) In Lines 14-15, HRRM calculates the final number of 871 872 replicas and the actual reliability value of the application, respectively. 873

Compared with MaxRe and RR algorithms, the main 874 improvement of the presented HRRM is that it recalculates 875 the sub-reliability requirement of each task based not only on 876 its previous assignments ($\{n_{seq(1)}, n_{seq(2)}, \ldots, n_{seq(j-1)}\}$), but 877 also on succeeding pre-assignments $\{n_{seq(j+1)}, n_{seq(j+2)}, \ldots, \}$ 878 $n_{seq(|N|)}$, whereas MaxRe algorithm has a fixed and equal 879 sub-reliability requirements for all tasks and RR algorithm is 880 merely based on previous assignments. 881

TABLE 9 Task Assignment of the Motivating Parallel Application Using the HRRM Algorithm

n_i	$R_{\rm req}(n_i)$	$R(n_i, u_1)$	$R(n_i, u_2)$	$R(n_i, u_3)$	num_i	$R(n_i)$
n_1	0.99383156	0.98609754	0.97628571	0.98393051	2	0.99977659
n_3	0.98792188	0.98906028	0.98068890	0.96637821	1	0.98906028
n_4	0.99268768	0.98708414	0.98807171	0.96986344	2	0.99984594
n_2	0.98671636	0.98708414	0.97190229	0.96811926	1	0.98708414
n_5	0.99346128	0.98807171	0.98068890	0.98216103	2	0.99978721
n_6	0.98754331	0.98708414	0.97628571	0.98393051	2	0.99979245
n_9	0.98165546	0.98216103	0.98216103	0.96464029	1	0.98216103
n_7	0.99331998	0.99302444	0.97775124	0.98039473	2	0.99986324
n_8	0.98732777	0.99501248	0.98363538	0.97511487	1	0.99501248
n_{10}	0.98615598	0.97921896	0.98955493	0.97161077	1	0.98955493
		NR(G) =	= 15, R(G) =	0.94323987		

6.3 Time Complexity of the HRRM Algorithm

The time complexity of the HRRM algorithm is analyzed as 883 follows: 884

- (1)Calculating the reliability of the application must 885 traverse all tasks, which can be done within O(|N|) 886 time (Lines 2-13). 887
- Calculating the sub-reliability requirement of the 888 (2)current task must traverse all tasks, which can be 889 done within O(|N|) time (Line 6). 890
- (3)The number of replicas must be lower or equal to the 891 number of processors, which can be done within 892 O(|U|) time (Lines 7-12). 893
- (4)Calculating the reliability value of the current task 894 must traverse all assigned processors, which can be 895 done in O(|U|) time (Line 11) 896

Considering that (2) and (3) are not nested in the 897 algorithm, the time complexity of the HRRM algorithm is 898 $O(|N|^2 + |N| \times |U|^2)$, which is similar to those of MaxRe and 899 RR algorithms. Thus, HRRM implements efficient fault- 900 tolerance without increasing time complexity. 901

6.4 Example of the HRRM Algorithm

Example 5. The same parameter values ($\lambda_1 = 0.0010$, 903) $\lambda_2 = 0.0015, \ \lambda_3 = 0.0018, \ \text{and} \ R_{\text{seq}}(G) = 0.94)$ with afore- 904 mentioned examples are used. Table 9 shows the task 905 assignment for each task of the motivating parallel appli-906 cation using HRRM algorithm. Each row shows the 907 selected processors (in red) and corresponding reliability 908 values. The sub-reliability requirement and task assign- 909 ment of n_1 using HRRM algorithm is similar to those 910 using MaxRe and RR algorithms. However, the remain- 911 ing tasks are different. For example, when assigning n_{3} , 912 the actual reliability value for n_1 is 0.99977659, and suc- 913 ceeding pre-assignments with reliability requirements are 914 $\sqrt[10]{0.94} = 0.99383156$, then the sub-reliability requirement 915 for n_3 should be $\frac{0.94}{0.99977659 \times 0.99383156^8} = 0.98792188$. Com- 916 pared with the RR algorithm, an obvious improvement 917 for the HRRM algorithm is that it shows relative fair reli- 918 ability requirements among tasks; furthermore, most sub- 919 reliability requirements of tasks using HRRM are less 920 than those using the RR algorithm. Finally, the number of 921 replicas and the actual reliability value of the application 922

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(a) Fast Fourier transform with ρ =4.

(b) Gaussian elimination with ρ =5

Fig. 3. Example of real parallel applications.

G are 15 and 0.94323987 (calculated by Eqs. (5) and (4)),
 respectively, which are lower than those with MaxRe and
 RR algorithms.

926 7 EXPERIMENTS

927 7.1 Experimental Metrics and Parameter Values

Considering that this study aims to implement redundancy 928 minimization with replication to satisfy application's reliabil-929 930 ity requirement, performance metrics selected for comparison should be the actual reliability value and total number of 931 replicas of the application. Meanwhile, computation time 932 933 should be included from a time complexity perspective. The computation time is measured from the start time to the end 934 time of an algorithm to schedule an application. 935

Algorithms compared with the proposed ERRM and HRRM algorithms are the state-of-the-art MaxRe [14] and RR [15] algorithms. MaxRe and RR algorithms address the same problem of minimizing resource redundancy of a parallel application to satisfy application's reliability requirement on heterogeneous distributed systems.

Considering that this study focuses on the design phase, the processor and application parameters used in this phase are known. In other words, these values have been obtained in the analysis phase and are as follows [15]: 10,000 s $\leq w_{i,k} \leq 100,000$ s, 10,000 s $\leq c_{i,j} \leq 100,000$ s, and 0.000001 $\leq \lambda_k \leq 0.000009$. The aforementioned values are generated with uniform distribution.

The parallel applications will be tested on a simulated het-949 erogeneous system based on the above real processor and 950 application parameter values to reflect a real deployment. A 951 952 main advantage of simulation is that it can greatly reduce development cost during the design phase and effectively 953 954 provide certain optimization guide to the implementation 955 phase. The simulated multiprocessor system is configured 64 heterogeneous processors by creating 64 processor objects 956 based known parameter values using Java on a standard 957 desktop computer with 2.6 GHz Intel CPU and 4 GB memory. 958

959 Meanwhile, real parallel applications with precedence constrained tasks, such as fast Fourier transform and Gauss-960 ian elimination applications, are widely used in distributed 961 systems [9], [15]. The Fourier transform and Gaussian elimi-962 963 nation application are two typical parallel applications with high and low parallelism, respectively. To verify the effec-964 tiveness and validity of the proposed algorithms, we use 965 the two types of real parallel applications to compare the 966 results of all the algorithms. 967

A new parameter ρ is used as the size of the fast Fourier transform application. The total number of tasks is



Fig. 4. Results of the small-scale fast Fourier transform application on different reliability requirements (Experiment 1).

 $|N| = (2 \times \rho - 1) + \rho \times \log_2 \rho$, where $\rho = 2^y$ for some integer 970 *y* [9]. Fig. 3a shows an example of the fast Fourier transform 971 application with ρ =4. Notably, ρ exit tasks exist in the fast 972 Fourier transform application with the size of ρ . To adopt 973 the application model of this study, we add a virtual exit 974 task, and the last ρ tasks are set as the immediate predecessor tasks of the virtual exit task. A new parameter ρ is used 976 as the matrix size of the Gaussian elimination application, 977 and the total number of tasks is $|N| = \frac{\rho^2 + \rho - 2}{2}$ [9]. Fig. 3b 978 shows an example of the Gaussian elimination parallel 979 application with ρ =5.

7.2 Fast Fourier Transform Application

Experiment 1. This experiment compares the actual reliability values and the total number of replicas of a smallscale fast Fourier transform application with $\rho = 32$ (i.e., 984 |N| = 223) for varying reliability requirements. $R_{seq}(G)$ is 985 changed from 0.9 to 0.99 with 0.01 increments. Note that 986 computation time values of all the algorithms are within 987 one second for the small-scale application and we no longer list such values in this experiment. 989

Note that the plotted values in Figs. 4a and 4b are obtained 990 by executing one run of the algorithms for one application. 991 Many applications with the same parameter values and 992 scales are tested and show the same regular pattern and rel-993 atively stable results as Figs. 4a and 4b. In other words, 994 experiments are repeatable and do not affect the consistency 995 of the results. Therefore, the plotted values are the actual 996 values rather than the average values during the runs. 997

Fig. 4a shows the actual reliability values of the small-998 scale fast Fourier transform application on different reliabil-999 ity requirements. We can see that all the algorithms can 1000 satisfy the given reliability requirements in all cases. Specifi- 1001 cally, MaxRe generates the maximum reliability values 1002 followed by RR, HRRM, and ERRM. The overrunning reli- 1003 ability values (i.e., $R_{seq}(G)-R(G)$) reach 0.0613 and 0.0246 1004 for MaxRe and RR, respectively. On the contrary, the over- 1005 running reliability values are very small for HRRM (0.0001- 1006 0.0008) and ERRM (0.0001-0.0006) in all cases. Considering 1007 no additional fees will be paid for the overrunning reliabil- 1008 ity values, more resources are wasted for resource providers 1009 in using MaxRe and RR. 1010

Fig. 4b shows the total number of replicas of the small- 1011 scale fast Fourier transform application on different reliabil- 1012 ity requirements. As expected, MaxRe generates the maxi- 1013 mum numbers of replicas followed by RR, HRRM, and 1014 ERRM in all cases. The reason is that MaxRe has obtained the 1015 maximum actual reliability values followed by RR, HRRM, 1016 and ERRM in Fig. 4a, whereas optimizing reliability and 1017



(c) Computation time (unit. second).

Fig. 5. Results of the large-scale fast Fourier transform application on different reliability requirements (Experiment 2).

redundancy is a bi-criteria optima problem as discussed in Section 1.2.

The same regular pattern for the actual reliability values is shown in Fig. 4a. As evident from Fig. 4b, the numbers of replicas using HRRM and ERRM are very similar and are much lower than those using MaxRe and RR, especially on relatively low reliability requirements. For example, when $R_{seq}(G) \leq 0.94$, both ERRM and HRRM outperform MaxRe and RR by about 18 and 7 percent, respectively.

1027 **Experiment 2.** This experiment compares the actual reli-1028 ability values, the total number of replicas, and the com-1029 putation time of a large-scale fast Fourier transform 1030 application with $\rho = 128$ (i.e., |N| = 1151) for varying reli-1031 ability requirements. $R_{seq}(G)$ is also changed from 0.9 to 1032 0.99 with 0.01 increments.

Fig. 5a shows the actual reliability values of the large-1033 scale fast Fourier transform application on different reli-1034 ability requirements. All the algorithms can satisfy the 1035 given reliability requirements in all cases. Similar to the 1036 results of the small-scale application in Fig. 4a, MaxRe still 1037 1038 generates the maximum reliability values followed by RR, HRRM, and ERRM. Maximum differences between actual 1039 reliability and given reliability requirement are 0.0747 1040 $(R_{seq}(G) = 0.9)$ and 0.0184 $(R_{seq}(G) = 0.90)$ for MaxRe and 1041 RR, respectively. On the contrary, in all cases the differen-1042 ces remain the minimum and close to application's reliabil-1043 ity requirements using HRRM (0.0001-0.0003) and ERRM 1044 (0.0001 - 0.0002).1045

Fig. 5b shows the total number of replicas of the largescale fast Fourier transform application on different reliability requirements. Similar to Fig. 4b in small-scale, MaxRe still generates the maximum numbers of replicas followed by RR, HRRM, and ERRM in all cases. The numbers of replicas using HRRM and ERRM are still very close and are much lower than those using MaxRe and RR in most cases.

Fig. 5c shows the computation time values of the largescale fast Fourier transform application for reliability requirements. The values show that computation time is within 2.1 second using MaxRe, RR, and HRRM, whereas those using ERRM are 80-120 times longer. Such results indicate that ERRM is time-consuming for large-scale applications, as analyzed earlier.



Fig. 6. Results of the small-scale Gaussian elimination application on different reliability requirements (Experiment 3).

An interesting phenomenon is that the computation time 1060 values using ERRM for large scale applications are not 1061 increased but reduced as the application's reliability requirements increase in most cases, shown in Fig. 5c. The reason is 1063 that when using ERRM, it first calls the LBR algorithm 1064 (Algorithm 1) to obtain the initial reliability values of the 1065 application. A higher reliability requirement of the application may lead to higher initial reliability values with very 1067 short time by using LBR in these cases, such that the total 1068 computation time is not increased, but reduced with the 1069 application's reliability requirements increase. 1070

The results of Figs. 4a, and 5c show that ERRM and 1071 HRRM algorithms generate less redundancy than the stateof-the-art MaxRe and RR algorithms. Specifically, results 1073 of HRRM algorithm are very similar to those of ERRM algo-1074 rithm indicating that HRRM implements approximate optimal redundancy with minimum time, whereas the enough 1076 optimal ERRM algorithm is time-consuming for large-scale 1077 parallel applications. 1078

7.3 Gaussian Elimination Application

Experiment 3. This experiment compares the actual reliability values and the total number of replicas of in a 1081 small-scale Gaussian elimination application with $\rho = 21$ 1082 (i.e., |N|=230). The total number of task for the Gaussian 1083 elimination is similar to that of the fast Fourier transform 1084 application for varying reliability requirements. $R_{seq}(G)$ 1085 is also changed from 0.9 to 0.99 with 0.01 increments. 1086 Similar to small-scale fast Fourier transform in Experiment 1, the computation time values using all the algorithms are also within one second for the small-scale 1089 Gaussian elimination. Therefore, we also no longer list 1090 such values in this experiment.

Figs. 6a and 6b show the actual reliability values and total 1092 number of replicas of the small-scale Gaussian elimination 1093 application on different reliability requirements. In general, 1094 Experiment 3 shows similar pattern and values as Experi-1095 ment 1 for the total number of replicas for all the algorithms. 1096

The results of Experiments 1 and 3 indicate that different 1097 parallelism degrees of applications in the same small-scale 1098 will generate similar actual reliability values and total number of replicas. In other words, parallelism degrees do not 1100 affect the scopes of actual reliability values and total number of replicas. The reason is that the reliability value of the 1102 application is the product of that of each task according to 1103 Eq. (4); considering that the number of tasks, the reliability 1104 requirement, and computation time are approximate equal, 1105 the actual reliability values and total number of replicas are also approximate equal. 1107



(c) Computation time (unit: second).

Fig. 7. Results of the large-scale Gaussian elimination application on different reliability requirements (Experiment 4).

Experiment 4. This experiment compares the actual reliability values, the total number of replicas, and the computation time of a large-scale Gaussian elimination application with $\rho = 47$ (i.e., |N| = 1, 127) for varying reliability requirements. $R_{seq}(G)$ is also changed from 0.9 to 0.99 with 0.01 increments.

Figs. 7a, 7b, and 7c show the actual reliability values, 1114 total number of replicas, and computation time of the large-1115 scale Gaussian elimination application on different reliabil-1116 ity requirements. Experiment 4 shows similar pattern and 1117 values as Experiment 2 in actual reliability values and total 1118 number of replicas for all the algorithms. The results of 1119 Experiments 2 and 4 further indicate that parallelism 1120 degrees do not affect the scopes of actual reliability values 1121 and total number of replicas. 1122

1123 7.4 Randomly Generated Parallel Application

To extensively demonstrate the benefits of the proposed 1124 algorithms, we consider randomly generated parallel appli-1125 cations by the task graph generator [40]. Considering that 1126 the objective platform is heterogeneous processors, heterogeneity degrees may also affect the redundancy of applica-1128 tion. Heterogeneity degree is easy to be implemented for 1129 randomly generated parallel applications as long as adjust 1130 the heterogeneity factor values. Randomly generated paral-1131 lel applications are generated depending on the following 1132 parameters: average computation time is 50,000 ms, com-1133 1134 munication to computation ratio (CCR) is 1, and shape parameter is 1. The heterogeneity degree (factor) values 1135 belong to the scope of (0,1] in the task graph generator, 1136 where 0 and 1 represent the lowest and highest heterogene-1137 ity factors, respectively. Without loss of generality, we use 1138 large-scale randomly generated parallel application with 1139 1,140 tasks, which are approximate equal to those of fast 1140 Fourier transform and Gaussian elimination applications in 1141 Experiments 2 and 4. 1142

1143**Experiment 5.** This experiment compares the actual reli-
ability values and the total number of replicas of a large-
scale low-heterogeneity (with the heterogeneity factor
0.1) randomly generated parallel application with
|N| = 1,140 for varying reliability requirements. $R_{seq}(G)$
is also changed from 0.9 to 0.99 with 0.01 increments.



Fig. 8. Results of the large-scale low-heterogeneity randomly generated parallel application on different reliability requirements (Experiment 5).

Figs. 8a and 8b show the actual reliability values and total 1149 numbers of replicas the large-scale low-heterogeneity randomly generated parallel application on different reliability 1151 requirements. It is easy to see that Experiment 5 shows similar pattern and values as Experiments 2 and 4 using all the algorithms. The main differences are as follows: 1154

- The actual reliability values and total numbers of replicas obtained by MaxRe in Experiment 5 are relatively stable for different reliability requirements. The reason is that the execution time values are relative stable on the same processor for a low-heterogeneity parallel application and the reliability requirement using MaxRe is the same for all tasks, such that the values for the application do not changed much.
- (2) The actual reliability values and total numbers of 1163 replicas obtained by RR, ERRM, and HRRM are relatively close in the same reliability requirement. The 1165 reason is still that the execution time values are relative stable on the same processor for a low-heterogeneity parallel application. 1168
- **Experiment 6.** This experiment compares the actual reli- 1169 ability values and the total number of replicas of a large- 1170 scale high-heterogeneity (with the heterogeneity factor 1) 1171 randomly generated parallel application with |N| = 1140 1172 for varying reliability requirements. $R_{seq}(G)$ is also 1173 changed from 0.9 to 0.99 with 0.01 increments. 1174

Figs. 9a and 9b show the actual reliability values and total 1175 numbers of replicas the large-scale high-heterogeneity randomly generated parallel application on different reliability 1177 requirements. It is easy to see that Experiment 6 shows similar pattern and values as Experiment 5 using all the algo-1179 rithms. The main difference is that the high-heterogeneity 1180 application needs fewer replicas than the low-heterogeneity 1181 application. The total numbers of replicas for the former is 1182 only 60 percent of those for the latter using all the algorithms. 1183 The reason is that the actual reliability values for a task on dif-1184 ferent processors change much in a high-heterogeneity appli-1185 cation, and these algorithms tend to choose the processor 1186



Fig. 9. Results of the large-scale high-heterogeneity randomly generated parallel application on different reliability requirements (Experiment 6).

with the maximum reliability value for each task replication.
Moreover, different from the low-heterogeneity application
where the total numbers of replicas obtained by RR, ERRM,
and HRRM are relatively close, ERRM and HRRM generate
much less replicas than RR for the high-heterogeneity application. The reason is still that the actual reliability values for a
task on different processors change much.

1194 7.5 Summary of Experiments

Based on the above experimental results, summarizations are as follows.

- 1197(1)The proposed redundancy minimization algorithms,
ERRM and HRRM, can generate less redundancy1198ERRM and HRRM, can generate less redundancy1199than the state-of-the-art MaxRe and RR algorithm at1200different scales, parallelism degrees, and heterogene-1201ity degrees.
- Results of the HRRM algorithm are very similar to those of the ERRM algorithm. HRRM implements approximate optimal redundancy with minimum computation time, whereas the enough optimal ERRM algorithm is time-consuming for large-scale parallel applications.
- (3) According to the analysis of the number of active processors, parallelism degrees do not affect the scopes of reliability values and total number of replicas for different types of applications in the same-scale.
- (4) If the parallel application is small, then ERRM can be utilized to minimize redundancy; otherwise HRRM is the preferred alternative for reducing redundancy with minimum computation time.
- (5) RR, ERRM, and HRRM obtain relatively close numbers of replicas for a low-heterogeneity application, whereas ERRM and HRRM obtain much less replicas than RR for the high-heterogeneity application. In other words, ERRM and HRRM are better suitable for high-heterogeneity applications than for low-heterogeneity applications.

1223 **8 CONCLUSION**

We developed enough and heuristic replication algorithms 1224 ERRM and HRRM to minimize the redundancy for a paral-1225 lel application in heterogeneous service-oriented systems. 1226 The ERRM algorithm can enough minimize redundancy by 1227 presenting two-stage replications. To decrease the time 1228 complexity of the time-consuming ERRM algorithm, the 1229 HRRM algorithm was also presented to deal with large-1230 scale parallel applications within a short time. The main 1231 advantage for HRRM is its capability to obtain lower sub-1232 reliability requirements for most tasks compared with 1233 MaxRe and RR, such that HRRM can generate less redun-1234 dancy than MaxRe and RR. Results of our experiments on 1235 real and random generaed parallel applications at different 1236 scales, parallelism degrees, and heterogeneity degrees vali-1237 date that both ERRM and HRRM generate less redundancy 1238 than the state-of-the-art MaxRe and RR algorithms. Experi-1239 ment results also show that the HRRM implements approxi-1240 mate optimal redundancy with a short computation time. 1241 We believe that the proposed algorithms can effectively 1242 facilitate a reliability-aware design for parallel applications 1243 in heterogeneous service-oriented systems. 1244

Resource usage and shortest schedule length are also 1245 important concern in high-performance computing systems. In fact, minimum redundancy does not mean minimum resource usage and shortest schedule length for a 1248 parallel application on heterogeneous systems because 1249 the same task has different execution time values on different processors. In our future work, we will consider 1251 the resource usage and schedule length minimization in such environment. 1253

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SUPPLEMENT MATERIAL

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The web page http://esnl.hnu.edu.cn/index.php/tsc/ publishes the experimental codes of the paper. 1267

REFERENCES

- Z. Cai, X. Li, and J. N. D. Gupta, "Heuristics for provisioning services to workflows in XaaS clouds," *IEEE Trans. Services Comput.*, 1270 vol. 9, no. 2, pp. 250–263, Mar./Apr. 2016.
- A. Zhou, S. Wang, B. Cheng, Z. Zheng, F. Yang, R. Chang, M. Lyu, 1272 and R. Buyya, "Cloud service reliability enhancement via virtual 1273 machine placement optimization," *IEEE Trans. Services Comput.*, 1274 vol. PP, no. 99, p. 1, Jan. 2016, doi: 10.1109/TSC.2016.2519898.
- [3] Z. Fu, F. Huang, X. Sun, A. Vasilakos, and C.-N. Yang, "Enabling 1276 semantic search based on conceptual graphs over encrypted outsourced data," *IEEE Trans. Services Comput.*, vol. PP, no. 99, p. 1, 1278 Oct. 2016, doi: 10.1109/TSC.2016.2622697. 1279
- Z. Xia, X. Wang, X. Sun, and Q. Wang, "A secure and dynamic multikeyword ranked search scheme over encrypted cloud data," *IEEE* 1281 *Trans. Parallel Distrib. Syst.*, vol. 27, no. 2, pp. 340–352, Feb. 2016.
- [5] Y. Kong, M. Zhang, and D. Ye, "A belief propagation-based 1283 method for task allocation in open and dynamic cloud environments," *Knowl.-Based Syst.*, vol. 115, pp. 123–132, Jan. 2017. 1285
- [6] F. Zhangjie, S. Xingming, L. Qi, Z. Lu, and S. Jiangang, "Achieving 1286 efficient cloud search services: Multi-keyword ranked search over 1287 encrypted cloud data supporting parallel computing," *IEICE* 1288 *Trans. Commun.*, vol. 98, no. 1, pp. 190–200, Jan. 2015.
- Q. Liu, W. Cai, J. Shen, Z. Fu, X. Liu, and N. Linge, "A speculative 1290 approach to spatial-temporal efficiency with multi-objective opti- 1291 mization in a heterogeneous cloud environment," *Secur. Commun.* 1292 *Netw.*, vol. 9, no. 17, pp. 4002–4012, Nov. 2016. 1293
- [8] Z. Tang, L. Qi, Z. Cheng, K. Li, S. U. Khan, and K. Li, "An 1294 energy-efficient task scheduling algorithm in DVFS-enabled 1295 cloud environment," J. Grid Comput., vol. 14, no. 1, pp. 55–74, 1296 Mar. 2016. 1297
- H. Topcuoglu, S. Hariri, and M.-Y. Wu, "Performance-effective 1298 and low-complexity task scheduling for heterogeneous 1299 computing," *IEEE Trans. Parallel Distrib. Syst.*, vol. 13, no. 3, 1300 pp. 260–274, Aug. 2002.
- M. A. Khan, "Scheduling for heterogeneous systems using constrained critical paths," *Parallel Comput.*, vol. 38, no. 4, pp. 175–1303 193, Apr. 2012.
- G. Xie, R. Li, and K. Li, "Heterogeneity-driven end-to-end 1305 synchronized scheduling for precedence constrained tasks and 1306 messages on networked embedded systems," *J. Parallel Distrib.* 1307 *Comput.*, vol. 83, pp. 1–12, May. 2015.

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- 1309 [12] G. Xie, X. Xiao, R. Li, and K. Li, "Schedule length minimization of 1310 parallel applications with energy consumption constraints using 1311 heuristics on heterogeneous distributed systems," Concurrency Com-1312 *put.-Parctice Experience*, pp. 1–10, Nov. 2016, doi: 10.1002/cpe.4024.
- 1313 [13] J.-S. Leu, C.-F. Chen, and K.-C. Hsu, "Improving heterogeneous 1314 SOA-Based iot message stability by shortest processing time 1315 scheduling," IEEE Trans. Services Comput., vol. 7, no. 4, pp. 575-1316 585, Oct.-Dec. 2014.
- 1317 [14] L. Zhao, Y. Ren, Y. Xiang, and K. Sakurai, "Fault-tolerant schedul-1318 ing with dynamic number of replicas in heterogeneous systems," 1319 in Proc. 12th IEEE Int. Conf. High Perform. Comput. Commun., 2010, pp. 434-441. 1320
- 1321 [15] L. Zhao, Y. Ren, and K. Sakurai, "Reliable workflow scheduling with less resource redundancy," Parallel Comput., vol. 39, no. 10, 1322 1323 pp. 567-585, Jul. 2013.
 - Z. Zheng, T. C. Zhou, M. Lyu, and I. King, "Component ranking [16] for fault-tolerant cloud applications," IEEE Trans. Services Comput., vol. 5, no. 4, pp. 540–550, Oct.-Dec. 2012.
 - [17] W. Qiu, Z. Zheng, X. Wang, X. Yang, and M. R. Lyu, "Reliabilitybased design optimization for cloud migration," IEEE Trans. Services Comput., vol. 7, no. 2, pp. 223–236, Apr.-Jun. 2014.
 - M. Silic, G. Delac, and S. Srbljic, "Prediction of atomic web [18] services reliability for QoS-Aware recommendation," IEEE Trans. Services Comput., vol. 8, no. 3, pp. 425-438, May/Jun. 2015.
- 1333 [19] A. Girault and H. Kalla, "A novel bicriteria scheduling heuristics providing a guaranteed global system failure rate," IEEE Trans. 1334 1335 Dependable Secure Comput., vol. 6, no. 4, pp. 241-254, Oct.-Dec. 2009.
 - [20] A. Benoit, M. Hakem, and Y. Robert, "Fault tolerant scheduling of precedence task graphs on heterogeneous platforms," in Proc.
 - 22th IEEE Int. Parallel Distrib. Process., 2008, pp. 1–8. [21] A. Benoit and M. Hakem, "Optimizing the latency of streaming applications under throughput and reliability constraints," in Proc. 45th Int. Conf. Parallel Process., 2009, pp. 325-332.
 - [22] [Online]. Available: https://en.wikipedia.org/wiki/IEC_61508
 - [Online]. Available: https://en.wikipedia.org/wiki/ISO_9000 [23]
 - G. Xie, L. Liu, L. Yang, and R. Li, "Scheduling trade-off of [24] dynamic multiple parallel workflows on heterogeneous distributed computing systems," Concurrency Comput.-Parctice Experi-ence, vol. 29, no. 2, pp. 1–18, Jan. 2017, doi: 10.1002/cpe.3782.
 - V. T'kindt and J.-C. Billaut, Multicriteria Scheduling: Theory, Models [25] and Algorithms. Berlin, Germany: Springer, Mar. 2006.
- 1349 [26] A. Doğan and F. Özgüner, "Biobjective scheduling algorithms for 1350 1351 execution time-reliability trade-off in heterogeneous computing 1352 systems," Comput. J., vol. 48, no. 3, pp. 300-314, Mar. 2005.
 - [27] M. Hakem and F. Butelle, "A bi-objective algorithm for scheduling parallel applications on heterogeneous systems subject to failures," in Proc. RenPar2006, 2006, pp. 25-35.
 - J. J. Dongarra, E. Jeannot, E. Saule, and Z. Shi, "Bi-objective sched-[28] uling algorithms for optimizing makespan and reliability on het-erogeneous systems," in Proc. 19th ACM Int. Symp. Parallel Algorithms Architectures, 2007, pp. 280–288.
 - [29] J. Broberg, S. Venugopal, and R. Buyya, "Market-oriented grids and utility computing: The state-of-the-art and future directions,"
- J. Grid Comput., vol. 6, no. 3, pp. 255–276, Sep. 2008. [30] [Online]. Available: https://en.wikipedia.org/wiki/Service-level_ 1362 1363 1364 agreement 1365
 - S. M. Shatz and J. P. Wang, "Models and algorithms for reliability-[31] oriented task-allocation in redundant distributed-computer systems," IEEE Trans. Rel., vol. 38, no. 1, pp. 16-27, Apr. 1989.
- J. Mei, K. Li, X. Zhou, and K. Li, "Fault-tolerant dynamic resched-1368 [32] uling for heterogeneous computing systems," J. Grid Comput., 1369 vol. 13, no. 4, pp. 507-525, Dec. 2015. 1370
- X. Qin, H. Jiang, and D. R. Swanson, "An efficient fault-tolerant 1371 [33] scheduling algorithm for real-time tasks with precedence con-1372 straints in heterogeneous systems," in Proc. 31th Int. Conf. Parallel 1373 Process., 2002, pp. 360-368.
- 1374 [34] X. Qin and H. Jiang, "A novel fault-tolerant scheduling algorithm 1375 for precedence constrained tasks in real-time heterogeneous 1376 systems," *Parallel Comput.*, vol. 32, no. 5, pp. 331–356, Jun. 2006. Q. Zheng, B. Veeravalli, and C.-K. Tham, "On the design of fault-1377
- 1378 [35] 1379 tolerant scheduling strategies using primary-backup approach for computational grids with low replication costs," IEEE Trans. Com-1380 put., vol. 58, no. 3, pp. 380–393, Mar. 2009. 1381
- 1382 [36] A. Benoit, L.-C. Canon, E. Jeannot, and Y. Robert, "Reliability of task graph schedules with transient and fail-stop failures: Complexity 1383 1384 and algorithms," J. Scheduling, vol. 15, no. 5, pp. 615–627, Oct. 2012.

- [37] A. Verma and N. Bhardwaj, "A review on routing information 1385 protocol (RIP) and open shortest path first (OSPF) routing proto- 1386 col," Int. J. Future Generation Commun. Netw., vol. 9, no. 4, pp. 161-1387 170, Apr. 2016. 1388
- [38] Q. Zheng and B. Veeravalli, "On the design of communication- 1389 aware fault-tolerant scheduling algorithms for precedence 1390 constrained tasks in grid computing systems with dedicated com-1391 munication devices," J. Parallel Distrib. Comput., vol. 69, no. 3, 1392 pp. 282–294, 2009. 1393
- [39] L. Zhao, Y. Ren, and K. Sakurai, "A resource minimizing schedul-1394 ing algorithm with ensuring the deadline and reliability in hetero-1395 geneous systems," in Proc. 25th IEEE Int. Conf. Adv. Inf. Netw. 1396 *Appl.,* 2011, pp. 275–282. 1397
- [40] [Ónline]. https://sourceforge.net/projects/ Available: 1398 taskgraphgen/ 1399



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