Supplementary Material for A Framework of Price Bidding Configurations for Resource Usage in Cloud Computing

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1 Notation Table

TABLE 1: Notations

Notation	Description
n	Number of cloud users
m	Number of servers in a zone in the cloud
$\mathcal{N}$	Set of the n cloud users
$\mathcal{M}$	Set of the m servers in the zone in the cloud
p_i	Bidding price of cloud user i
$\underline{p}$	Minimal bidding price for a server in one time slot
$\bar{p}_i$	Maximal possible bidding price of cloud user i
$\mathcal{P}_i$	The set of price bidding strategies of cloud user i
t_i	Reserved time slots of cloud user i
b_i	Bidding strategy of cloud user i
$\boldsymbol{b}$	Bidding strategy of all cloud users
$\boldsymbol{b}_{-i}$	Bidding strategy profile of all users except that of user i
λ_i^t	Request arrival rate of cloud user i in t-th time slot
$\boldsymbol{\lambda}_i^{t_i}$	User i's request profile over the t_i future time slots
m_i	Allocated number of servers for cloud user i
$\boldsymbol{m}$	Allocated server vector for all cloud users
μ_i	Processing rate of a server for requests from user i
$\bar{T}_i^t$	Average response time of cloud user i in t-th time slot
$\Xi_{\mathcal{S}}$	Aggregated payment from users in $\mathcal{S}$ for using a server
P_i^t	Payment of cloud users i in t-th time slot
P_T	Total payment from all cloud users
r_i	Benefit obtained by user i by finishing one task request
u_i^t	Utility of cloud user i in t-th time slot
u_i	Total utility of cloud user i over t_i future time slots
$\boldsymbol{u}$	Utility vector of all cloud users

2 Proofs of Theorem 4.2 and 4.3

Proof of Theorem 4.2: As mentioned in the theorem, both of the functions $\mathcal{K}_1(x)$ and $\mathcal{K}_2(x)$ are convex in $x \in \mathcal{X}$. Then we have $\forall x_1, x_2 \in \mathcal{X}$,

$$\mathcal{K}_1(\theta x_1 + (1-\theta) x_2) \leq \theta \mathcal{K}_1(x_1) + (1-\theta)\mathcal{K}_1(x_2),$$

and

$$\mathcal{K}_2(\theta x_1 + (1-\theta) x_2) \leq \theta \mathcal{K}_2(x_1) + (1-\theta)\mathcal{K}_2(x_2),$$

where $0 < \theta < 1$. We further obtain $\forall x_1, x_2 \in \mathcal{X}$,

$$\begin{aligned}
&\mathcal{K}_3(\theta x_1 + (1-\theta) x_2) \\
&\quad = \mathcal{K}_1(\theta x_1 + (1-\theta) x_2) + \mathcal{K}_2(\theta x_1 + (1-\theta) x_2) \\
&\quad \leq \theta(\mathcal{K}_1(x_1) + \mathcal{K}_2(x_1)) + (1-\theta)(\mathcal{K}_1(x_2) + \mathcal{K}_2(x_2)) \\
&\quad = \theta \mathcal{K}_3(x_1) + (1-\theta)\mathcal{K}_3(x_2).
\end{aligned}$$

Thus, we can conclude that $\mathcal{K}_3(x)$ is also convex in $x \in \mathcal{X}$ and the result follows. □

Proof of Theorem 4.3: According to [1], [2], the proof of this theorem follows if the following two conditions are satisfied. (1) For each cloud user i ($i \in \mathcal{N}$), the set $\mathcal{P}_i$ is convex and compact, and each disutility function $f_i(p_i, \boldsymbol{b}_{-i}, \boldsymbol{\lambda}_i^{t_i})$ is continuous in $p_i \in \mathcal{P}_i$. (2) For each fixed tuple $\boldsymbol{b}_{-i}$, the function $f_i(p_i, \boldsymbol{b}_{-i}, \boldsymbol{\lambda}_i^{t_i})$ is convex in p_i over the set $\mathcal{P}_i$.

It is obvious that the statements in the first part hold. We only need to prove the convexity of $f_i(p_i, \boldsymbol{b}_{-i}, \boldsymbol{\lambda}_i^{t_i})$ in p_i for every fixed $\boldsymbol{b}_{-i}$. By Theorem 4.1, we know that if $r_i \geq w_i / (\sigma^2 \mu_i^2)$ ($i \in \mathcal{N}$), then each of the functions $\psi_i^t(p_i, \boldsymbol{b}_{-i}, \lambda_i^t)$ ($t \in \mathcal{T}_i$) is convex in $p_i \in \mathcal{P}_i$. In addition, according to the property presented in Theorem 4.2, it is easy to deduce that

$$f_i(p_i, \boldsymbol{b}_{-i}, \boldsymbol{\lambda}_i^{t_i}) = \sum_{t=1}^{t_i} \psi_i^t(p_i, \boldsymbol{b}_{-i}, \lambda_i^t),$$

is also convex in $p_i \in \mathcal{P}_i$. Thus, the result follows. □

References

[1] H. Chen, Y. Li, R. H. Louie, and B. Vucetic, "Autonomous demand side management based on energy consumption scheduling and instantaneous load billing: An aggregative game approach," *Smart Grid, IEEE Transactions on*, vol. 5, no. 4, pp. 1744–1754, 2014.

[2] G. Scutari, D. Palomar, F. Facchinei, and J.-S. Pang, "Convex optimization, game theory, and variational inequality theory," *Signal Processing Magazine, IEEE*, vol. 27, no. 3, pp. 35–49, May 2010.