Information Sciences 574 (2021) 567–589

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

DLEA: A dynamic learning evolution algorithm for many-objective optimization

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ARTICLE INFO

Article history:
Received 22 January 2021
Received in revised form 20 May 2021
Accepted 26 May 2021
Available online 6 June 2021

Keywords:
Evolutionary algorithms (EAs)
Dynamic learning strategy
Many-objective optimization
Performance indicators

ABSTRACT

For many-objective problems, how to maintain the diversity and convergence of the distribution of the solution set over Pareto front (PF) has always been the research emphasis. In the iteration process, the state of population is critical to improve the level of evolution. Therefore, this paper will use two convergence and diversity indicators to further strengthen the usage of evolutionary state information and propose a dynamic learning strategy. In addition, a dynamic learning strategy based many-objective evolutionary algorithm (MaOEA) is proposed, called dynamic learning evolution algorithm (DLEA), which continuously changes the direction of learning: convergence and diversity in the iteration process. The purpose is to make the algorithm prefer to convergence in the early iteration and prefer to diversity when it is close to PF in the late iteration, so that the convergence and diversity of the final solution set can be well maintained. And then, the performance of DLEA is measured by two indicators. Meanwhile, DLEA will be compared with four state-of-the-art algorithms on the DTLZ and MaF, and its performance will be verified on a many-objective combinatorial problem. And the experimental results and Friedman test show that DLEA has great advantages.

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1. Introduction

In the real world, there are many problems which contain multiple objectives that are required be optimized by tradeoffs at the same time. We call them multi-objective optimization problems (MOPs). In general, these problems having more than three objectives are called many-objective optimization problems (MaOPs). The solutions of such problems are not like numerical optimization problems that only consider a single objective, but optimize each objective in the problem at the same time. A MaOP can be represented as follows:

$$\begin{aligned} \min F(x) &= (f_1(x), f_2(x), \dots, f_M(x))^T \\ &\textit{subject to } x \in \Omega \end{aligned} \tag{1}$$

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https://doi.org/10.1016/j.ins.2021.05.064
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where M is the objective number, and Ω is the decision space which can be mapped to objective space R^M by function F

For MaOPs, due to the conflicting objectives, a solution set is often obtained instead of a single optimal solution. Each solution is called Pareto optimal solution, and the whole is called Pareto front (PF). The Pareto optimal solution is closely related to Pareto domination which is defined as follows: for the minimization problem, taking two solutions x_1 and x_2 in Ω, if and only if for any i in {1, 2, ..., M} satisfies $f_i(x_1) \leq f_i(x_2)$, and there exists j in {1, 2, ..., M} satisfies $f_j(x_1) < f_j(x_2)$. Let's call it $F(x_1)$ Pareto dominates $F(x_2)$, and the notation is $F(x_1) \succ F(x_2)$. And if and only if no solution $\times$ in Ω to satisfy $F(x) \succ F(x^*)$, x^* is called Pareto optimal solution.

Because of its unique population-based characteristics, evolutionary algorithms (EAs) are widely used to deal with optimization problems. After years of development, more and more EAs have been proposed. Considering the characteristics of MaOPs, a mass of many-objective evolutionary algorithms (MaOEAs) is proposed, which can effectively deal with all kinds of MaOPs. Some MaOEAs are improved on the basis of the existing EAs [35–37], such as competitive mechanism based multi-objective particle swarm optimization (CMOPSO) [50] and evolutionary multi-objective seagull optimization algorithm (EMoSOA) [4]. Though different MaOEAs have their own advantages, all MaOEAs can be classified into the following categories.

The first kind is the MaOEAs based on the dominance relationship. This MaOEAs first defines the dominance relationship between the solutions, then use it to classify the solution in the population, and selects the solution according to the demand. Generally, this MaOEAs will be combined with the second indicator to assist the selection. Among them, the most classical algorithm is non-dominated sorting genetic algorithm II (NSGA-II) [7], which uses Pareto dominance to perform non-dominated sorting to classify the solutions, and uses crowding distance to assist selection and increases the diversity of the solution sets. However, the disadvantage of NSGA-II is that as the number of objectives increases, the number of non-dominated solutions in the population will also increase, and even the solutions in the whole population are non-dominated solutions. As a result, NSGA-II is not as effective in handling MaOPs as MOPs because it reduces the selection pressure when the objective number is high. NSGA-III [9,15] uses reference points to increase selection pressure and the diversity of solution sets. In addition, some researchers have proposed other dominance relationships. For example, ϵ-dominance [12,20] and fuzzy dominance [38], both have good performance in handling MaOPs.

The second type of MaOEAs are indicator-based algorithms, which rely on at least one evaluation indicator. However, the defined indicators can make the population converge towards PF and there is no need for the second indicator to implement additional operations to increase diversity. By controlling the evolution of the whole population towards a better indicator, the final solution set can maintain convergence and diversity at the same time. For example, Fast hypervolume-based algorithm (HypE) [2], uses HV [33,42] as the evaluation indicator of each generation of population. It is easy to understand in this way, but some indicators also have the disadvantage of high computational complexity. In addition, a metaheuristic algorithm based on R2 indicator is proposed (MOMBI-II) [14]. This indicator has low computational cost and weak-Pareto compatibility. It can use certain computational resources to deal with many-objective optimization problems, and it will not reduce the selection pressure with the increase of the number of objectives.

The third category is MaOEAs based on decomposition. It is not based on the established indicator, but combines the mathematical decomposition idea with EAs to decompose a MaOP into multiple sub-problems for simultaneous optimization. Multi-objective evolutionary algorithm based on decomposition (MOEA/D) was first proposed by Zhang *et al.* [48] in 2007. The whole algorithm introduced three decomposition methods, namely weighted sum approach, Tchebycheff approach, and penalty-based boundary intersection approach. Each decomposition approach has its own unique advantages and disadvantages in handling various types of MOPs. After more than ten years of development, some new decomposition-based MaOEAs have been proposed, such as by combining decomposition and domination (MOEA/DD) [23], as well as decomposition and DE operator (MOEA/DDE) [22]. An algorithm combining the decomposition idea and Pareto adaptive scalarizing methods (MOEA/D-PaS) [41] was proposed to balance the selection pressure toward the Pareto optimal and the algorithm robustness to Pareto optimal front (PF) geometries. In addition, considering the relationship between sub-problems, Wang *et al.* [40] proposed a new algorithm to allocate computational resources according to the optimization difficulty of sub-problems. And recently the evolutionary multi-objective optimization algorithm (DDEA) based on dynamic decomposition was proposed by He *et al.* [13]. These algorithms also have good performance in handling MaOPs.

The algorithms mentioned above have relatively good performance in MaOPs or MOPs, but most MOEAs (MaOEAs) do not consider the state information of the population during iteration. The state information is very important for improving the convergence and diversity of the population. At the beginning of the search, most individuals are generated randomly, therefore, it is preferred to converge towards PF, so that the whole population converges into a good population. In the late stage of the search, more attention should be paid to the diversity of the population. Since the convergence effect of individuals in the late stage is relatively good, it is more preferred to select individuals that can increase the diversity of the population when selecting new individuals. Based on this, a dynamic learning evolution algorithm (DLEA) is proposed in this paper, which uses separate convergence and diversity indicators to change the selection ratio of individuals in the iterative process. It means that individuals having better convergence are more likely to be selected in the early stage of the search, while individuals having better diversity are more likely to be selected in the late stage of the search.

The rest of the paper is arranged as follows. Section 2 will introduce the related work of dynamic learning evolution algorithm (DLEA). The framework of DLEA is introduced in Section 3. The parameter sensitivity analysis and strategy selection of DLEA and the results of comparison between DLEA and four state-of-the-art MOEAs will be described in Section 4, and Section 5 is a summary of this work.

2. Related work

In recent years, various improved EAs have been used to deal with various dynamic optimization problems (DOPs) and dynamic multi-objective optimization problems (DMOPs). At the same time, many EAs and MOEAs with dynamic updating strategies have been proposed to solve numerical optimization problems (NOPs) and MOPs/MaOPs, all of which have their own advantages.

For DOPs and DMOPs, some novel methods have been proposed. For example, external archive and multi-population strategy are used to improve harmony search (HS) algorithm to deal with DOPs [34]. This enables HS to adapt to the changing environment of DOPs. In addition, multi-population strategies were also used in [19,30,34] to deal with DOPs. Cao *et al.* [3] proposed a particle swarm optimization (PSO) [18] algorithm based on short-term, long-term memory, and neighborhood learning. Short-term memory and long-term memory are used to store the individual in the current environment and the best individual in the history, respectively. At the same time, the neighborhood learning strategy is used to improve the way of speed updating. These strategies can be combined to handle DOPs well. Liu *et al.* [27] also proposed a dynamic multi-population particle swarm optimization algorithm (DP-DMPPSO) based on decomposition and prediction. Using the archive update mechanism based on the objective space decomposition and the population prediction mechanism to accelerate the convergence, the results show that the algorithm has a good effect in processing DMOPs. Finally, various strategies [10,11,29,31] are applied to implement a dynamic multi-objective evolutionary algorithm (DMOEAs) for DMOPs.

Various MOPs/MaOPs also have different characteristics, such as multi-mode and PF discontinuity. Therefore, it is difficult to find a MOEA that performs well on all MOPs/MaOPs. Recently, many researchers proposed many dynamic optimization strategies [25,43,47], which enabled MOEAs to make adaptive adjustments according to the characteristics of MOPs/MaOPs. For example, neural network is associated with objective function [21,45], and weighted functions (such as weighted Tchebycheff function) are used to handle objective and decompose the decision space to achieve better results. In addition, a Pareto optimal solutions calculation method based on switching topology is proposed. In [26,44], dynamic population size, external archives, greedy strategy, and local search strategy are also used to make adjustments. These strategies combined with MOEAs can better compensate for the disadvantage that MOEAs cannot flexibly adapt to handle different types of MOPs/MaOPs. Some researchers combined decomposition with dynamic resource allocation strategy and proposed a multi-objective multifactorial optimization algorithm (MFEA/D-DRA) [46] to deal with multi-objective and multifactorial optimization (MO-MFO). At the same time, dynamic decomposition is proposed in [13,28] to deal with different MOPs/MaOPs. Different from traditional decomposition-based MOEAs [48], these algorithms do not use preset fixed reference vectors to guide MOEAs to select solutions, but dynamically change the reference vectors according to the PF shape of MOPs/MaOPs during the execution of MOEAs (for example, some Pareto optimal solutions are used as reference vectors). By using dynamic decomposition, as long as the diversity of the selected reference vector can be guaranteed, the Pareto optimal solution set can be obtained with a good diversity and convergence as well as a good approximation to the Pareto-optimal front. Jiang *et al.* [16] also proposed the combination of dynamic covariance matrix learning and multi-objective differential evolution to deal with MOPs/MaOPs with variable linkages.

The above MOEAs with dynamic optimization strategy can make corresponding adjustments according to the characteristics of the problem, but no MOEAs focus on the state information of population in iterative process. This is a very important point, because the convergence of the whole population is very poor at the beginning of the iteration, especially when the population initialization is just completed. In this case, MOEAs should pay more attention to convergence than diversity. As the convergence of the population becomes better, MOEAs should prefer to maintain the diversity of the population. By using this dynamic selection mechanism to maintain convergence and diversity, Pareto optimal solution can better represent Pareto-optimal front.

3. The framework of DLEA

This section will first introduce the general framework of DLEA and then describe each step in detail. Some of the advantages and disadvantages of DLEA compared with the traditional approach can be found in Table 1.

3.1. The general framework of DLEA

As shown in Algorithm 1, the general framework of DLEA is very similar to most MOEAs. The population P is first initialized (Line 1), and then the iterative update process begins. On the basis of satisfying the iteration condition, tournament selection is performed firstly (Line 3), and the newly generated offspring *Offs* is generated by crossover operator and mutation operator (Line 4). Finally, population P and offspring *Offs* are selected through environmental selection to obtain the new population P_{new} with population size N. Among them, environment selection is different between DLEA and NSGA-II, which is also the key point. This introduction will be given in Section 3.2.

As shown in Fig. 1(a), after the population initialization, the distribution of these individuals in the objective space is very chaotic. In other words, the convergence and diversity of the population are poor. According to the current population, the priority is to urge these individuals to converge to PF as soon as possible. This will be guided by indicator-based method. For example, in a practical engineering problem, the individuals on PF are those who can minimize the cost. In this case, more

Table 1
The advantage and disadvantage of dynamic learning strategy and other traditional methods.

	Advantage	Disadvantage
Non-dominated methods	1. Processing works better when there are fewer objectives; 2. A second indicator (such as crowding distance) is often used as the selection criterion for two non-dominated solutions to improve the performance of the algorithm.	1. The convergence effect becomes worse and worse as the number of objectives increases; 2. Convergence speed is slow.
Indicator-based methods	1. Convergence speed is fast; 2. Only a single indicator is used to select solutions, which is easy to understand and operate; 3. The time cost is relatively small to make algorithm implement efficiently.	1. The distribution of convergent population is bad; 2. Each indicator focuses on only one convergence advantage and may not be more effective for certain problems.
Decomposition methods	1. It is better to deal with the problem which has uniform distribution solutions in Pareto front; 2. Problems of various types (such as non-convex problems) can be better handled by rational use of decomposition methods.	1. The problem with uneven distribution solution in Pareto front is not well handled; 2. The production mode of weight vector is relatively simple. When the problem is complex, the weight vector uniformly generated cannot well guide the solution to converge towards the entire Pareto front (convergence and diversity).
Dynamic learning strategies	1. Putting different emphasis in the early and late stages of the algorithm according to the characteristics of population evolution; 2. The entire evolutionary process can be accelerated or slowed down for other purposes by controlling convergence factors; 3. Different evaluation indicators can be flexibly chosen to improve the performance of the algorithm.	1. Time cost is high, and two indicators are required be calculated for each environmental selection; 2. The calculation time of different strategy is also different, so the running time is uncertain when the indicators are uncertain.

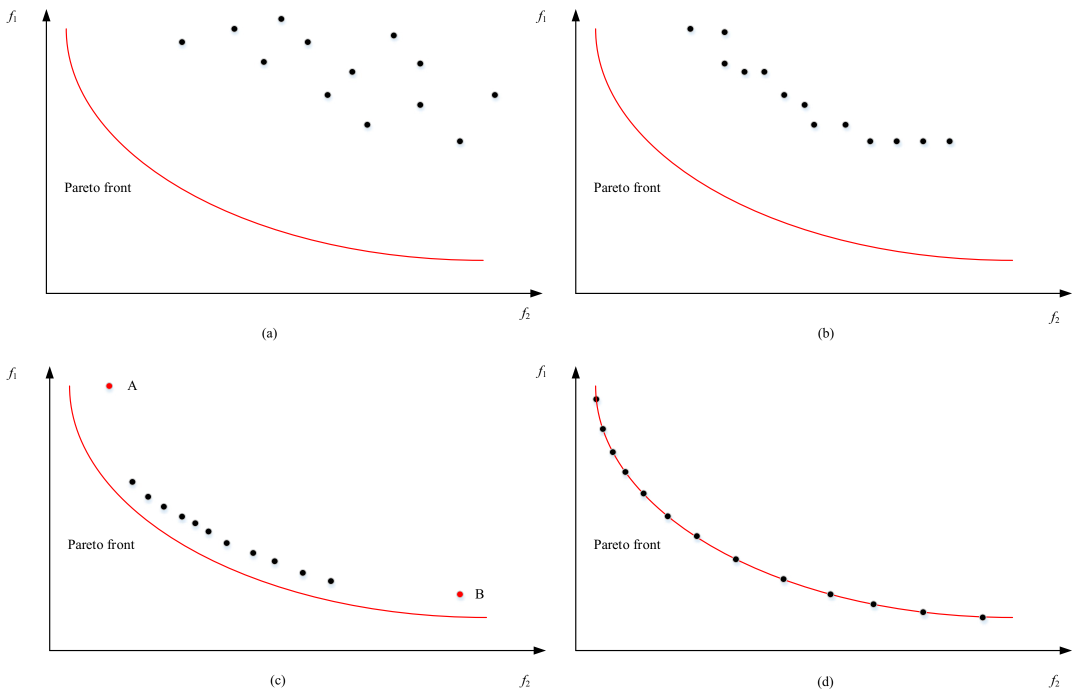

Fig. 1. The process of dynamic learning strategy. (a) The population state just after initialization. (b) The population state after initial evolution. (c) The population distribution after a certain number of times guided by the dynamic learning strategy, and the diversity at this time is relatively poor. (d) That is the final stage of the algorithm, the diversity-related solutions (such as individuals A and B in (c)) are guided to be selected, so that the convergence and diversity on PF are very good in the end.

computational resources should be allocated to the process of convergence-related operations to achieve rapid convergence of the population. And then, a small part of the computational resources is allocated to operations that increase the diversity of the population to ensure that the population has a good diversity.

After implementing the operations mentioned above, the distribution of individuals in population will gradually move towards the state in Fig. 1(b). However, the convergence level of the whole population is not enough at this time, so high selection pressure is still required to promote convergence. As the iteration goes on, the distribution of individuals in population will gradually move towards certain promising areas, as shown in Fig. 1(c). As introduced in Section 1, the indicator-based algorithms converge quickly but lose diversity easily. The example in Fig. 1(c) shows that these individuals are close to PF under the guidance of the indicator, but the convergence position is more inclined to the central region of PF. At this point, more computational resources need to be tilted to increase the diversity of the population, such as the preference to keep the individual A and the individual B in Fig. 1(c) into the next generation. By changing the computational resource allocation according to the evolutionary state of the population, individuals in population can maintain good convergence and diversity. As shown in Fig. 1(d), the individuals in the resulting solution set are uniformly distributed on PF.

Algorithm 1 Framework of DLEA

Input: Population size: N; The number of objective: M; Parameter of p-norm: p
Output: The final population : P

$P = Initialization()$
while $gen < maxgen$ **do**
  $MatingPool = TournamentSelection()$
  $Offs = Variation()$
  $P = EnvironmentalSelection([P, Offs], N, gen, maxgen, p)$
end while

3.2. Environmental selection

The environmental selection of DLEA can be found in Algorithm 2. Here, the input parameter P_{2N} refers to the combined population of *P* and *Offs* in Algorithm 1, whose population size is $2 \times N$. The purpose of environmental selection is to select *N* best individuals from the combined population P_{2N} as the parent population (P_{new}) of the next iteration.

As shown in Line 1 of Algorithm 2, the non-dominated sorting of the population P_{2N} is performed to obtain the non-dominated layer *FrontNo* and the maximum layer (the layer requiring the selection individuals) *MaxFNo*. Then, individuals whose number of layers is less than *MaxFNo* are put into P_{new} (Line 2). The number of convergence-related variables C_n and diversity-related variables D_n were calculated to select individuals in layer *MaxFNo*. The calculation of C_n can be shown in Eq. (2).

$$C_n = \left\lceil nd_i \times \alpha \times \left(1 - \frac{gen}{maxgen}\right) \right\rceil \tag{2}$$

where nd_i is the number of individuals that need to be selected in layer *MaxFNo* at generation *i*, $\alpha \in (0, 1)$ is a convergence factor to control the convergence rate of the algorithm, the outermost sign is the integer function, *gen* is the current iteration, and *maxgen* is the preset maximum iteration. And the calculation of D_n can be shown as follows.

$$D_n = nd_i - C_n \tag{3}$$

Algorithm 2 Environmental Selection

Input: Population: P_{2N}; Population size: N; Current iteration: gen; Maximum iteration: $maxgen$; Parameter of p-norm: p
Output: Next generation population : P_{new}

1: $[FrontNo, MaxFNo] = NDSort()$
2: $P_{new} = P_{2N}(FrontNo < MaxFNo)$
3: Calculate Cn by Eq. (2)
4: Calculate Dn by Eq. (3)
5: //Calculate the $I_{\varepsilon+}$ indicator of individuals
6: $I_{\varepsilon+} = IndicatorI_{\varepsilon+}()$
7: //Calculate the $L_p - norm$ distance between individuals
8: $PNormDis = IndicatorDis()$
9: **for** $i = 1 : Cn$ **do**
10:   Choose the individual by $I_{\varepsilon+}$ and put into P_{new}
11: **end for**
12: **for** $i = 1 : Dn$ **do**
13:   Choose the individual by $PNormDis$ and put into P_{new}
14: **end for**

The next step calculates the $I_{\varepsilon+}$ [39] of all individuals in population P_{2N} (Line 6), which is the minimum distance required to describe a solution in the objective space in order to dominate another solution, as shown in Eq. (4). It is mainly used to measure the convergence and diversity of individuals in a population, but it pays more attention to convergence. Here, we use $I_{\varepsilon+}$ as the indicator to select convergence-related individuals in the layer *MaxFNo*. According to the characteristics (the smaller the indicator value is the better) of the indicator, we select C_n smallest individuals and put it into P_{new} (Lines 9–11). In order to maintain the diversity of the population, we use L_p-norm-based diversity maintenance mechanism [39] (Line 8), and the L_p-norm-based distance has been proved to be superior to the use of Euclidean distance or Manhattan distance as the distance measurement method in diversity maintenance strategy [39]. After calculating this distance, we select D_n individuals in the layer *MaxFNo* according to the L_p-norm-based distance and put them into P_{new} (Lines 12–14).

The DLEA uses the $I_{\varepsilon+}$ and L_p-norm-based diversity maintenance mechanism as selection criteria of the layer *MaxFNo* for the following reasons: this algorithm is to pay more attention to the convergence-related individuals in search upfront. While, in the later stages, the algorithm pays more attention to the diversity of the population. Therefore, at least one indicator is necessary to evaluate convergence and one indicator to evaluate diversity (diversity maintenance mechanism). Since non-dominated sorting may lose selection pressure on MaOPs, so $I_{\varepsilon+}$ is selected here. And L_p-norm-based distance is proved to be better than Euclidean distance and Manhattan distance, therefore, DLEA uses L_p-norm-based distance instead of the other two distances to measure the distance between individuals in the diversity maintenance mechanism. The calculation of $I_{\varepsilon+}$ can be shown in Eq. (4).

$$I_{\varepsilon+}(x_1, x_2) = \min_{\varepsilon}(f_i(x_1) - \varepsilon \leqslant f_i(x_2), 1 \leqslant i \leqslant M) \tag{4}$$

In addition, Eq. (5) is used in [39] as the cost required to delete an individual in population, which is equivalent to fitness function.

$$F(x_1) = \sum_{x_2 \in P \setminus \{x_1\}} -e^{-I_{\varepsilon+}(x_2, x_1)/0.05} \tag{5}$$

where x_1 and x_2 are two different individuals in population, and M is the number of objectives. For L_p-norm-based distance, the comparative experiment of parameter p in [39] shows that L_p-norm-based distance performs best when the parameter $p = 1/M$. Moreover, compared with other values, the robustness is also very high when $p = 1/M$, because the value p at this time can be adjusted with the change of the number of objectives.

3.3. Time complexity analysis

In DLEA, the time complexity of the tournament selection (Line 3 in Algorithm 1) and variation (Line 4 in Algorithm 1) operations are $O(N)$. Processing individuals using non-dominated sorting (Lines 1–2 in Algorithm 2) requires $O(N\log^{M-2}N)$ computations. The calculation of $I_{\varepsilon+}$ indicator (Line 6 in Algorithm 2) and L_p-norm-based distance (Line 8 in Algorithm 2) requires the calculation of $O(N^2M)$ and $O(N^2)$, respectively. The time complexity of selecting convergence-related individuals (Lines 9–11 in Algorithm 2) by $I_{\varepsilon+}$ and selecting diversity-related individuals (Lines 12–14 in Algorithm 2) by L_p-norm-based distance are $O(N)$. So, the worst-case time complexity at one generation of DLEA is max{$O(N\log^{M-2}N)$, $O(N^2M)$}.

4. Experiments

In this section, the parameter settings and running environment of the comparative experiments will be introduced. The optimal setting of the parameter α in Eq. (2) will also be given experimentally. And why did DLEA choose the strategies described in Section 3 will also be explained in this section. Finally, DLEA was compared with five state-of-the-art MOEAs on DTLZ test suite [8].

4.1. Parameter settings

In the experiments on optimal setting of parameter α, α will take ten equally spaced values from 0.1 to 1. And 30 times were run independently on 3, 5, 8, 10, and 15-objective DTLZ test suite. In the experiments on DLEA with different strategies, four versions of DLEA will be compared, and the settings are shown in Table 6. The second experiment was run on 3-objective DTLZ test suite. In the comparative experiments, DLEA will adopt the conclusions drawn in the first two experiments for parameter setting and strategy selection. And then, DLEA will be compared to KnEA [23], I-DBEA [1], hpaEA [5], and SPEA/R [17]. In this case, Das and Dennis's approach [6] is used to generate the original reference points on the hyperplane, while the other algorithms should have the same initial population size to ensure fairness. In addition, the number of generated reference points is the same as NSGA-III [9,15]. So, the corresponding population size N is set to 91, 210, 156, 275, and 135, respectively. The corresponding number of FEs is $10^{4}*M/2$.

In addition, the running device is PC, the system version is Windows 10, the processor is Intel(R) Core(TM) i3-8100 CPU 3.6 GHz, and the RAM is 8 GB.

4.2. Experiments on optimal setting of parameter α

The reason that affects the performance of MaOEAs is not only the structure of the algorithm itself, but also the setting of parameters, which may greatly affect the performance of the algorithm.

In DLEA, there are two parameters worth studying, one is the setting of parameter p in the L_p-norm-based distance, but this has been proved in [39] that the best parameter p should be set to $1/M$. The other is the parameter α used to calculate the C_n in Eq. (2), which is a convergence factor and $\alpha \in (0, 1)$. If the parameter is not well controlled, the performance of the whole DLEA may become worse. If α is set too small, the algorithm may converge too slowly, and the final effect may not reach the best in a limited time, and if α is set too large, premature convergence and local optimization may occur. Therefore, parameter analysis of parameter α will be conducted next.

Table 2 shows the average IGD [24,49] value (the smaller IGD value, the better result) of DELA after running on 3-objective DTLZ test suite. In total, 10 different α values are tested for comparison, and the last row of Table 2 is the average Friedman ranking (the smaller the average ranking, the better the performance) of DLEA with different α values on DTLZ1-9. As can be seen from Table 2, the average IGD value shows a smaller trend when the value of α changed from 0.1 to 1, but some fluctuations are also included. The average ranking results can also prove this point. On the 3-objective DTLZ1-9, the best performance is α = 0.9, followed by α = 0.2 and α = 0.4, and others perform relatively poorly.

Tables 3-5 are the average IGD results of DLEA under 5, 8, and 10-objective DTLZ test suite, respectively. According to the average ranking in each table, the best performance in Tables 3-5 is α = 1.0, α = 1.0, and α = 0.9, while the other values are relatively poor, which is similar to the results presented in Table 2. Therefore, it can be concluded that the closer the value of α is to 0.9, the better the performance of DLEA will be.

Fig. 2 shows the Friedman ranking (the smaller the average ranking, the better the performance) of average IGD [47,48] value (the smaller IGD value, the better result) of DELA after running on 3, 5, 8, and 10-objective DTLZ test suite. In total, 10 different α values are tested for comparison. As can be seen from Fig. 2, the Friedman ranking of average IGD value shows a smaller trend when the value of α changed from 0.1 to 1, but some fluctuations are also included. On the 3-objective DTLZ1-9, the best performance is α = 0.9, followed by α = 0.2 and α = 0.4, and others perform relatively poorly.

According to the average ranking in Fig. 2, the best performance on the 5-, 8-, and 10-objective DTLZ test suite is α = 1.0, α = 1.0, and α = 0.9, while the other values are relatively poor, which is similar to the results on 3-objective DTLZ. DLEA has the best performance when the value of α is in the interval [0.8, 1]. The reason may be that a larger convergence factor α will lead to a larger value of C_n, so more individuals having better convergence can be selected at one iteration, so that the population can converge faster. With the continuous iteration, *gen* gradually increases, and the number of searching diversity-related individuals increase so that the population also has a good diversity. The population obtained by using a smaller convergence factor α will also have a good diversity, but due to the slower convergence rate, the final result is not as good as the former.

In addition, this paper further analyzes the performance of DLEA with the value of α in the interval [0.8, 1]. Fig. 3 shows the Friedman test results of the IGD values obtained by DLEA on the DTLZ test suite with different objective numbers. The final subgraph in Fig. 3 is an average of the results for the five objectives, highlighted specifically in red. It can be seen from Fig. 3 that all the curves are generally concave. The lowest point (optimal result) of each curve is in the interval [0.9, 0.93]. From the average results, the average performance is the best when α = 0.9. The average performance of α = 0.91 is slightly

Table 2
Average IGD values of DLEA (α = 0.1, 0.2, ..., 1.0) on DTLZ1-9 with 3 objectives. The last row is average ranking of DLEA on DTLZ1-9 under different α.

Problem #	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
DTLZ1	2.13E-01	1.47E-01	1.74E-01	1.46E-01	2.10E-01	1.53E-01	1.62E-01	1.25E-01	1.17E-01	1.89E-01
DTLZ2	7.85E-02	7.59E-02	7.71E-02	7.69E-02	7.70E-02	7.74E-02	7.58E-02	7.75E-02	7.56E-02	7.76E-02
DTLZ3	7.37E+00	5.99E+00	6.96E+00	6.89E+00	7.15E+00	7.61E+00	6.16E+00	6.43E+00	6.80E+00	7.48E+00
DTLZ4	2.14E-01	1.69E-01	1.51E-01	1.84E-01	7.55E-02	2.13E-01	1.38E-01	2.56E-01	1.95E-01	1.67E-01
DTLZ5	9.97E-03	9.62E-03	1.03E-02	1.02E-02	9.89E-03	9.81E-03	9.74E-03	9.77E-03	9.77E-03	9.76E-03
DTLZ6	8.39E-03	8.96E-03	8.74E-03	9.01E-03	9.02E-03	8.93E-03	9.35E-03	9.70E-03	9.25E-03	9.51E-03
DTLZ7	1.16E-01	9.09E-02	1.00E-01	8.98E-02	1.00E-01	8.61E-02	1.21E-01	8.92E-02	1.07E-01	1.18E-01
DTLZ8	7.94E-02	7.41E-02	7.36E-02	6.91E-02	6.95E-02	6.76E-02	7.37E-02	6.68E-02	6.79E-02	6.43E-02
DTLZ9	4.56E-01	5.34E-01	5.65E-01	4.93E-01	4.96E-01	5.93E-01	5.38E-01	5.09E-01	3.95E-01	4.82E-01
Rank	7.33	4.22	6.22	4.89	5.67	5.89	5.33	5.22	4.11	6.11

Table 3
Average IGD values of DLEA (α = 0.1, 0.2, ..., 1.0) on DTLZ1-9 with 5 objectives. The last row is average ranking of DLEA on DTLZ1-9 under different α.

Problem #	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
DTLZ1	6.99E-01	4.97E-01	4.86E-01	4.92E-01	4.07E-01	3.73E-01	4.33E-01	3.86E-01	3.89E-01	4.61E-01
DTLZ2	2.61E-01	2.56E-01	2.56E-01	2.55E-01	2.52E-01	2.53E-01	2.50E-01	2.50E-01	2.50E-01	2.50E-01
DTLZ3	1.36E+01	1.12E+01	9.48E+00	9.60E+00	9.32E+00	9.66E+00	7.90E+00	8.58E+00	8.22E+00	8.49E+00
DTLZ4	3.71E-01	3.37E-01	3.68E-01	3.26E-01	3.66E-01	3.38E-01	3.13E-01	3.18E-01	3.10E-01	3.07E-01
DTLZ5	1.84E-01	1.84E-01	1.75E-01	1.75E-01	1.59E-01	1.71E-01	1.73E-01	1.66E-01	1.52E-01	1.59E-01
DTLZ6	9.61E-01	6.86E-01	4.85E-01	4.60E-01	3.50E-01	2.86E-01	3.32E-01	2.85E-01	3.17E-01	3.16E-01
DTLZ7	5.73E-01	5.33E-01	4.84E-01	5.31E-01	3.96E-01	4.42E-01	4.93E-01	4.87E-01	4.00E-01	4.27E-01
DTLZ8	2.46E-01	2.40E-01	2.30E-01	2.18E-01	2.13E-01	2.27E-01	2.20E-01	2.17E-01	2.22E-01	2.10E-01
DTLZ9	5.03E+00	4.51E+00	4.01E+00	4.21E+00	3.59E+00	3.56E+00	3.64E+00	3.28E+00	3.31E+00	3.29E+00
Rank	10.00	8.67	7.33	6.78	4.33	4.89	4.67	3.11	2.67	2.56

Table 4
.Average IGD values of DLEA (α = 0.1, 0.2, ..., 1.0) on DTLZ1-9 with 8 objectives. The last row is average ranking of DLEA on DTLZ1-9 under different α.

Problem #	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
DTLZ1	2.62E+00	1.74E+00	1.23E+00	1.06E+00	9.60E-01	7.51E-01	6.96E-01	6.86E-01	8.90E-01	7.88E-01
DTLZ2	5.16E-01	5.05E-01	4.95E-01	4.84E-01	4.89E-01	4.74E-01	4.71E-01	4.71E-01	4.69E-01	4.62E-01
DTLZ3	6.59E+01	4.60E+01	3.89E+01	2.93E+01	2.22E+01	1.71E+01	1.82E+01	1.92E+01	1.67E+01	1.52E+01
DTLZ4	6.37E-01	6.12E-01	5.92E-01	5.87E-01	5.81E-01	5.75E-01	5.77E-01	5.76E-01	5.58E-01	5.43E-01
DTLZ5	3.49E-01	3.58E-01	3.39E-01	3.29E-01	3.37E-01	3.34E-01	3.31E-01	3.05E-01	3.18E-01	3.17E-01
DTLZ6	5.46E+00	4.14E+00	3.10E+00	2.34E+00	1.78E+00	1.68E+00	1.11E+00	9.06E-01	9.96E-01	8.66E-01
DTLZ7	2.98E+00	2.34E+00	1.92E+00	1.70E+00	1.45E+00	1.69E+00	1.57E+00	1.34E+00	1.19E+00	1.35E+00
DTLZ8	4.95E-01	4.96E-01	4.76E-01	4.77E-01	4.54E-01	4.77E-01	4.54E-01	4.43E-01	4.52E-01	4.57E-01
DTLZ9	1.41E+01	1.27E+01	1.20E+01	1.11E+01	1.07E+01	1.02E+01	1.01E+01	9.57E+00	9.42E+00	9.13E+00
Rank	9.78	9.22	7.78	6.56	5.67	4.89	4.11	2.44	2.44	2.11

worse than α = 0.9. However, when the value of α is in the interval [0.8, 0.87] or the interval [0.95, 1], the performance of DLEA is relatively poor.

In this paper, in order to ensure optimal performance of DLEA, α = 0.9 is selected as the parameter setting of subsequent experiments.

4.3. Experiments on DLEA with different strategies

In order to analyze the effect of each part in DLEA on the results, this paper adjusts each part to obtain four versions of DLEA. As shown in Table 6, *gr* in the second column is an additional parameter which is used to control the proportion of the convergence-related solutions. When this parameter is used, the calculation of C_n is as follows:

$$C_n = \left[gr \times nd_i \times \alpha \times \left(1 - \frac{gen}{max\,gen}\right)\right] \tag{6}$$

where *gr* is set to the golden ratio. Since the golden ratio is used as the dividing standard in many fields, here we try to add *gr* to control the proportion of convergence-related individuals. The third column is the adjustment of parameter α. The parameter α is not used in DLEA1, and it is recommended that α = 0.9 in DLEA2-4. In the fourth column, the selection criteria of

Table 5
Average IGD values of DLEA (α = 0.1, 0.2, ..., 1.0) on DTLZ1-9 with 10 objectives. The last row is average ranking of DLEA on DTLZ1-9 under different α.

Problem #	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
DTLZ1	3.17E+00	2.34E+00	1.68E+00	1.76E+00	1.37E+00	1.26E+00	1.21E+00	9.42E-01	8.60E-01	9.55E-01
DTLZ2	6.77E-01	6.31E-01	6.30E-01	6.06E-01	5.94E-01	5.95E-01	5.93E-01	5.78E-01	5.79E-01	5.75E-01
DTLZ3	9.97E+01	6.87E+01	4.97E+01	3.52E+01	3.05E+01	2.29E+01	2.18E+01	1.81E+01	1.70E+01	1.89E+01
DTLZ4	7.94E-01	7.66E-01	7.65E-01	7.35E-01	7.17E-01	7.10E-01	7.00E-01	6.88E-01	6.78E-01	6.55E-01
DTLZ5	4.11E-01	4.05E-01	4.19E-01	4.36E-01	3.80E-01	4.20E-01	3.89E-01	4.06E-01	3.84E-01	4.14E-01
DTLZ6	6.73E+00	5.36E+00	5.49E+00	4.21E+00	3.89E+00	2.57E+00	2.28E+00	2.23E+00	1.63E+00	1.46E+00
DTLZ7	8.22E+00	4.99E+00	4.03E+00	3.36E+00	2.86E+00	2.73E+00	2.59E+00	2.26E+00	2.28E+00	2.34E+00
DTLZ8	6.37E-01	6.03E-01	6.39E-01	5.91E-01	5.93E-01	5.82E-01	6.00E-01	5.58E-01	5.74E-01	5.67E-01
DTLZ9	1.88E+01	1.72E+01	1.63E+01	1.55E+01	1.48E+01	1.45E+01	1.40E+01	1.3452e+1	1.33E+01	1.30E+01
Rank	9.44	8.22	8.22	7.22	5.33	5.44	4.22	2.44	2.00	2.44

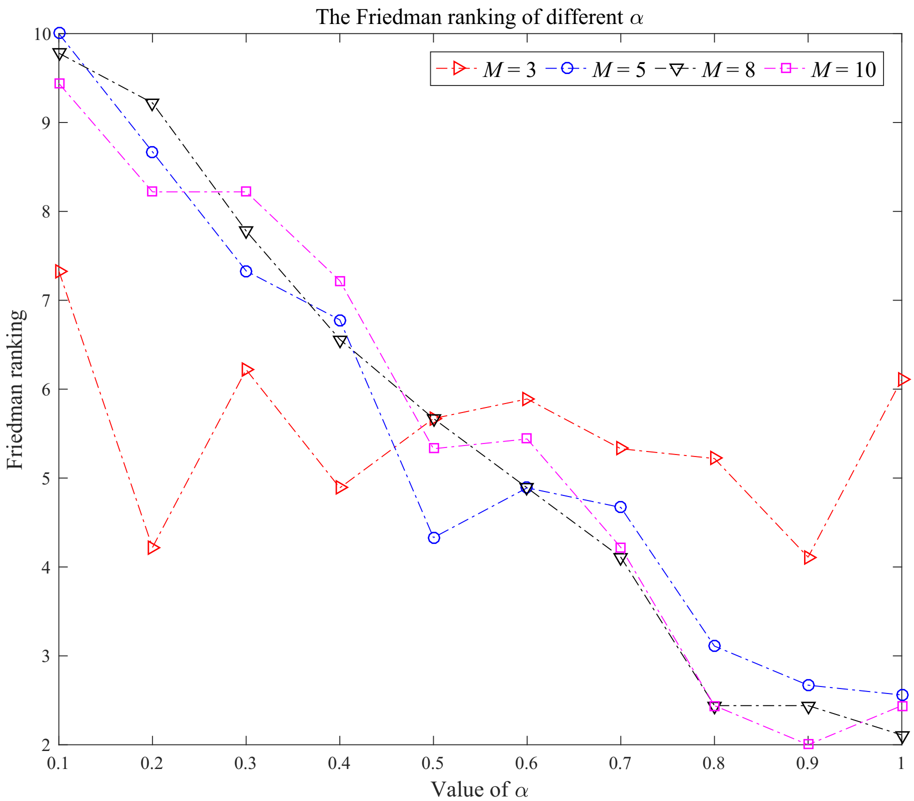

Fig. 2. Friedman test results for different α (α = 0.1, 0.2, ..., 1.0) under different objective numbers. The overall trend is that as the value of convergence factor α increases, the value of Friedman ranking becomes smaller (the result is better).

Table 6
DLEA using different strategies, where *gr* is an additional parameter to control the proportion of the selected convergence-related solution.

Algorithm	*gr*	α	Tournament Selection	Convergence	Diversity
DLEA1	TRUE	FALSE	*FrontNo*	$I_{\epsilon+}$, choose min individual	L_p-norm-based
DLEA2	FALSE	0.9	$I_{\epsilon+}$	$I_{\epsilon+}$, delete max individual	L_p-norm-based
DLEA3	FALSE	0.9	$I_{\epsilon+}$	$I_{\epsilon+}$, choose min individual	L_p-norm-based
DLEA4	TRUE	0.9	$I_{\epsilon+}$	$I_{\epsilon+}$, choose min individual	L_p-norm-based

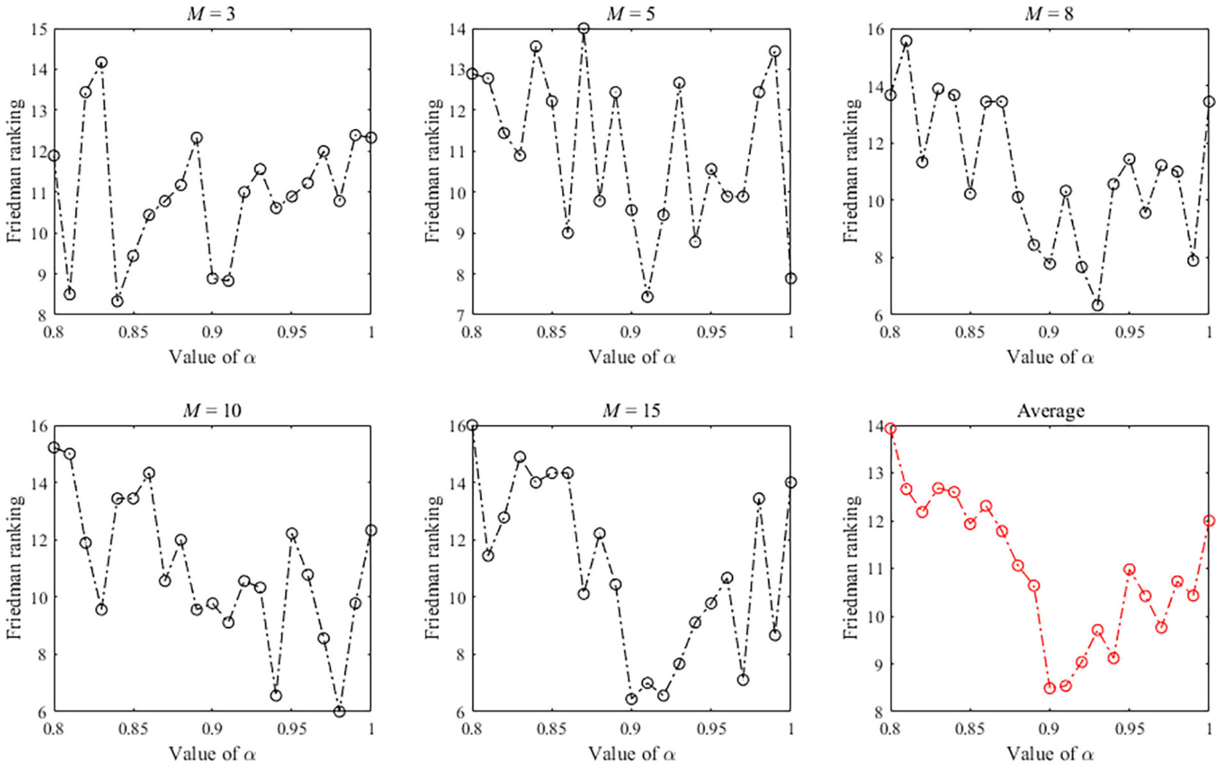

Fig. 3. Friedman test results for different α (α = 0.8, 0.81, . . ., 1.0) under different objective numbers. All curves are concave in general and reach the lowest point (the result is better) when α = 0.9, especially the average result.

Table 7
Average IGD values of DLEA1-4 on 3-objective DTLZ test suite.

Problem #	DLEA1	DLEA2	DLEA3	DLEA4
DTLZ1	2.35E-01	2.71E-01	2.17E-01	2.07E-01
DTLZ2	8.08E-02	3.63E-01	7.71E-02	7.62E-02
DTLZ3	7.62E+00	7.47E+00	6.30E+00	7.79E+00
DTLZ4	2.31E-01	3.27E-01	1.85E-01	2.11E-01
DTLZ5	9.96E-03	1.71E-01	9.41E-03	1.04E-02
DTLZ6	3.73E-02	3.38E-01	9.61E-03	9.33E-03
DTLZ7	1.23E-01	6.47E-01	8.79E-02	8.85E-02
DTLZ8	6.42E-02	1.91E-01	6.59E-02	6.94E-02
DTLZ9	8.73E-02	1.17E-01	1.04E-01	1.25E-01
Rank	2.44	3.67	1.56	2.33

tournament selection are adjusted, in which DLEA1 used the layer number obtained after the non-dominated ranking of individuals, while DLEA2-4 used the $I_{\varepsilon+}$ as the selection criteria. The fifth and sixth columns are respectively the indicators of selective convergence-related individuals and diversity-related individuals in environmental selection. The selection of diversity-related individuals is based on L_p-norm-based distance, while the selection of convergence-related individuals has two selection strategies. The first is to select C_n individuals with the lowest $I_{\varepsilon+}$. The second is to delete the individuals with the highest $I_{\varepsilon+}$ until the last C_n individuals are left. However, this strategy will calculate the cost of deletion after deleting the individual with the minimum $I_{\varepsilon+}$.

Table 8
Average DM results obtained by DLEA and 4 state-of-the-art MOEAs on 3-objective DTLZ test suite.

Problem	*M*	KnEA	hpaEA	I-DBEA	SPEA/R	DLEA
DTLZ1	3	5.9477e-1 (8.28e-2)	5.3286e-1 (8.55e-2)	4.5960e-1 (1.55e-1)	5.1766e-1 (9.44e-2)	7.0917e-1 (4.68e-2)
DTLZ2	3	5.5540e-1 (2.46e-2)	5.8539e-1 (1.92e-2)	5.6811e-1 (2.88e-2)	5.4506e-1 (5.12e-3)	7.7422e-1 (2.15e-2)
DTLZ3	3	2.8893e-1 (8.48e-2)	2.3035e-1 (1.02e-1)	2.0813e-1 (5.46e-2)	3.0200e-1 (1.18e-1)	3.2199e-1 (1.24e-1)
DTLZ4	3	5.5560e-1 (3.56e-2)	5.8932e-1 (1.78e-1)	4.9074e-1 (1.54e-1)	5.7399e-1 (1.88e-2)	7.4838e-1 (7.30e-2)
DTLZ5	3	5.9695e-1 (5.89e-2)	8.3749e-1 (1.43e-2)	5.5738e-1 (5.62e-2)	3.2606e-1 (2.85e-2)	8.7486e-1 (1.54e-2)
DTLZ6	3	5.9746e-1 (8.13e-2)	8.4529e-1 (1.38e-2)	4.8841e-1 (1.32e-1)	6.1228e-1 (2.80e-2)	8.6184e-1 (1.74e-2)
DTLZ7	3	6.8486e-1 (4.67e-2)	7.0389e-1 (1.68e-2)	6.3774e-1 (7.45e-2)	5.9562e-1 (4.15e-2)	8.0211e-1 (6.42e-2)

Table 9
Average HV results obtained by DLEA and 4 state-of-the-art MOEAs on 3-objective DTLZ test suite.

Problem	*M*	KnEA	hpaEA	I-DBEA	SPEA/R	DLEA
DTLZ1	3	1.2076e-1 (2.06e-2)	1.2563e-1 (3.45e-2)	8.0472e-2 (5.34e-2)	8.4410e-2 (4.92e-2)	1.2578e-1 (2.46e-2)
DTLZ2	3	7.1173e-1 (6.19e-3)	7.4061e-1 (9.79e-4)	7.3665e-1 (1.81e-3)	7.3800e-1 (2.50e-4)	7.0272e-1 (5.21e-3)
DTLZ3	3	1.1003e-2 (4.92e-2)	6.1863e-3 (1.96e-2)	0.0000e+0 (0.00e+0)	2.0246e-4 (6.40e-4)	0.0000e+0 (0.00e+0)
DTLZ4	3	7.1492e-1 (4.18e-3)	5.7769e-1 (2.28e-1)	5.1005e-1 (2.09e-1)	7.3740e-1 (4.51e-4)	6.8386e-1 (7.69e-2)
DTLZ5	3	1.2418e-1 (6.07e-3)	1.3261e-1 (6.51e-5)	1.2603e-1 (1.56e-3)	1.2075e-1 (6.22e-4)	1.3241e-1 (1.36e-4)
DTLZ6	3	1.1904e-1 (7.98e-3)	1.3291e-1 (3.93e-5)	1.2513e-1 (3.42e-3)	1.1780e-1 (2.30e-3)	1.3302e-1 (3.36e-5)
DTLZ7	3	1.5974e+0 (6.81e-2)	1.5816e+0 (2.92e-2)	1.4263e+0 (3.00e-1)	1.5455e+0 (9.06e-3)	1.5329e+0 (6.93e-2)

Table 10
Average DM results obtained by DLEA and 4 state-of-the-art MOEAs on 5-objective DTLZ test suite.

Problem	*M*	KnEA	hpaEA	I-DBEA	SPEA/R	DLEA
DTLZ1	5	3.6705e-1 (8.80e-2)	4.0983e-1 (5.74e-2)	2.3837e-1 (7.18e-2)	3.5473e-1 (5.38e-2)	4.6674e-1 (4.16e-2)
DTLZ2	5	4.3652e-1 (1.60e-2)	5.1658e-1 (1.41e-2)	4.7294e-1 (2.32e-2)	4.5443e-1 (1.19e-2)	6.4734e-1 (1.37e-2)
DTLZ3	5	1.3660e-1 (3.02e-2)	1.3602e-1 (2.53e-2)	1.5478e-1 (3.56e-2)	1.3066e-1 (3.27e-2)	1.7789e-1 (7.51e-2)
DTLZ4	5	4.5446e-1 (2.37e-2)	5.0066e-1 (3.58e-2)	4.4576e-1 (9.15e-2)	4.8313e-1 (1.66e-2)	5.9235e-1 (2.34e-2)
DTLZ5	5	3.4275e-1 (2.56e-2)	3.6642e-1 (1.18e-2)	4.0917e-1 (1.19e-2)	3.1713e-1 (2.80e-2)	3.9227e-1 (1.98e-2)
DTLZ6	5	3.3102e-1 (3.12e-2)	3.7290e-1 (3.20e-2)	3.6259e-1 (3.02e-2)	2.9440e-1 (3.32e-2)	4.0426e-1 (2.10e-2)
DTLZ7	5	4.8837e-1 (5.68e-2)	5.2583e-1 (6.78e-2)	4.6351e-1 (6.68e-2)	3.9056e-1 (7.13e-2)	6.3942e-1 (7.01e-2)

After 30 independent runs, average IGD values of DLEA1-4 on 3-objective DTLZ test suite are recorded, as shown in Table 7. As can be seen in Table 7, the number of the best results of DLEA1 on 9 test problems is 2, 0, 4, and 3, respectively. The results of the Friedman test on DLEA1-4 are also shown in the last row. According to Friedman test, the top performers were DLEA3 (1.56 on average), followed by DLEA4 and DLEA1 (2.33 and 2.44 on average), and the worst performer is DLEA2 (3.67 on average). This indicates that compared with DLEA1, DLEA2, and DLEA4, using the strategy selected in DLEA3 can achieve the best performance. Based on this, DLEA3 is compared with other state-of-the-art MOEAs in the following experiments, and DLEA3 is referred to simply as DLEA below for convenience.

4.4. Comparative experiments

Tables 8–9 are the average diversity metric (DM) [32] and HV results of DLEA and 4 other state-of-the-art MOEAs running 30 times independently on 3-objective DTLZ test suite, respectively. From the results of DM values, DLEA performed the best

Table 11
Average HV results obtained by DLEA and 4 state-of-the-art MOEAs on 5-objective DTLZ test suite.

Problem	M	KnEA	hpaEA	I-DBEA	SPEA/R	DLEA
DTLZ1	5	3.6397e-2 (1.53e-2)	3.3820e-2 (1.97e-2)	1.9079e-2 (1.47e-2)	1.6880e-2 (1.62e-2)	4.6017e-2 (7.39e-4)
DTLZ2	5	1.2716e+0 (7.84e-3)	1.2810e+0 (4.38e-3)	1.2785e+0 (6.01e-3)	1.2858e+0 (2.07e-3)	1.1836e+0 (8.89e-3)
DTLZ3	5	0.0000e+0 (0.00e+0)	0.0000e+0 (0.00e+0)	0.0000e+0 (0.00e+0)	0.0000e+0 (0.00e+0)	1.4188e-2 (5.86e-2)
DTLZ4	5	1.2806e+0 (5.50e-3)	1.2298e+0 (7.05e-2)	1.1374e+0 (1.92e-1)	1.2788e+0 (3.37e-3)	1.1882e+0 (1.34e-2)
DTLZ5	5	2.5137e-3 (1.93e-3)	2.5328e-3 (9.48e-4)	8.4559e-3 (1.40e-4)	3.4144e-3 (7.14e-4)	6.6792e-3 (7.49e-4)
DTLZ6	5	1.5710e-3 (2.59e-3)	5.8034e-4 (1.06e-3)	3.7554e-3 (2.61e-3)	4.8614e-7 (1.54e-6)	6.6007e-3 (2.95e-4)
DTLZ7	5	2.2481e+0 (3.12e-2)	2.0824e+0 (5.58e-2)	2.1009e+0 (7.68e-2)	1.9528e+0 (4.81e-2)	2.1089e+0 (4.42e-2)

Table 12
Average DM results obtained by DLEA and 4 state-of-the-art MOEAs on 8-objective DTLZ test suite.

Problem	M	KnEA	hpaEA	I-DBEA	SPEA/R	DLEA
DTLZ1	8	2.7169e-1 (3.00e-2)	3.6413e-1 (5.67e-2)	2.1720e-1 (3.22e-2)	3.5734e-1 (5.18e-2)	3.8272e-1 (2.92e-2)
DTLZ2	8	1.6482e-1 (2.17e-2)	3.2537e-1 (2.45e-2)	2.0992e-1 (2.05e-2)	2.6250e-1 (2.29e-2)	4.3185e-1 (3.15e-2)
DTLZ3	8	1.1546e-1 (1.87e-2)	1.3347e-1 (2.54e-2)	5.2138e-2 (2.41e-2)	9.8757e-2 (1.36e-2)	2.3524e-1 (3.73e-2)
DTLZ4	8	1.9192e-1 (2.41e-2)	3.2194e-1 (2.71e-2)	2.3909e-1 (2.99e-2)	2.8448e-1 (3.70e-2)	2.9493e-1 (3.07e-2)
DTLZ5	8	3.0828e-1 (2.83e-2)	4.1056e-1 (2.04e-2)	1.0719e-2 (4.28e-5)	2.0584e-1 (1.21e-2)	3.4101e-1 (2.59e-2)
DTLZ6	8	2.2936e-1 (3.72e-2)	2.5969e-1 (2.75e-2)	2.4953e-1 (1.88e-1)	1.7570e-1 (9.49e-3)	3.3444e-1 (4.08e-2)
DTLZ7	8	6.0362e-1 (4.84e-2)	6.5194e-1 (6.37e-2)	5.6919e-1 (5.90e-2)	6.4713e-1 (6.34e-2)	6.3385e-1 (3.37e-2)

Table 13
Average HV results obtained by DLEA and 4 state-of-the-art MOEAs on 8-objective DTLZ test suite.

Problem	M	KnEA	hpaEA	I-DBEA	SPEA/R	DLEA
DTLZ1	8	1.3776e-3 (2.60e-3)	1.6312e-3 (2.67e-3)	5.8086e-4 (1.27e-3)	1.8755e-3 (2.45e-3)	7.5871e-3 (2.37e-4)
DTLZ2	8	1.8961e+0 (1.71e-2)	1.9043e+0 (3.61e-2)	1.9656e+0 (2.93e-3)	1.9480e+0 (6.32e-3)	1.6076e+0 (3.40e-2)
DTLZ3	8	0.0000e+0 (0.00e+0)	0.0000e+0 (0.00e+0)	1.3700e-2 (3.42e-2)	0.0000e+0 (0.00e+0)	3.2696e-1 (4.35e-1)
DTLZ4	8	1.9348e+0 (6.74e-3)	1.8795e+0 (4.93e-2)	1.7251e+0 (9.09e-2)	1.9406e+0 (7.12e-3)	1.5417e+0 (4.56e-2)
DTLZ5	8	3.5238e-6 (5.06e-6)	8.5857e-7 (2.39e-6)	7.5663e-6 (7.29e-6)	2.9080e-6 (5.65e-6)	8.9693e-6 (5.20e-6)
DTLZ6	8	0.0000e+0 (0.00e+0)	0.0000e+0 (0.00e+0)	1.1666e-5 (7.79e-6)	0.0000e+0 (0.00e+0)	5.3318e-6 (7.13e-6)
DTLZ7	8	1.3240e+0 (1.90e-1)	1.8263e+0 (9.70e-2)	2.2000e+0 (1.94e-1)	1.6796e+0 (1.40e-1)	1.7522e+0 (2.04e-1)

on all the test problems, followed by hpaEA, KnEA and SPEA/R performed equally on each of the seven test problems. The worst performer was I-DBEA, which performed worst on almost all problems. The results of HV showed that KnEA, hpaEA, and DLEA performed equally on the 3-objective DTLZ test suite, and all performed best on the two instances. Secondly, SPEA/R had the highest HV value on DTLZ4, but its overall performance was similar to that of I-DBEA.

Tables 10 and 11 are the average DM and HV results of DLEA and 4 other state-of-the-art MOEAs running 30 times independently on 5-objective DTLZ test suite, respectively. As can be seen from Table 10, DLEA achieved the best DM values on the 5-objective DTLZ test suite except for DTLZ5. I-DBEA obtained the best DM value on DTLZ5. However, in terms of overall

Table 14
Average DM results obtained by DLEA and 4 state-of-the-art MOEAs on 10-objective DTLZ test suite.

Problem	*M*	KnEA	hpaEA	I-DBEA	SPEA/R	DLEA
DTLZ1	10	2.8537e-1 (3.60e-2)	3.6349e-1 (3.53e-2)	1.9752e-1 (4.87e-2)	2.5597e-1 (6.14e-2)	2.8411e-1 (4.64e-2)
DTLZ2	10	1.8277e-1 (1.96e-2)	2.3910e-1 (2.13e-2)	1.5563e-1 (2.41e-2)	1.7154e-1 (1.61e-2)	3.9516e-1 (2.52e-2)
DTLZ3	10	1.1771e-1 (1.13e-2)	1.5541e-1 (2.28e-2)	4.1759e-2 (3.61e-4)	1.1242e-1 (1.38e-2)	1.3861e-1 (3.82e-2)
DTLZ4	10	2.1744e-1 (2.24e-2)	2.1545e-1 (2.21e-2)	1.7967e-1 (2.69e-2)	1.8293e-1 (1.94e-2)	2.2928e-1 (2.48e-2)
DTLZ5	10	2.5026e-1 (2.98e-2)	4.2844e-1 (1.53e-2)	7.2609e-3 (0.00e+0)	1.7416e-1 (6.36e-3)	2.9811e-1 (1.54e-2)
DTLZ6	10	1.3730e-1 (3.12e-2)	2.6056e-1 (2.43e-2)	7.2609e-3 (0.00e+0)	1.6247e-1 (6.50e-3)	3.0823e-1 (3.30e-2)
DTLZ7	10	6.4138e-1 (1.13e-2)	6.2249e-1 (3.28e-2)	8.5420e-1 (4.20e-2)	6.0845e-1 (2.51e-2)	8.3639e-1 (5.94e-2)

Table 15
Average HV results obtained by DLEA and 4 state-of-the-art MOEAs on 10-objective DTLZ test suite.

Problem	*M*	KnEA	hpaEA	I-DBEA	SPEA/R	DLEA
DTLZ1	10	0.0000e+0 (0.00e+0)	6.4433e-6 (2.22e-5)	8.4379e-4 (7.91e-4)	6.0895e-7 (1.91e-6)	2.0276e-3 (3.50e-4)
DTLZ2	10	2.4718e+0 (1.17e-2)	2.3470e+0 (4.47e-2)	2.4401e+0 (1.55e-1)	2.4396e+0 (1.22e-2)	2.1482e+0 (2.81e-2)
DTLZ3	10	0.0000e+0 (0.00e+0)	0.0000e+0 (0.00e+0)	1.0391e-1 (9.02e-2)	0.0000e+0 (0.00e+0)	3.5599e-2 (9.88e-2)
DTLZ4	10	2.4921e+0 (5.71e-3)	2.2689e+0 (7.92e-2)	2.4116e+0 (5.42e-2)	2.4075e+0 (2.71e-2)	2.0267e+0 (5.11e-2)
DTLZ5	10	9.8928e-9 (1.47e-8)	3.1415e-10 (1.40e-9)	0.0000e+0 (0.00e+0)	0.0000e+0 (0.00e+0)	2.4476e-8 (2.06e-8)
DTLZ6	10	0.0000e+0 (0.00e+0)	0.0000e+0 (0.00e+0)	3.5136e-9 (1.41e-8)	0.0000e+0 (0.00e+0)	7.3531e-9 (1.63e-8)
DTLZ7	10	1.1273e+0 (3.67e-1)	1.2910e+0 (1.96e-1)	2.1133e+0 (1.38e-1)	1.3783e+0 (2.48e-1)	1.8844e+0 (2.94e-1)

Table 16
Average DM results obtained by DLEA and 4 state-of-the-art MOEAs on 15-objective DTLZ test suite.

Problem	*M*	KnEA	hpaEA	I-DBEA	SPEA/R	DLEA
DTLZ1	15	3.6253e-1 (2.35e-2)	3.4274e-1 (3.25e-2)	2.1103e-1 (3.81e-2)	2.5708e-1 (4.57e-2)	2.7746e-1 (2.15e-2)
DTLZ2	15	1.0951e-1 (1.37e-2)	2.5568e-1 (2.05e-2)	7.6836e-2 (8.52e-3)	1.2409e-1 (1.23e-2)	3.9954e-1 (1.11e-2)
DTLZ3	15	2.7175e-1 (2.30e-2)	2.7306e-1 (3.09e-2)	7.7591e-2 (6.57e-3)	1.3998e-1 (1.18e-2)	2.0792e-1 (2.70e-2)
DTLZ4	15	1.2651e-1 (2.03e-2)	1.7027e-1 (1.49e-2)	1.2314e-1 (2.24e-2)	1.2929e-1 (1.16e-2)	2.1240e-1 (2.95e-2)
DTLZ5	15	6.8917e-2 (4.88e-2)	4.5021e-1 (2.32e-2)	1.3923e-2 (2.62e-5)	1.6182e-1 (6.88e-3)	2.2662e-1 (4.49e-2)
DTLZ6	15	1.2274e-1 (2.35e-2)	2.9735e-1 (2.21e-2)	1.3952e-2 (8.69e-5)	9.0258e-2 (6.36e-3)	1.5412e-1 (5.95e-2)
DTLZ7	15	8.6543e-1 (1.84e-2)	9.0077e-1 (1.30e-2)	8.7313e-1 (0.00e+0)	9.1210e-1 (1.21e-2)	8.9579e-1 (1.57e-2)

effect, hpaEA has the best performance except for DLEA. KnEA and SPEA/R are similar in performance. As you can see from Table 11, DLEA got the highest HV values on the three instances. The second is KnEA that obtained the maximum HV value on the two instances. IBDEA and SPEA/R obtained the maximum HV values on DTLZ5 and DTLZ2, respectively. Although hpaEA did not get the maximum HV result on any one instance, its overall performance was better than I-DBEA and SPEA/R.

Tables 12 and 13 are the average DM and HV results of DLEA and 4 other state-of-the-art MOEAs running 30 times independently on 8-objective DTLZ test suite, respectively. It can be seen from Tables 12 and 13 that DLEA performs best under the measurement of two indicators, obtaining the optimal DM results on 4 instances and the optimal HV results on 3 instances, respectively. The hpaEA got the best DM on three instances, and I-DBEA got the best HV on three instances. However, the performance of SPEA/R is relatively poor.

Table 17
Average HV results obtained by DLEA and 4 state-of-the-art MOEAs on 15-objective DTLZ test suite.

Problem	M	KnEA	hpaEA	I-DBEA	SPEA/R	DLEA
DTLZ1	15	0.0000e+0 (0.00e+0)	5.7614e-5 (5.56e-5)	3.2486e-6 (1.42e-5)	1.6193e-5 (3.35e-5)	1.8516e-5 (2.21e-5)
DTLZ2	15	4.0966e+0 (9.27e-3)	3.5624e+0 (1.30e-1)	3.7975e-1 (1.38e-7)	4.1188e+0 (6.45e-3)	2.5415e+0 (1.55e-1)
DTLZ3	15	0.0000e+0 (0.00e+0)	0.0000e+0 (0.00e+0)	3.6611e-1 (1.07e-2)	0.0000e+0 (0.00e+0)	3.2227e-1 (1.39e-1)
DTLZ4	15	4.1018e+0 (1.05e-2)	4.0052e+0 (5.13e-2)	3.7231e+0 (1.14e+0)	4.1095e+0 (6.63e-3)	1.9180e+0 (3.45e-1)
DTLZ5	15	9.2744e-18 (2.05e-17)	1.1401e-18 (5.10e-18)	0.0000e+0 (0.00e+0)	0.0000e+0 (0.00e+0)	2.3127e-17 (2.90e-17)
DTLZ6	15	0.0000e+0 (0.00e+0)	0.0000e+0 (0.00e+0)	0.0000e+0 (0.00e+0)	0.0000e+0 (0.00e+0)	1.8143e-17 (3.42e-17)
DTLZ7	15	2.1529e-2 (6.40e-2)	2.0000e+0 (1.29e-1)	1.4239e+0 (5.40e-1)	1.7818e+0 (2.42e-1)	1.8519e+0 (2.36e-1)

Tables 14 and 15 are the average DM and HV results of DLEA and 4 other state-of-the-art MOEAs running 30 times independently on 10-objective DTLZ test suite, respectively. It can be seen from Tables 14 and 15 that DLEA performs best under the measurement of two indicators, and obtains the optimal DM results and the optimal HV results on three instances. In addition to DLEA, hpaEA and I-DBEA performed better, in which hpaEA got the best DM on the three instances and I-DBEA got the best DM on DTLZ7. Both I-DBEA and KnEA obtained the maximum HV on two instances.

Tables 16 and 17 are the average DM and HV results of DLEA and 4 other state-of-the-art MOEAs running 30 times independently on 15-objective DTLZ test suite, respectively. As can be seen from Table 16, hpaEA achieved the best DM on three instances on the 15-objective DTLZ test suite. The second is DLEA, which has the best DM on the two instances. The optimal DM results were obtained by KnEA and SPEA/R on DTLZ1 and DTLZ7, respectively. As can be seen from Table 17, hpaEA, SPEA/R, and DLEA respectively obtained the optimal HV results on the two instances, while I-DBEA also obtained the maximum HV on DTLZ3. By these two measures, DLEA and hpaEA performed best, while I-DBEA and KnEA performed relatively poorly.

Based on the results of DM and HV on the 3, 5, 8, 10, and 15-objective DTLZ test suite, DLEA performs best among several MOEAs, regardless of whether the number of objectives is low (MOPs) or high (MaOPs). hpaEA performs well in these two indicators, which is largely related to the evolution based on hypervolume. The results of KnEA, I-DBEA, and SPEA/R have competitiveness in HV results, but there is almost no advantage in DM results, which indicates that these algorithms are not strong in maintaining diversity. This has a great relationship with the characteristics of three algorithms, and the specific reasons will be analyzed below.

From the characteristics of the problem, both DTLZ1 and DTLZ3 have local Pareto-optimal front, which may cause MOEAs to fall into these local Pareto-optimal front. However, according to the experimental results, DLEA obtains the best DM on most DTLZ1 and DTLZ3, and the best HV on DTLZ1. This indicates that DLEA has certain advantages when dealing with MOPs/MaOPs with local Pareto-optimal front. Maybe because $I_{\varepsilon+}$ is used to select the convergence-related solution, and the L_p-norm-based diversity maintenance mechanism can make the solution distribution more uniform. This has advantages over using reference point or knee point in dealing with such problems.

The Pareto-optimal front of DTLZ2 is a part of a sphere in the quadrant, uniformly distributed and without local optimal Pareto-optimal front. As can be seen from the results in Tables 8–17, DLEA obtained the best DM on all instances of DTLZ2. This shows that DLEA has a good ability to preserve diversity in dealing with problems of uniform distribution, which depends on the L_p-norm-based diversity maintenance mechanism in DLEA. However, at the same time, the HV result is relatively poor, because the solution selected by $I_{\varepsilon+}$ is centrally distributed in a certain part in evolution. In addition, I-DBEA and SPEA/R performed better on HV on DTLZ2. In the process of evolution, the decomposition-based method in I-DBEA leads the

Table 18
Comparison of runtime (s) among the five algorithms on 3-, 5-, 8-, 10-, and 15-objective DTLZ2.

Problem	M	KnEA	I-DBEA	hpaEA	SPEA/R	DLEA
DTLZ2	3	8.4951e-1 (2.52e-1)	9.3675e + 0 (1.38e-1)	1.0896e + 0 (9.30e-2)	3.3234e + 0 (3.74e-2)	5.0654e + 0 (6.02e-2)
	5	9.5127e-1 (8.13e-3)	1.0383e + 1 (1.32e-1)	1.2577e + 0 (1.15e-2)	3.3014e + 0 (9.30e-3)	5.1105e + 0 (1.36e-2)
	8	1.0895e + 0 (9.87e-3)	1.0492e + 1 (1.21e-1)	1.4116e + 0 (1.79e-2)	2.9415e + 0 (1.17e-2)	5.1721e + 0 (1.04e-2)
	10	1.1384e + 0 (1.27e-2)	9.3445e + 0 (3.35e-1)	1.4158e + 0 (2.25e-2)	2.7958e + 0 (1.34e-2)	5.2056e + 0 (1.36e-2)
	15	1.2756e + 0 (8.68e-2)	8.7883e + 0 (7.15e-1)	1.6180e + 0 (1.33e-1)	2.0690e + 0 (1.80e-1)	5.4031e + 0 (1.84e-1)
Rank		1	5	2	3	4

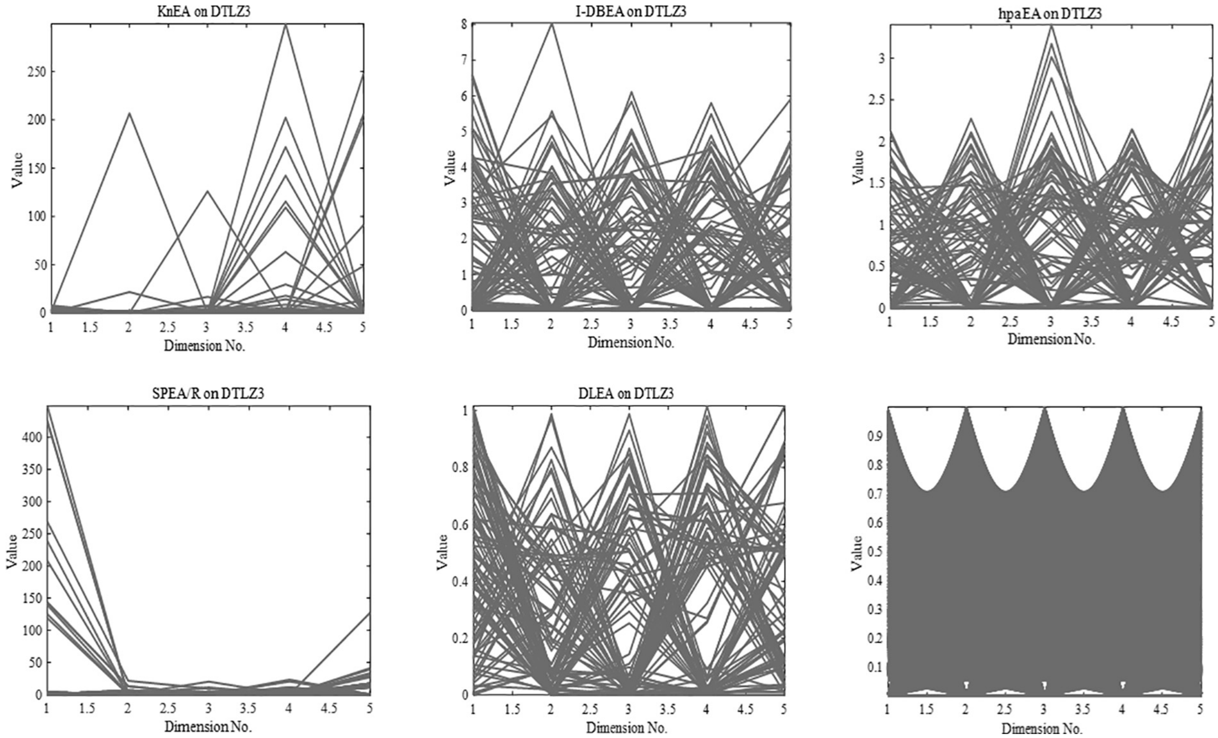

Fig. 4. The final solution set obtained by the five MOEAs on 5-objective DTLZ3, shown by parallel coordinates. The last subfigure is the Pareto-optimal front of 5-objective DTLZ3.

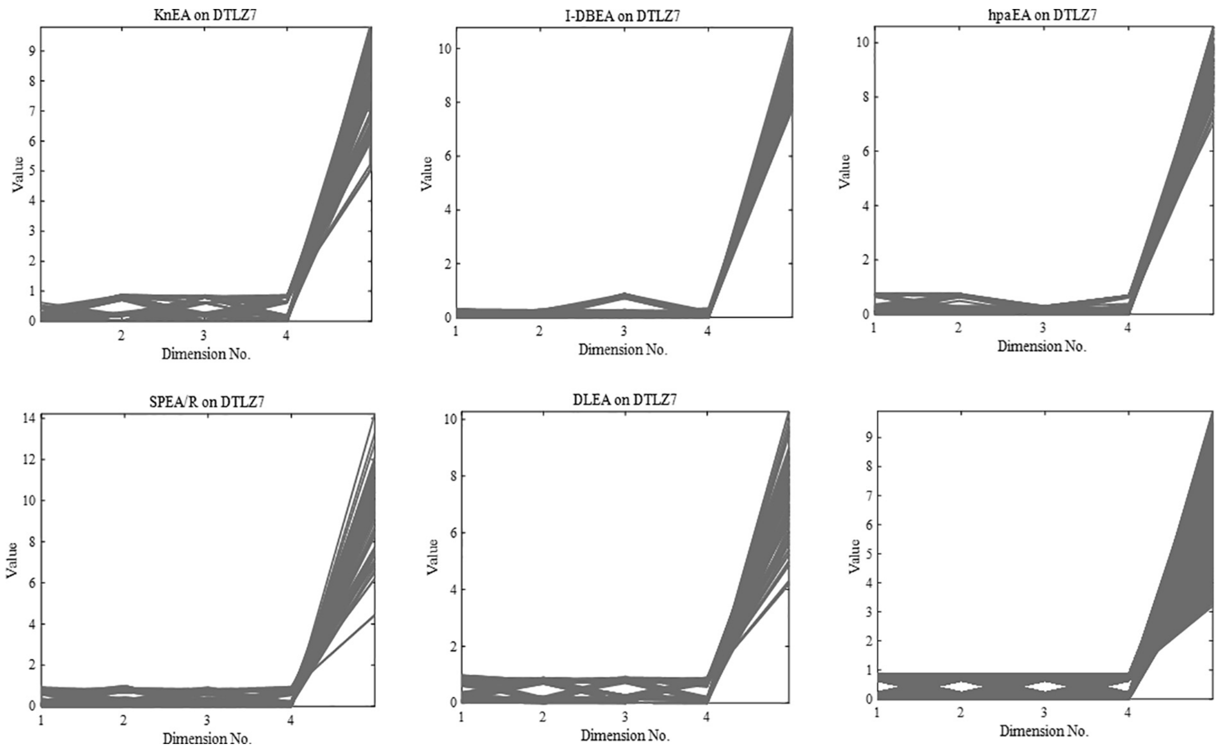

Fig. 5. The final solution set obtained by the five MOEAs on 5-objective DTLZ7, shown by parallel coordinates. The last subfigure is the Pareto-optimal front of 5-objective DTLZ7.

Table 19
Average DM results obtained by DLEA and 4 state-of-the-art MOEAs on 3-objective MaF test suite.

Problem	*M*	KnEA	I-DBEA	hpaEA	SPEA/R	DLEA
MaF1	3	6.6315e-1 (2.23e-2)	4.8486e-1 (1.80e-2)	6.4524e-1 (2.79e-2)	3.7442e-1 (4.38e-2)	7.1885e-1 (1.71e-2)
MaF2	3	7.2193e-1 (2.42e-2)	6.5734e-1 (2.18e-2)	6.7117e-1 (2.79e-2)	5.6697e-1 (3.01e-2)	7.6828e-1 (1.75e-2)
MaF3	3	1.2764e-1 (8.46e-2)	1.2951e-1 (5.93e-2)	1.0760e-1 (5.02e-2)	9.2771e-2 (4.95e-2)	1.5219e-1 (1.04e-1)
MaF4	3	2.3699e-1 (1.17e-1)	1.5066e-1 (7.15e-2)	2.8464e-1 (1.27e-1)	2.2270e-1 (1.28e-1)	3.5866e-1 (1.60e-1)
MaF5	3	5.4412e-1 (6.79e-2)	5.3702e-1 (8.11e-2)	5.8150e-1 (2.23e-1)	5.7204e-1 (1.36e-2)	7.1747e-1 (1.18e-1)
MaF6	3	4.0861e-1 (8.19e-2)	5.2289e-1 (1.64e-1)	8.5130e-1 (1.48e-2)	4.5160e-1 (7.20e-2)	8.6447e-1 (1.50e-2)
MaF7	3	6.7417e-1 (5.93e-2)	6.2765e-1 (8.66e-2)	6.9582e-1 (2.46e-2)	5.7946e-1 (3.33e-2)	8.4106e-1 (4.02e-2)
MaF8	3	2.8201e-1 (1.50e-1)	6.9219e-1 (1.58e-1)	4.8717e-1 (1.49e-1)	6.0000e-1 (1.29e-1)	6.5883e-1 (7.12e-2)
MaF9	3	2.0217e-1 (1.20e-1)	4.4576e-1 (5.65e-2)	4.9894e-1 (9.58e-2)	2.0300e-1 (6.96e-2)	5.1329e-1 (9.38e-2)
MaF10	3	5.8487e-1 (2.75e-2)	5.4837e-1 (2.39e-2)	3.4697e-1 (1.76e-1)	5.3051e-1 (3.51e-2)	6.1138e-1 (3.15e-2)
MaF11	3	5.4049e-1 (2.54e-2)	5.5131e-1 (2.07e-2)	5.3671e-1 (2.42e-2)	5.7119e-1 (1.41e-2)	6.3824e-1 (2.08e-2)
MaF12	3	6.7452e-1 (1.94e-2)	6.7728e-1 (2.33e-2)	6.6709e-1 (1.11e-2)	6.5022e-1 (1.51e-2)	7.7917e-1 (2.60e-2)
MaF13	3	4.5737e-1 (4.89e-2)	5.4173e-1 (7.12e-2)	6.2381e-1 (5.22e-2)	5.2976e-1 (4.53e-2)	5.6973e-1 (3.17e-2)
MaF14	3	1.3478e-1 (6.83e-2)	1.9102e-1 (6.43e-2)	1.1092e-1 (6.08e-2)	1.4601e-1 (5.35e-2)	1.2345e-1 (7.75e-2)
MaF15	3	2.5658e-1 (5.67e-2)	2.5194e-1 (6.17e-2)	3.6299e-1 (4.46e-2)	2.5547e-1 (3.75e-2)	2.9451e-1 (3.06e-2)

solution to converge to PF through uniform distribution of reference points, and it is easy to lead the improvement of diversity for such problems like DTLZ2 with uniform distributed PF. However, SPEA/R preserves the diversity of the solution set through a method based on the density estimation of the reference direction.

The Pareto-optimal front of DTLZ4 is part of the sphere of the first quadrant, but is not uniformly distributed, so it can be used to test the ability of MOEAs to keep the solutions well distributed. The results show that DLEA has the best DM on DTLZ4, indicating that the diversity preserving mechanism used in DLEA can effectively maintain the diversity of solutions. While the HV result of SPEA/R on DTLZ4 was the best, which was the effect of density estimation based on the reference direction. By contrast, the results of I-DBEA on both indicators are not very good. The reason is that the uniformly generated reference points in I-DBEA are not well adapted to such problems as DTLZ4. Due to the different densities of solutions in each region of PF, good diversity cannot be maintained by relying on uniformly distributed reference points.

The Pareto-optimal front of DTLZ5 and DTLZ6 are missing. For example, in the case of three objectives, their Pareto-optimal front is a curve instead of a surface. But they also have their own characteristics, it's harder for DTLZ6 to converge to PF than DTLZ5. From the results, no matter HV indicator or DM indicator, DLEA has the best comprehensive performance on these two problems. At the same time, hpaEA has shown some competition in some DTLZ5 test cases. However, DLEA has more advantages than hpaEA in dealing with DTLZ6 which is difficult to converge. KnEA, I-DBEA, and SPEA/R did not obtain promising results on DTLZ5 and DTLZ6.

The Pareto-optimal front of DTLZ7 is disconnected and contains 2^{M-1} Pareto-optimal front. This problem is mainly used to test the ability of MOEAs to hold solutions in multiple subregions. The results showed that DLEA had the best DM on 3 and 5-objective DTLZ7. At the same time, the results of DLEA were basically the second best on HV results. This shows that DLEA has some advantages in dealing with such a very complex PF problem. In addition, hpaEA and I-DBEA show similar performance on the other instances and can handle some problems like DTLZ7 with multiple subregions. KnEA also got the best HV results on the 3 and 5-objective DTLZ7. However, as the number of objectives increases, the performance also decreases, indicating that the performance of knee-point based guidance is poor when the number of objectives is large. Although the SPEA/R achieved the highest HV results on the 15-objective DLTZ7, the overall performance of the SPEA/R was almost the worst. This is because density estimation strategy based on reference direction in SPEA/R is not easy to determine the reference direction on PF with multiple subregions.

For DTLZ3, its PF belongs to the interval [0, 1] on each objective and contains some locally optimal Pareto fronts, so it will bring great difficulties to optimization process. Fig. 4 shows the optimal solution set obtained by each algorithm running in accordance with the same function evaluations on the 5-objective DTLZ3. It can be seen that the solution set obtained by DLEA algorithm has the best convergence and diversity. DLEA converges to PF for each objective and performs well in diversity maintenance. The results of hpaEA and I-DBEA showed good diversity maintenance, but neither of them could converge

Table 20
Average DM results obtained by DLEA and 4 state-of-the-art MOEAs on 5-objective MaF test suite.

Problem	M	KnEA	I-DBEA	hpaEA	SPEA/R	DLEA
MaF1	5	4.2115e-1 (5.42e-2)	3.4501e-1 (4.32e-2)	4.6012e-1 (5.66e-2)	1.9117e-1 (2.85e-2)	4.8634e-1 (3.55e-2)
MaF2	5	6.4040e-1 (2.52e-2)	5.5016e-1 (3.60e-2)	6.9985e-1 (2.19e-2)	5.4025e-1 (2.35e-2)	6.9149e-1 (3.33e-2)
MaF3	5	9.9236e-2 (3.17e-2)	1.1372e-1 (3.28e-2)	1.1438e-1 (2.99e-2)	1.0424e-1 (2.37e-2)	1.3839e-1 (3.06e-2)
MaF4	5	1.7232e-1 (1.44e-1)	9.6148e-2 (4.91e-2)	1.9397e-1 (1.36e-1)	8.4531e-2 (3.49e-2)	2.7777e-1 (1.33e-1)
MaF5	5	4.6396e-1 (2.83e-2)	4.1795e-1 (8.63e-2)	5.0046e-1 (4.07e-2)	4.9785e-1 (1.60e-2)	5.9014e-1 (2.50e-2)
MaF6	5	6.7801e-1 (1.90e-2)	4.6429e-1 (1.15e-1)	8.5751e-1 (1.25e-2)	3.8514e-1 (4.98e-2)	8.7680e-1 (1.56e-2)
MaF7	5	4.7049e-1 (3.96e-2)	4.7919e-1 (6.81e-2)	4.7515e-1 (6.87e-2)	3.4446e-1 (7.45e-2)	6.4882e-1 (5.98e-2)
MaF8	5	5.2261e-1 (6.18e-2)	6.4164e-1 (1.56e-1)	6.0692e-1 (5.55e-2)	6.0000e-1 (1.75e-1)	6.6133e-1 (2.66e-2)
MaF9	5	4.5903e-1 (7.94e-2)	3.8660e-1 (5.28e-2)	5.6971e-1 (7.74e-2)	1.6219e-1 (3.83e-2)	6.0461e-1 (6.36e-2)
MaF10	5	5.0262e-1 (3.14e-2)	4.4532e-1 (2.50e-2)	3.0965e-1 (8.36e-2)	4.2430e-1 (3.66e-2)	4.5316e-1 (4.47e-2)
MaF11	5	6.9485e-1 (1.91e-2)	6.6959e-1 (2.61e-2)	7.0663e-1 (1.57e-2)	6.6046e-1 (2.19e-2)	6.8502e-1 (1.50e-2)
MaF12	5	5.5982e-1 (2.21e-2)	5.6162e-1 (1.08e-2)	6.0410e-1 (9.58e-3)	5.6914e-1 (1.17e-2)	6.9435e-1 (1.21e-2)
MaF13	5	5.5734e-1 (3.16e-2)	4.3420e-1 (4.55e-2)	6.5954e-1 (1.41e-2)	3.5599e-1 (6.39e-2)	5.9033e-1 (2.46e-2)
MaF14	5	1.3726e-1 (3.14e-2)	1.3064e-1 (4.04e-2)	1.2271e-1 (2.54e-2)	1.4862e-1 (4.11e-2)	1.9706e-1 (5.36e-2)
MaF15	5	1.4934e-1 (3.81e-2)	3.1011e-2 (1.76e-2)	8.9811e-2 (2.94e-2)	8.3865e-2 (2.53e-2)	2.0885e-1 (2.03e-2)

Table 21
Average DM results obtained by DLEA and 4 state-of-the-art MOEAs on 8-objective MaF test suite.

Problem	M	KnEA	I-DBEA	hpaEA	SPEA/R	DLEA
MaF1	8	4.1004e-1 (3.17e-2)	2.7682e-1 (1.60e-2)	4.0677e-1 (3.27e-2)	2.8455e-1 (2.58e-2)	4.2215e-1 (3.88e-2)
MaF2	8	4.2306e-1 (2.67e-2)	5.1101e-1 (1.30e-2)	4.9957e-1 (2.58e-2)	3.3842e-1 (2.07e-2)	6.0844e-1 (1.36e-2)
MaF3	8	1.5062e-1 (1.97e-2)	7.6265e-2 (2.45e-2)	1.7120e-1 (2.47e-2)	1.5009e-1 (2.05e-2)	2.3588e-1 (7.11e-2)
MaF4	8	2.2033e-1 (1.66e-1)	2.7484e-1 (1.03e-1)	3.1547e-1 (1.86e-1)	1.0858e-1 (2.85e-2)	3.4942e-1 (1.54e-1)
MaF5	8	1.8865e-1 (2.18e-2)	2.3888e-1 (2.86e-2)	3.2895e-1 (2.92e-2)	3.1225e-1 (2.30e-2)	3.0371e-1 (2.95e-2)
MaF6	8	4.6655e-1 (3.05e-1)	1.3933e-1 (6.67e-2)	7.4569e-1 (2.52e-1)	1.8786e-1 (1.56e-1)	8.4620e-1 (1.71e-1)
MaF7	8	6.0516e-1 (4.56e-2)	5.6464e-1 (7.14e-2)	6.5005e-1 (5.80e-2)	6.9447e-1 (4.67e-2)	6.1662e-1 (5.89e-2)
MaF8	8	5.7373e-1 (2.34e-2)	4.9806e-1 (1.74e-1)	6.2982e-1 (3.86e-2)	6.2500e-1 (1.32e-1)	6.5027e-1 (1.14e-2)
MaF9	8	2.3903e-1 (2.42e-2)	3.6837e-1 (1.39e-1)	3.3411e-1 (8.46e-2)	8.9690e-2 (3.01e-2)	3.0215e-1 (1.22e-2)
MaF10	8	5.5697e-1 (5.28e-2)	4.6629e-1 (5.90e-2)	3.2676e-1 (6.13e-2)	5.0148e-1 (4.06e-2)	4.6861e-1 (3.87e-2)
MaF11	8	5.9861e-1 (3.16e-2)	6.1453e-1 (3.93e-2)	6.2895e-1 (2.03e-2)	5.9712e-1 (3.13e-2)	6.4057e-1 (2.35e-2)
MaF12	8	2.5155e-1 (1.87e-2)	3.2645e-1 (2.36e-2)	3.8272e-1 (3.81e-2)	3.4528e-1 (2.59e-2)	5.3975e-1 (2.67e-2)
MaF13	8	5.6532e-1 (3.20e-2)	2.6718e-2 (3.59e-2)	6.6189e-1 (2.05e-2)	1.7419e-1 (5.95e-2)	5.3331e-1 (2.14e-2)
MaF14	8	2.7202e-1 (3.75e-2)	2.7748e-1 (2.50e-1)	3.0177e-1 (3.82e-2)	2.9592e-1 (3.46e-2)	2.7124e-1 (3.35e-2)
MaF15	8	7.0379e-2 (3.43e-2)	6.0934e-2 (1.13e-2)	8.5268e-2 (1.68e-2)	3.9602e-2 (7.10e-3)	1.2922e-1 (2.20e-2)

Table 22
Average DM results obtained by DLEA and 4 state-of-the-art MOEAs on 10-objective MaF test suite.

Problem	M	KnEA	I-DBEA	hpaEA	SPEA/R	DLEA
MaF1	10	3.9355e-1 (3.32e-2)	2.3846e-1 (2.52e-2)	3.7161e-1 (3.20e-2)	2.1844e-1 (5.50e-2)	3.9138e-1 (2.64e-2)
MaF2	10	4.3750e-1 (2.85e-2)	2.1149e-2 (5.94e-3)	5.3676e-1 (1.80e-2)	4.8801e-1 (1.06e-2)	6.5788e-1 (8.09e-3)
MaF3	10	1.7981e-1 (2.65e-2)	7.6712e-2 (3.31e-2)	2.0464e-1 (2.68e-2)	1.7168e-1 (3.16e-2)	1.6340e-1 (3.89e-2)
MaF4	10	2.3595e-1 (1.57e-1)	3.7478e-1 (8.49e-2)	3.7773e-1 (1.10e-1)	9.3962e-2 (1.84e-2)	2.8131e-1 (1.33e-1)
MaF5	10	2.2606e-1 (1.72e-2)	1.7479e-1 (3.35e-2)	2.0254e-1 (2.09e-2)	1.7308e-1 (1.68e-2)	2.3220e-1 (1.94e-2)
MaF6	10	1.6421e-2 (8.29e-3)	2.1350e-2 (2.46e-2)	1.4564e-2 (7.97e-3)	1.4296e-1 (1.77e-2)	6.3112e-1 (3.81e-1)
MaF7	10	6.3934e-1 (4.73e-3)	8.3275e-1 (5.36e-2)	6.1425e-1 (2.52e-2)	5.9054e-1 (2.34e-2)	8.6441e-1 (6.30e-2)
MaF8	10	5.7031e-1 (8.78e-2)	7.0717e-1 (1.10e-1)	6.5325e-1 (1.15e-2)	6.3333e-1 (1.58e-1)	6.5373e-1 (1.18e-2)
MaF9	10	2.3018e-2 (2.16e-2)	4.7214e-1 (4.17e-2)	4.0536e-1 (1.16e-1)	7.9148e-2 (3.01e-2)	3.5554e-2 (6.33e-3)
MaF10	10	5.1767e-1 (2.11e-2)	4.4770e-1 (3.39e-2)	3.6338e-1 (4.46e-2)	3.9524e-1 (4.91e-2)	4.7167e-1 (3.22e-2)
MaF11	10	5.7861e-1 (2.73e-2)	2.7888e-1 (1.34e-1)	6.2001e-1 (2.54e-2)	4.9442e-1 (3.85e-2)	6.1906e-1 (2.35e-2)
MaF12	10	2.1337e-1 (1.96e-2)	2.0689e-1 (1.02e-1)	3.5517e-1 (2.25e-2)	2.8881e-1 (1.84e-2)	4.9563e-1 (2.24e-2)
MaF13	10	5.6590e-1 (4.10e-2)	1.1625e-2 (4.25e-3)	6.5893e-1 (3.02e-2)	1.4744e-1 (5.74e-2)	5.2922e-1 (2.04e-2)
MaF14	10	2.7741e-1 (2.76e-2)	1.5506e-1 (3.86e-2)	3.0519e-1 (3.50e-2)	2.5981e-1 (2.08e-2)	2.3903e-1 (3.41e-2)
MaF15	10	4.8995e-2 (1.53e-2)	7.3734e-2 (2.70e-2)	5.9303e-2 (1.55e-2)	3.7153e-2 (2.11e-3)	1.0632e-1 (3.15e-2)

Table 23
Average DM results obtained by DLEA and 4 state-of-the-art MOEAs on 15-objective MaF test suite.

Problem	M	KnEA	I-DBEA	hpaEA	SPEA/R	DLEA
MaF1	15	3.8541e-1 (2.99e-2)	1.0869e-1 (3.65e-2)	3.0345e-1 (2.06e-2)	1.9590e-1 (4.07e-2)	3.5820e-1 (1.48e-2)
MaF2	15	1.2257e-1 (1.82e-2)	3.0270e-2 (6.45e-4)	2.6885e-1 (1.76e-2)	3.9780e-1 (2.56e-2)	5.9415e-1 (1.07e-2)
MaF3	15	1.9533e-1 (2.56e-2)	1.1267e-1 (2.34e-2)	2.8733e-1 (3.32e-2)	2.4689e-1 (3.35e-2)	1.9890e-1 (3.21e-2)
MaF4	15	2.6026e-1 (8.42e-2)	1.0509e-1 (1.43e-2)	1.5440e-1 (1.84e-2)	1.6699e-1 (6.08e-2)	4.2247e-1 (3.36e-2)
MaF5	15	1.2381e-1 (1.68e-2)	1.2908e-1 (2.08e-2)	1.8320e-1 (2.36e-2)	1.5049e-1 (1.40e-2)	2.2069e-1 (2.44e-2)
MaF6	15	2.0443e-2 (1.08e-2)	9.8048e-2 (2.63e-1)	5.5504e-2 (1.80e-2)	1.0265e-1 (1.83e-2)	6.5340e-2 (5.02e-2)
MaF7	15	8.6271e-1 (1.64e-2)	8.7313e-1 (0.00e+0)	8.9851e-1 (1.40e-2)	9.2751e-1 (1.48e-2)	9.0213e-1 (1.46e-2)
MaF8	15	6.0844e-1 (1.35e-2)	1.2193e-1 (2.90e-2)	6.2221e-1 (6.84e-2)	4.3360e-1 (2.27e-1)	6.5440e-1 (1.30e-2)
MaF9	15	3.6667e-1 (1.54e-1)	3.8579e-2 (2.52e-2)	7.0208e-1 (6.06e-3)	8.5381e-2 (2.91e-2)	6.1285e-1 (3.54e-2)
MaF10	15	4.3261e-1 (2.64e-2)	2.2925e-1 (6.07e-2)	3.8337e-1 (4.58e-2)	1.2589e-1 (3.96e-3)	3.7137e-1 (2.90e-2)
MaF11	15	8.7223e-1 (2.79e-3)	8.7867e-1 (8.53e-3)	8.7156e-1 (4.26e-3)	9.0924e-1 (1.53e-2)	8.7087e-1 (4.03e-3)
MaF12	15	2.3655e-1 (4.13e-2)	8.1735e-2 (3.53e-3)	3.8523e-1 (1.80e-2)	2.7033e-1 (4.35e-2)	4.9441e-1 (2.05e-2)
MaF13	15	5.9792e-1 (3.34e-2)	2.7508e-2 (1.50e-2)	6.4374e-1 (2.74e-2)	1.0942e-1 (3.77e-2)	5.1215e-1 (1.31e-2)
MaF14	15	2.9343e-1 (3.13e-2)	3.9263e-1 (3.32e-1)	2.9165e-1 (2.78e-2)	2.4678e-1 (2.55e-2)	3.0335e-1 (2.32e-2)
MaF15	15	8.3058e-2 (1.86e-2)	9.7773e-2 (2.59e-2)	6.6988e-2 (1.35e-3)	6.4809e-2 (1.46e-17)	9.4350e-2 (1.36e-2)

to PF. This also shows the inefficiency of decomposition-based algorithms for such problems with some locally optimal Pareto front. In addition, both KnEA and SPEA/R have poor performance in convergence and diversity, and even do not converge to PF at all and have poor diversity.

DTLZ7 is a particular problem. Its PF is discontinuous and contains 2 $^{M-1}$ Pareto front subregions. This problem can well test the ability of the algorithm to preserve the solution in each subregion. Fig. 5 shows the optimal solution set obtained by the five algorithms running on the 5-objective DTLZ7 according to the same function evaluations. It can be seen from the above that every algorithm can converge to PF successfully, except that SPEA/R does not converge on the fifth objective. However, from the point of diversity of solution sets, the performance of the algorithms is quite different. Among them, the diversity of DLEA is the best, and it can maintain the diversity well on each objective. The second is SPEA/R. Although SPEA/R does not converge on the fifth objective, it has good convergence and diversity in the first four objectives, which is second only to DLEA. In addition, the diversity of KnEA is poor on the first objective and good on the other objectives. The worst performers in diversity are I-DBEA and hpaEA, both of which have poor diversity maintenance on each objective. According to the convergence and diversity of the obtained solutions and the simulation of PF, DLEA is the best among the six algorithms.

In addition, Table 18 shows the average runtime (the data in parentheses are standard deviations) of the five algorithms on DTLZ2. It can be seen from the results, the fastest is KnEA. This is because KnEA leads individuals to converge to the Pareto front through the knee points in the population without additional diversity maintenance mechanisms, so the time complexity is reduced. The runtime of hpaEA and SPEA/R is lower than DLEA. This is because DLEA will first implement a non-dominated ranking for the whole population, and then it needs to calculate the $I_{\varepsilon+}$ and L_p-norm-based distance of all individuals. On the other hand, I-DBEA runs the longest because it uses decomposition-based framework. However, DLEA has the best performance under the same population size and *FEs*. This indicates that using the same computational resources, DLEA contributes more to maintaining the convergence and diversity of the population.

Fig. 6 shows the memory footprint of the five MaOEAs, and it can be seen that DLEA has the smallest memory footprint. KnEA also has a smaller memory footprint because it has no additional diversity maintenance mechanisms. The memory footprint of hpaEA and SPEA/R is similar, but hpaEA is relatively small. The worst is I-DBEA, which has the largest memory footprint due to its decomposition-based framework.

4.5. Experiments on CEC 2018

In order to further analyze the performance of the five algorithms in dealing with many-objective optimization problems, this section will analyze the results of the five algorithms on the MaF (CEC 2018) test suite. In addition, MaF7 and DTLZ7 are

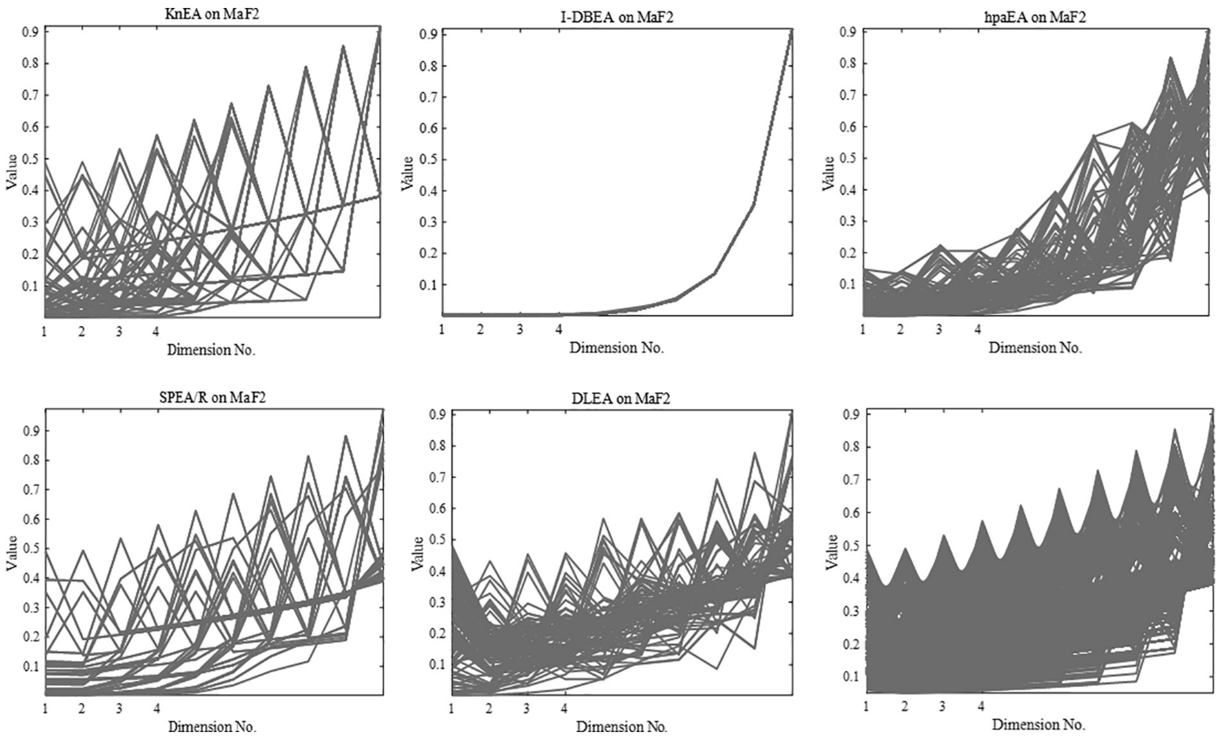

Fig. 7. The final solution set obtained by the five MOEAs on 10-objective MaF2, shown by parallel coordinates. The last subfigure is the Pareto-optimal front of 10-objective MaF2.

the same problem, but the results of MaF7 are also placed in the table for completeness. Finally, MaF8 is a multi-point distance minimization problem, used to calculate the Euclidean distance from a two-dimensional point $\times$ to a set of M two-dimensional vertices of a polygon, where M refers to the number of objectives. And then, the optimal solution set of each algorithm on MaF8 is also shown here using parallel coordinates in Fig. 9.

Tables 19–23 shows the DM results of the five algorithms on the 3, 5, 8, 10, and 15-objective MaF test suites. As can be seen from the DM results, DLEA has good DM results on most instances. In particular, the best DM results are achieved on 31 MaF test cases having 3, 5, and 8 objectives among 45 test cases. With the increase in the number of objectives, DLEA still holds certain competitiveness among the five algorithms. hpaEA showed the second-best performance, with 16 of the 75 instances and the best results on all instances of MaF13. KnEA and I-DBEA showed the similar performance, which have 6 and 7 best results, respectively. But the performance of the two algorithms was lower. Although SPEA/R only achieved four best results, its performance improved as the number of objectives increased. In particular, three of the best results were obtained on 15-objective MaF test cases. This shows that SPEA/R has the promise to handle MaOPs.

MaF2, also known as DTLZ2BZ, is a modification of DTLZ2. Fig. 7 shows the optimal solution set of the five algorithms on MaF2 and can be shown in parallel coordinates. It can be seen from the results that the optimal solution set obtained by each algorithm can converge to PF, but there are some differences in diversity maintenance. For example, DLEA does well in overall diversity, but it has less diversity on the lower part on each objective. The diversity of KnEA and SPEA/R remains similar, but is generally inferior to that of DLEA. hpaEA maintains poor diversity on the first five objectives and good diversity on the other objectives. However, I-DBEA performed poorly in diversity on all objectives and was the worst of the five algorithms.

Fig. 8 shows the optimal solution set obtained by the five algorithms on the 10-objective MaF5. MaF5 is concave badly-scaled DTLZ4, so it also has the characteristics of uneven PF distribution. From the results, the convergence of several algorithms on 10-objective MaF5 is similar. However, DLEA did not fully converge on the second objective, but it showed a very good diversity. Both KnEA and SPEA/R fully converge to PF, but have less diversity on each objective than DLEA. The diversity in the second objective of I-DBEA and hpaEA is relatively poor, especially I-DBEA, which hardly maintains the diversity in the second objective. Overall, the performance of DLEA on 10-objective MaF5 is still superior.

Fig. 9 shows the results of the optimal solution set obtained by the five algorithms on 10-objective MaF8. In terms of the results, SPEA/R had the worst performance on this problem, failing to converge on each objective and showing the worst diversity. The second one with poor performance is I-DBEA, which is distributed in several intervals [0.75, 1.25] on each objective, so the diversity is also poor. hpaEA performs better than KnEA which has good convergence but poor diversity. The convergence and diversity of hpaEA remains good, but not as good as that of DLEA.

In summary, DLEA still shows a competitive advantage over the other four comparative algorithms in the MaF test suite.

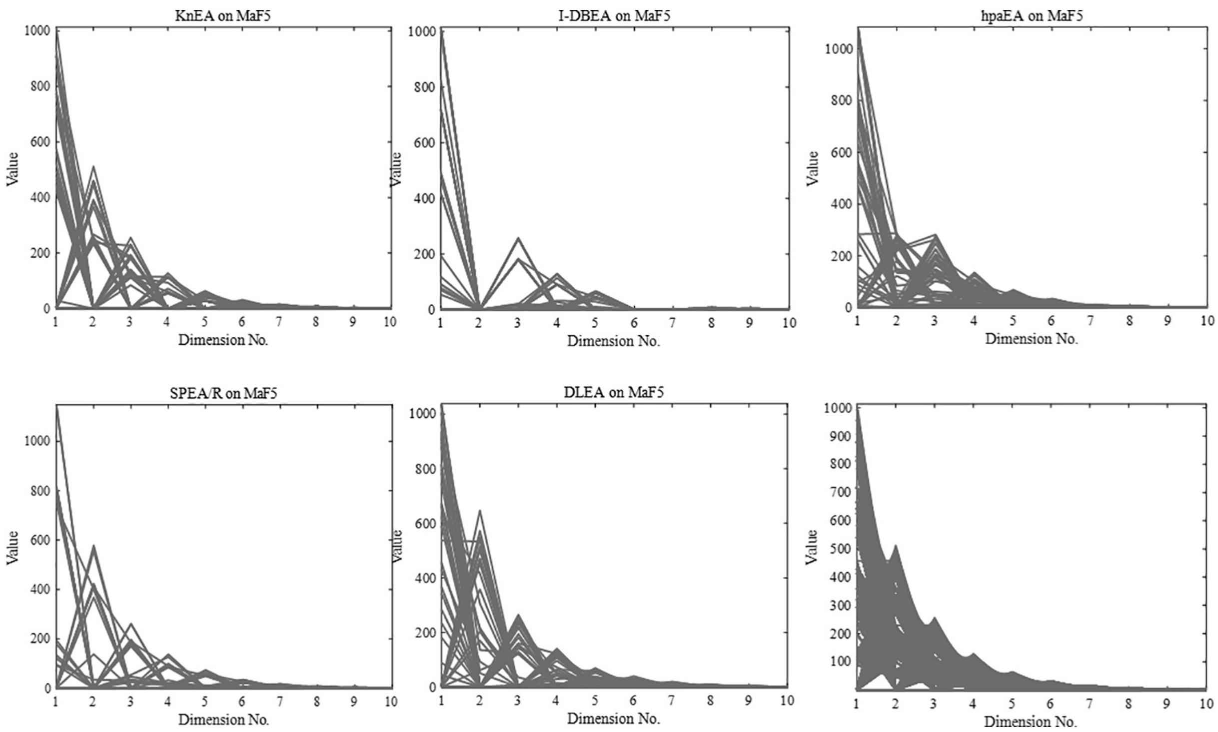

Fig. 8. The final solution set obtained by the five MOEAs on 10-objective MaF5, shown by parallel coordinates. The last subfigure is the Pareto-optimal front of 10-objective MaF5.

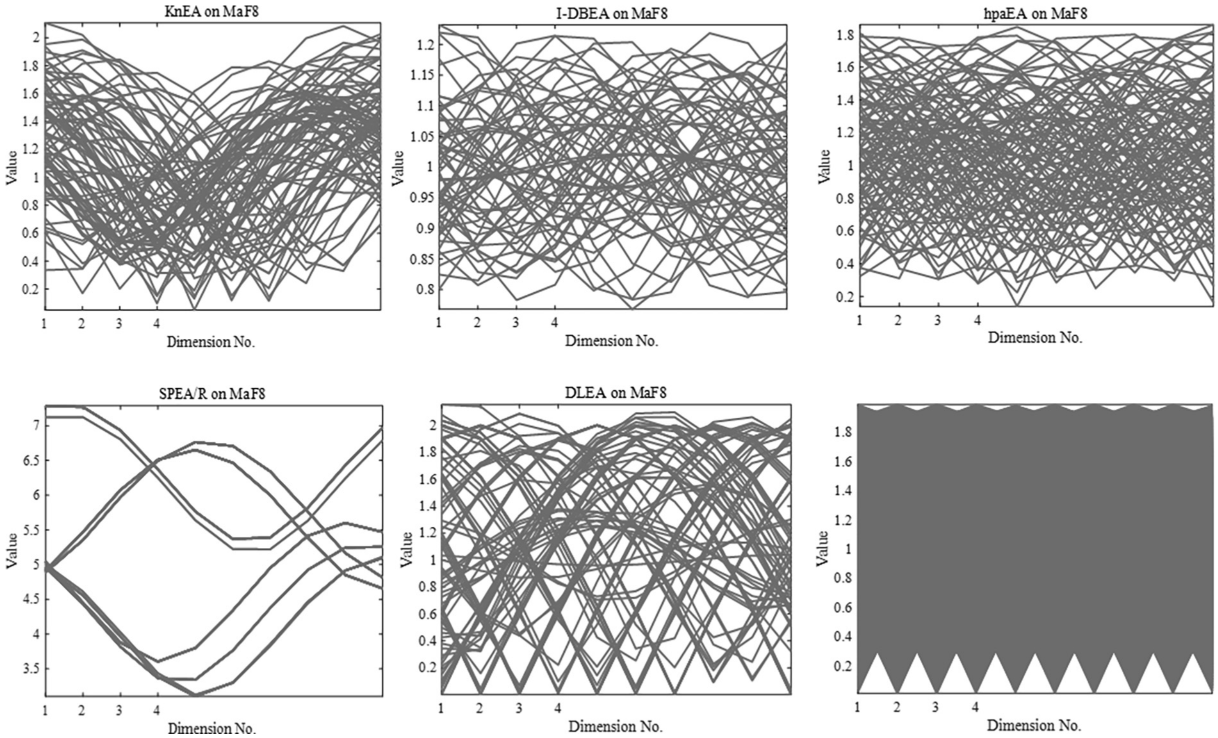

Fig. 9. The final solution set obtained by the five MOEAs on 10-objective MaF8, shown by parallel coordinates. The last subfigure is the Pareto-optimal front of 10-objective MaF8.

4.6. Experiments on many-objective combinatorial problems

In this part, DLEA will be used to deal with many-objective knapsack problems (MOKPs) to verify the ability of DLEA when dealing with many-objective combinatorial problems and compare it with the four state-of-the-art MOEAs. In this paper, five different objectives of MOKPs are set for comparative experiments. The HV results of each algorithm running on MOKP can be shown in Table 24.

The difference between MOKPs and traditional 0–1 knapsack problems is that the traditional 0–1 knapsack problems only need to satisfy the weight constraint to maximize the value of goods in knapsack. In addition to satisfying the weight constraints, MOKPs also need to consider the optimization of multiple objectives. Assuming a MOKP with two objectives, namely value and volume, which need to be optimized so that the goods can have a higher value and a smaller volume. In addition, each knapsack has p different value of goods in MOKP. Each objective function $f_i(x)$ calculated the total profit which depends on profit i (i = 1, 2, ..., p). This raises the question of how to maximize all objective functions while satisfying the weight constraint. Moreover, because the optimization direction of each objective conflict with each other, the PF of MOKPs is more complex, which poses a great challenge to the performance of MOEAs.

As can be seen from Table 24, DLEA performed best on five sets of MOKPs with different objectives compared with the four MOEAs. Compared with the other four algorithms, the HV results obtained by DLEA have great advantages. In addition,

Table 24
The HV results of DLEA and four state-of-the-art MOEAs run on MOKPs, where the two values are the mean and the standard deviation, the best results are shown in blue.

Problem	M	KnEA	I-DBEA	hpaEA	SPEA/R	DLEA
MOKP	3	1.0785e+12 (9.62e+9)	1.0613e+12 (1.28e+10)	1.0821e+12 (1.41e+10)	1.1024e+12 (1.52e+10)	1.0948e+12 (1.19e+10)
	5	8.7795e+19 (1.18e+18)	8.2530e+19 (1.38e+18)	8.5841e+19 (7.69e+17)	8.4131e+19 (1.17e+18)	8.7984e+19 (9.80e+17)
	8	6.5356e+31 (9.24e+29)	6.5604e+31 (9.38e+29)	6.6837e+31 (1.21e+30)	5.2963e+31 (1.77e+30)	6.8021e+31 (1.94e+30)
	10	5.3913e+39 (1.16e+38)	5.3278e+39 (7.64e+37)	5.6432e+39 (1.52e+38)	4.8449e+39 (1.02e+38)	5.7303e+39 (1.43e+38)
	15	1.6311e+59 (1.11e+58)	2.4273e+59 (6.40e+57)	1.8861e+59 (7.80e+57)	1.7072e+59 (4.92e+57)	2.1877e+59 (7.14e+57)

I-DBEA and SPEA/R obtained the highest HV results on 15-objective MOKP and 3-objective MOKP, respectively. But DLEA also obtained the second highest HV results on these two instances. At the same time, the results obtained by SPEA/R on all the examples except 3-objective MOKP were poor, followed by KnEA. The hpaEA performed well overall, second only to DLEA.

The possible reasons for such a result are as follows. For combinatorial MaOPs like MOKPs, the difficulty lies in how to generate as many effective solutions as possible, so that the final solution set can better represent the real PF. In other words, solution set shows a good advantage in both convergence and diversity. In these comparative algorithms, I-DBEA uses uniformly distributed reference points to guide convergence, which cannot well adapt to such complex combinatorial optimization problems. On the other hand, KnEA guides the evolution of other solutions in population by constantly identifying the knee points in the generated solution, but whether many knee points can be generated in the evolutionary process is unknown. Similarly, the reference direction of the SPEA/R seems to take little effect here, and the reason for the poor performance of hpaEA is similar to that of SPEA/R. While DLEA adapts to the evolutionary characteristics of the population by constantly changing the direction of evolution throughout the evolution process, so that the convergence and diversity of the finally obtained population are well maintained.

5. Conclusions

In recent years, in order to improve the performance of MOEAs when dealing with various characteristics of MOPs/MaOPs, researchers have proposed many MOEAs. However, these MOEAs fail to pay attention to the iterative process of the algorithm itself. In this paper, a dynamic learning multi-objective evolutionary algorithm namely DLEA is proposed. DLEA is able to accelerate convergence by paying more attention to convergence-related individuals in the early search process. In the late stage of the search, more attention is paid to the diversity-related individuals, so that the obtained individuals are more evenly distributed in PF. In addition, the parameter settings and strategy selection of DLEA are also demonstrated through experiments to achieve the best performance. Through comparing with four state-of-the-art MOEAs, it is found that DLEA has significant advantages no matter the number of objectives. In addition, the advantages of DLEA on solving combinatorial many-objective problems were also verified in this paper.

In addition, $I_{\varepsilon+}$ is used in this paper to maintain individual convergence and L_p-norm-based diversity maintenance mechanism is used to maintain diversity, but there are still many excellent strategies that can be used to maintain convergence and diversity, which are not studied in this paper. The orientation of future work can start from this point and be improved in the framework of DLEA to achieve better results. At present, new evolutionary algorithms based on agent model are gradually developed. It is hoped that the dynamic learning strategy can be used to improve the performance of the evolutionary algorithm in the future work. At the same time, using dynamic learning strategy to deal with the parameter adjustment of neural network is also worth working for.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grant No. 41576011, No. U1706218, No. 41706010, No. 41927805, and No. 61503165.

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